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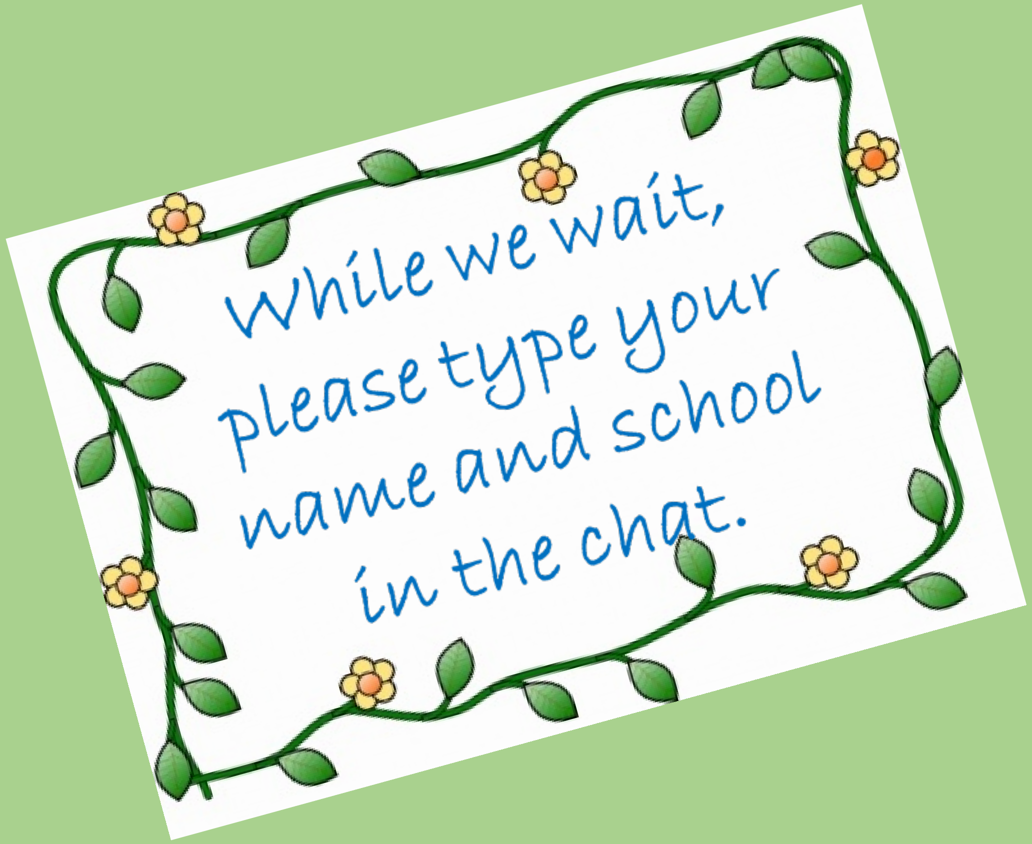
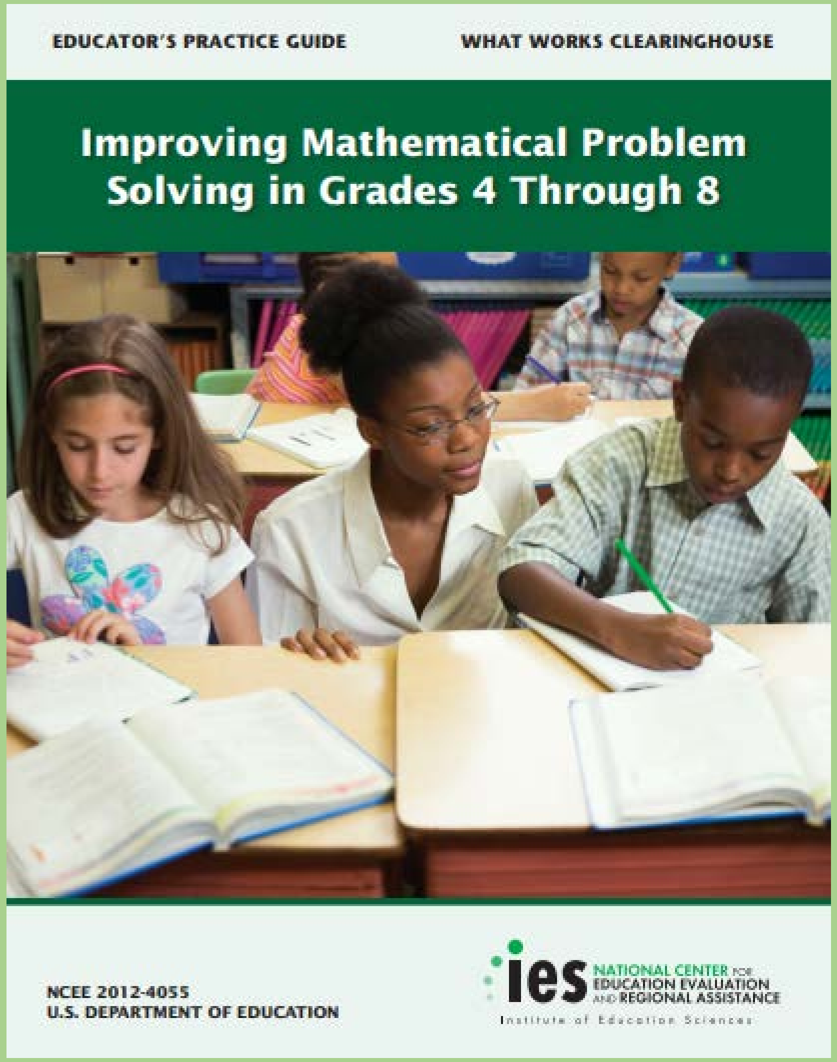




March 9, 2021

Improving Mathematical Problem Solving in Grades 4 - 5

Recommendation 1

Prepare problems and use them in whole-class instruction

1. Include both routine and non-routine problems in problem-solving activities

Routine Problems

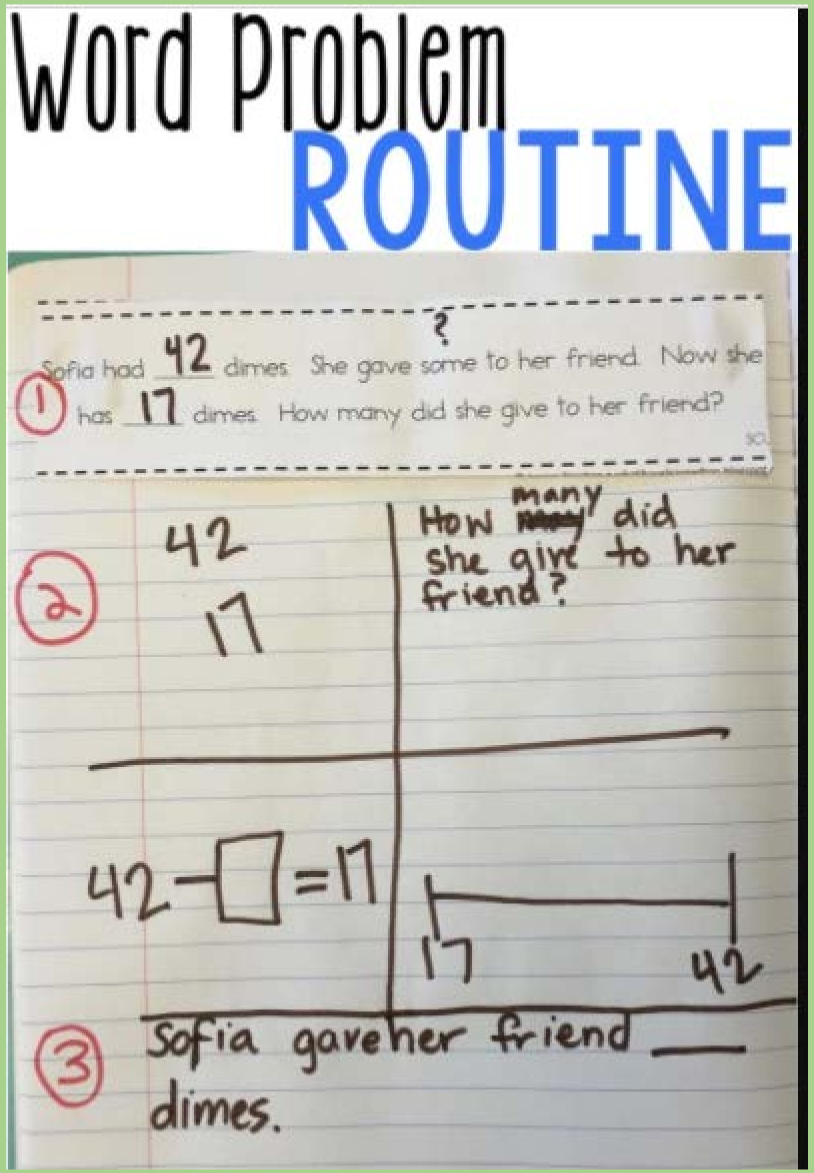
Can be solved using methods familiar to students by replicating previously learned methods in a step-by-step fashion.

Non-routine Problems

Problems for "which there is not a predictable, task instructions, or worked-out example.

Routine Problems

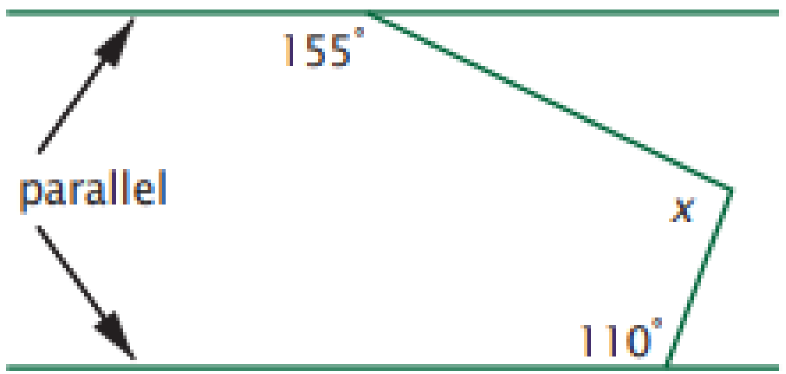
- Use these if goal is to help students understand the meaning of an operation or mathematical idea.
- These can also be cognitively demanding multistep problems that require methods familiar to students.
- Requires little or no transfer from previously modeled or worked problems.



Non-routine Problems

- If primary goal of instruction is to develop students' ability to think strategically, teachers should choose non-routine problems.
- This will force students to apply what they have learned in a new way.

1. Determine angle *x* without measuring. Explain your reasoning.



This problem is likely non-routine for a student who has only studied simple geometry problems involving parallel lines and a transversal.

2. There are 20 people in a room. Everybody high-fives with everybody else. How many high-fives occurred?

This problem is likely non-routine for students in beginning algebra.

2. Ensure that students will understand the problem by addressing issues students might encounter with the problem's context or language.

- Choose problems with familiar contexts.
- Clarify unfamiliar language and contexts
- Reword problems, drawing upon students' experiences.

Example 3. One teacher's efforts to clarify vocabulary and context

Mary, a 5th-grade teacher, identified the following vocabulary, contextual terms, and content for clarification, based on the background of her students.

Example Problem	Vocabulary	Context
In a factory, 54,650 parts were made. When they were tested, 4% were found to be defective. How many parts were working?	Students need to understand the term *defective* as being the opposite of *working* and the symbol % as *percent* to correctly solve the problem.	What is a *factory*? What does *parts* mean in this context?
At a used-car dealership, a car was priced at $7,000. The salesperson then offered a discount of $350. What percent discount, applied to the original price, gives the offered price?	Students need to know what *offered* and *original price* mean to understand the goal of the problem, and they need to know what *discount* and *percent discount* mean to understand what mathematical operators to use.	What is a *used-car dealership*?

Language and contexts need to be clarified in order for student to understand the problem

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3. Consider students' knowledge of mathematical content when planning lessons.

- Consider looking at skills taught in prior units or grade levels.
- Struggling students are likely to benefit from a quick review.
- A brief review of skills learned earlier also help students see how this knowledge applies to challenging problems.

Problem

Two vertices of a triangle are located at (0,4) and (0,10). The area of the triangle is 12 square units. What are all possible positions for the third vertex?

Mathematical language that needs to be reviewed

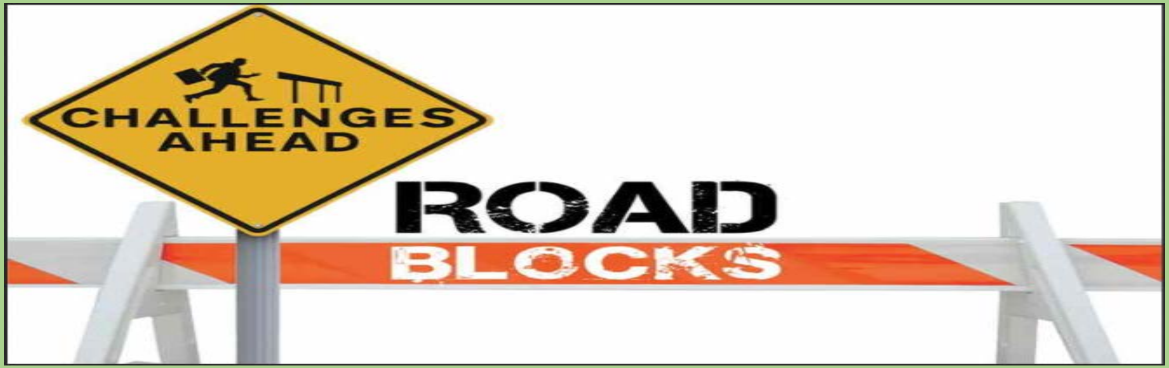
- *vertices*
- *triangle*
- *area square units*
- *vertex*

Potential Roadblocks and Solutions

Teachers are having trouble finding problems for the problem-solving activities.

Teachers have no time to add problem-solving activities to their mathematics instruction.

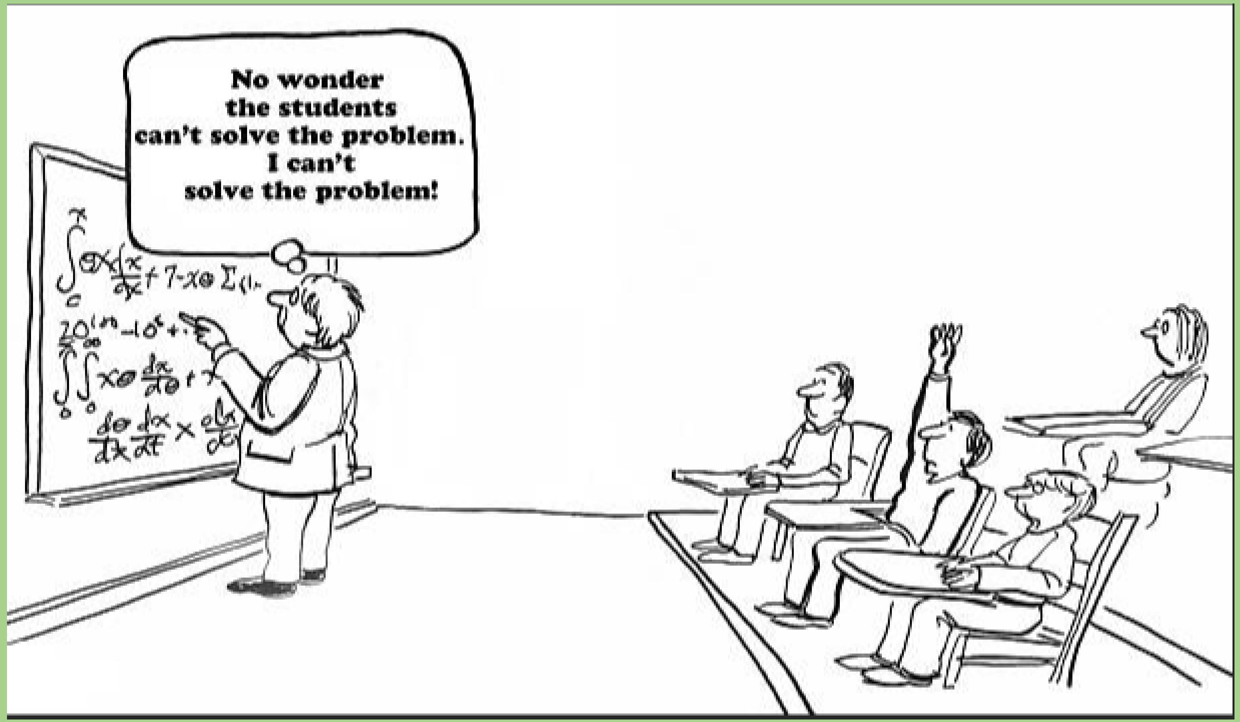
Teachers are not sure which words to teach when teaching problem solving.



Recommendation 2

Assist students in monitoring and reflecting on the problem-solving process.

1. Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.



The prompts that teachers provide can either be questions that students should ask and answer as they solve problems or task lists that help students complete steps in the problem-solving process.

- What is the story in this problem about?
- What is the problem asking?
- What do I know about the problem so far? What information is given to me? How can this help me?
- Which information in the problem is relevant?
- In what way is this problem similar to problems I have previously solved?
- What are the various ways I might approach the problem?
- Is my approach working? If I am stuck, is there another way I can think about solving this problem?
- Does the solution make sense? How can I verify the solution?
- Why did these steps work or not work?
- What would I do differently next time?

Note: These are examples of the kinds of questions that a teacher can use as prompts to help students monitor and reflect during the problem-solving process. Select those that are applicable for your students, or formulate new questions to help guide your students.

- Identify the givens and goals of the problem.
- Identify the problem type.
- Recall similar problems to help solve the current problem.
- Use a visual to represent and solve the problem.
- Solve the problem.
- Check the solution.

2. Model how to monitor and reflect on the problem-solving process.

What is this story about, and what do I need to find out?

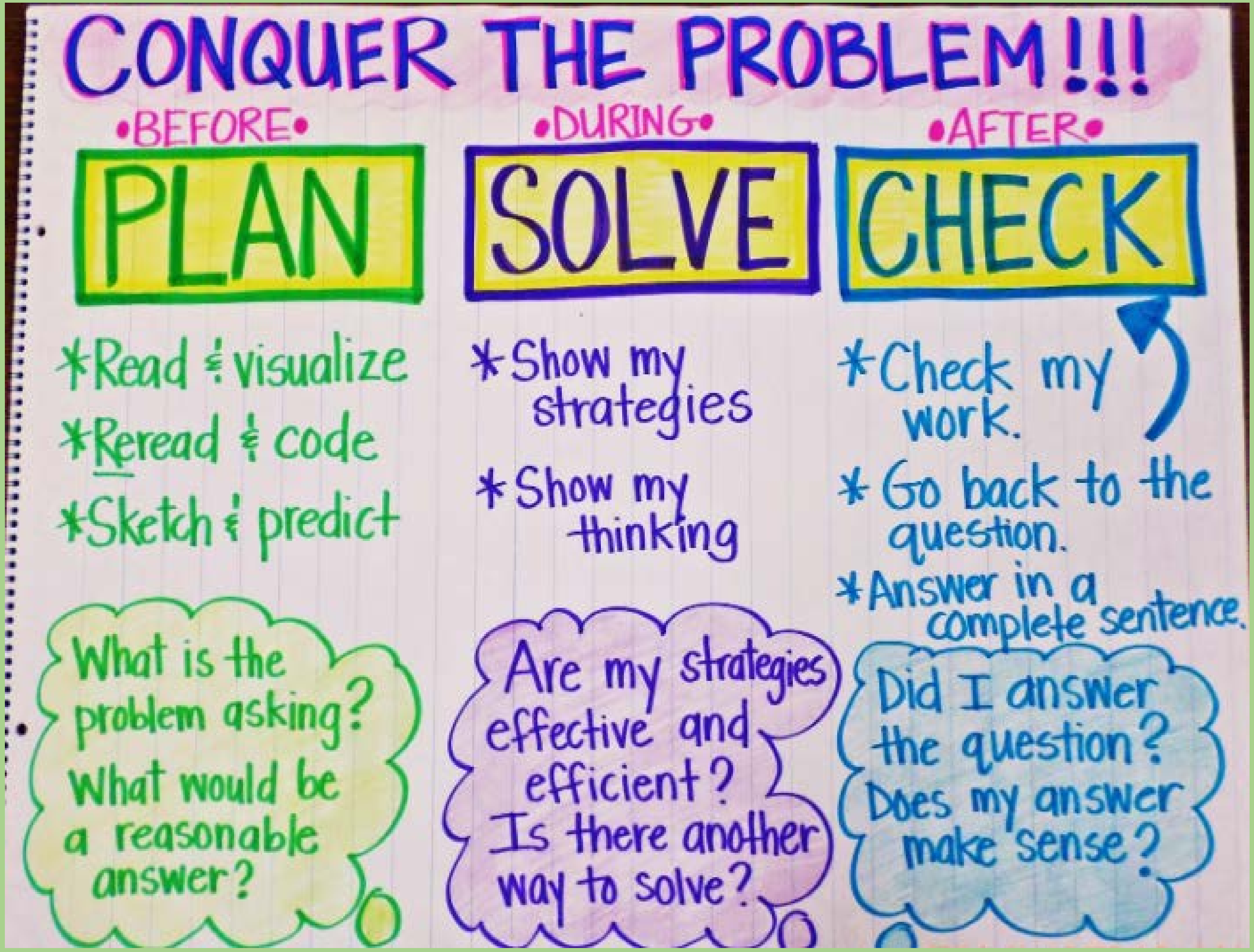
What steps should I take to solve this problem?

Have I ever seen a problem like this before?

Does this answer make sense when I reread the problem?

3. Use student thinking about a problem to develop students' ability to monitor and reflect.

- Build on students' ideas
- Clarify and refine how they monitor and reflect as they solve problems



Recommendation 3

Teach students how to use visual representations

- A major task for any student engaged in problem solving is to translate the quantitative information in a problem into a symbolic equation.

- Visual representations help students solve problems by linking the relationships between quantities in the problem with the mathematical operations needed to solve the problem.

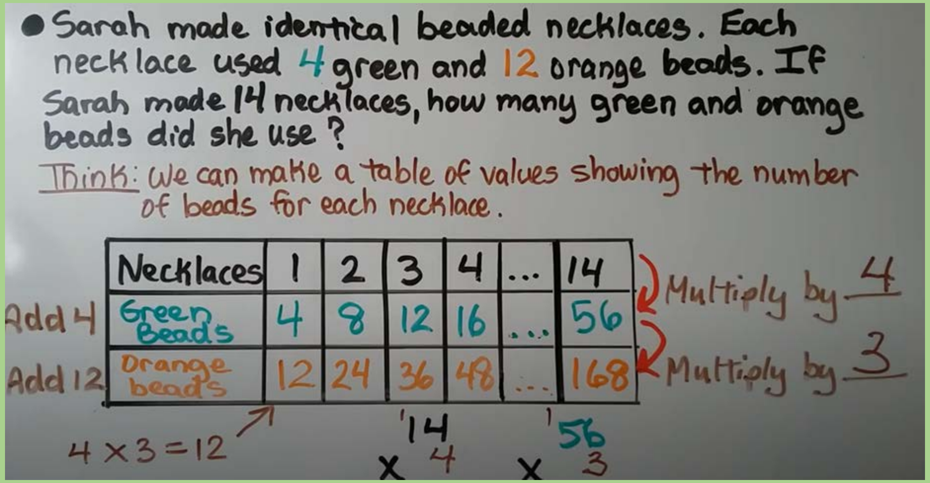
- Students who learn to visually represent the mathematical information in problems prior to writing an equation are more effective at problem solving.

Visual representations include:

Tables

Graphs

Number Lines



George practises the piano for 15 minutes each day. This t-chart shows how many minutes he practises in increasing numbers of days.

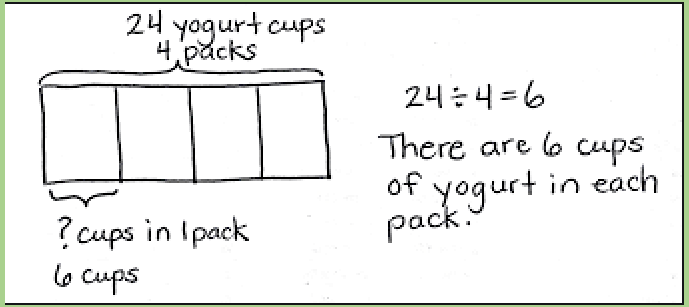
Days	Total number of minutes
1	15
2	30
3	45
4	60
5	75

<u>Diagrams</u>

Strip Diagrams

Percent Bars

Schematic Diagrams



Provide instruction in multiple strategies.

- Select the visual representations that will work best for students and use it consistently for similar problems.
- Keep in mind, students may need time to practice using visual representations and may struggle before achieving success with them.
- If, after a reasonable amount of time and further instruction, the representation still is not working for individual students or the whole class, consider teaching another type of visual representation.

Use think-alouds and discussions to teach students how to represent problems visually.

- Think aloud the decisions being made as you connect the problem to the representation
- Express **<u>why</u>** the decisions are being made
- Explain how you identified the type of problem—such as proportion, ratio, or percent—based on mathematical ideas in the problem and why you think a certain visual representation is most appropriate.
- Think-alouds are more than just the telling students what the teacher is doing.

Promote discussions by asking students guiding questions as they practice representing problems visually.

<u>For example:</u>

- What kind of problem is this? How do you know?
- What is the relevant information in this problem? Why is this information relevant?
- Which visual representation did you use when you solved this type of problem last time?
- What would you do next? Why?

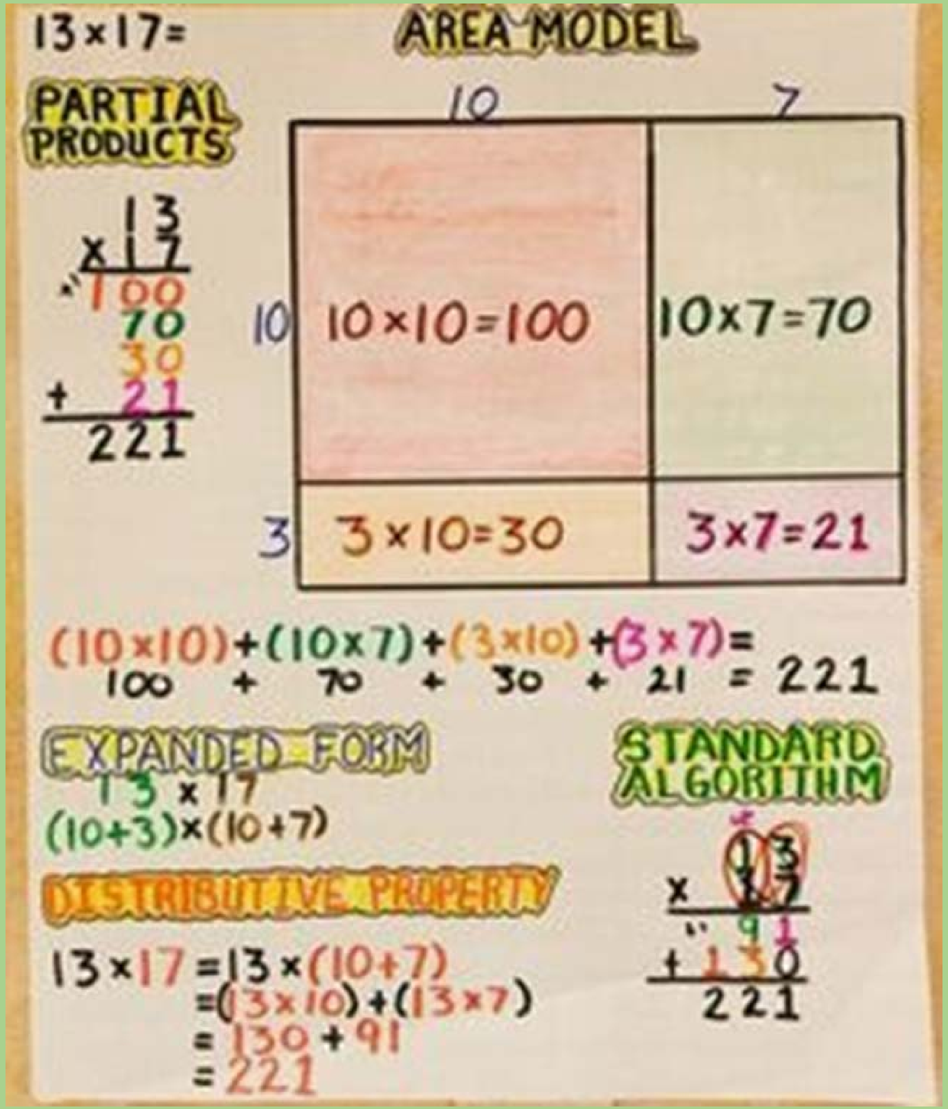
One study showed that if teachers help students design, develop, and improve their own visual representations, student achievement improves more than if students simply use teacher- or textbook-developed visuals.

Visual Representations	Verbal Representations	Contextual Representations	Physical Representations	Symbolic Representations
Illustrate, show, or work with mathematical ideas using diagrams, pictures, number lines, graphs, and other math drawings.	Use language (words and phrases) to interpret, discuss, define, or describe mathematical ideas, bridging informal and formal mathematical language.	Situate mathematical ideas in everyday, real-world, or imaginary situations, using a variety of discrete and continuous measures (e.g., people, meters, yards).	Use concrete objects to show, study, act upon, or manipulate mathematical ideas (e.g., cubes, counters, tiles, paper strips).	Record or work with mathematical ideas using numerals, variables, tables, and other symbols.

Recommendation 4

Expose students to multiple problem-solving strategies

Teach students that problems can be solved in more than one way and that they should learn to choose between strategies.

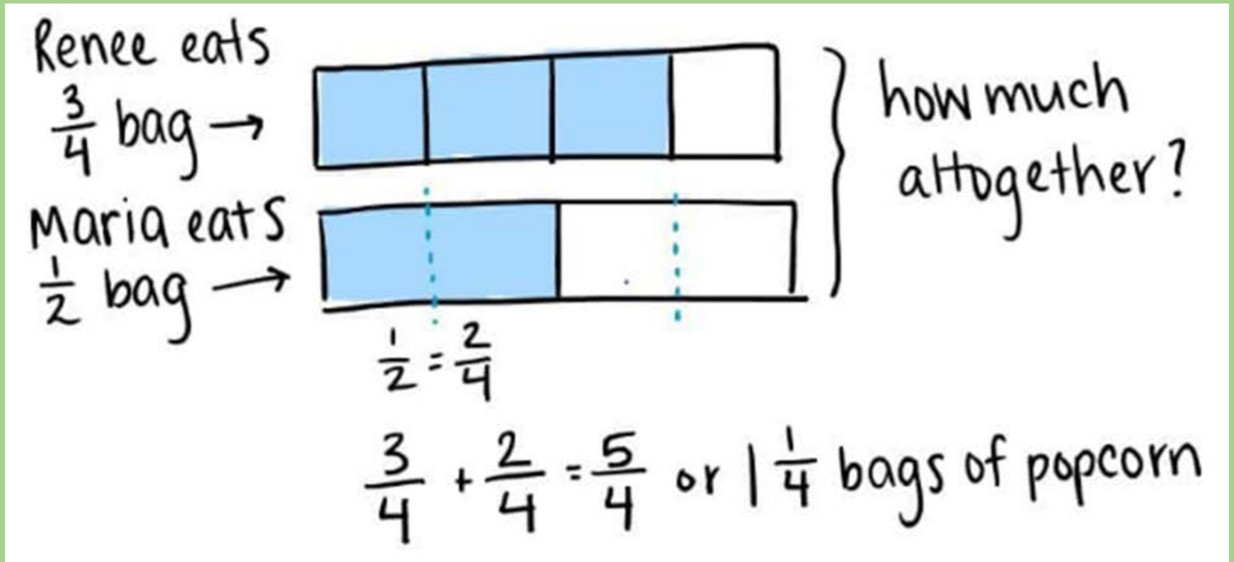


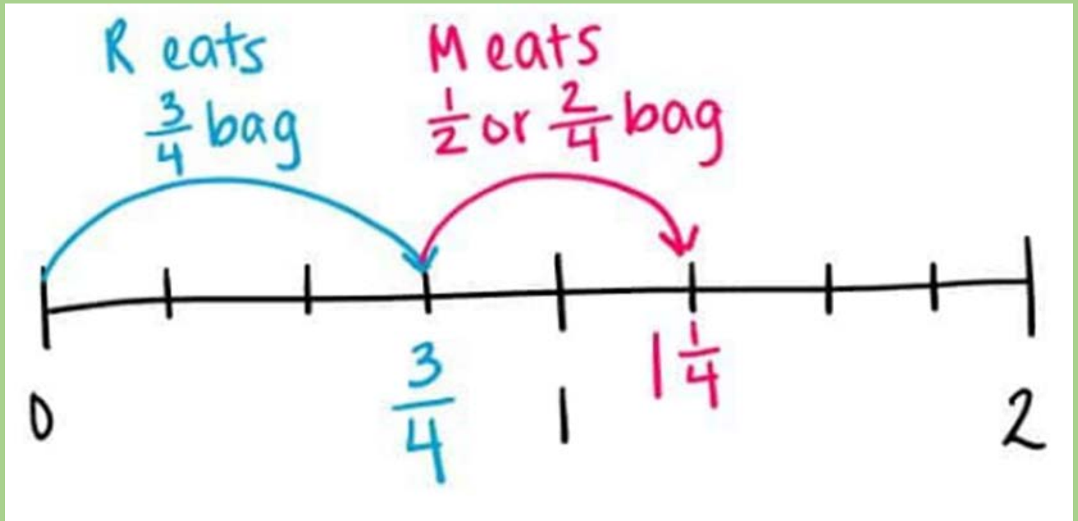
Instruct students in a variety of strategies for solving problems and provide opportunities for students to use, share, and compare the strategies.

Teachers should consider emphasizing the clarity and efficiency of different strategies when they are compared as part of a classroom discussion.

https://www.youtube.com/watch?v=3_Gzof5p_RM

When regularly exposed to problems that require different strategies, students learn different ways to solve problems. As a result, students become more efficient in selecting appropriate ways to solve problems and can approach and solve them with greater ease and flexibility.





Provide opportunities for students to compare multiple strategies in worked examples.

Mandy's solution		Erica's solution	
$5(y+1) = 3(y+1) + 8$		$5(y+1) = 3(y+1) + 8$	
$5y + 5 = 3y + 3 + 8$	Distribute	$2(y+1) = 8$	Subtract on both
$5y + 5 = 3y + 11$	Combine	$y + 1 = 4$	Divide on both
$2y + 5 = 11$	Subtract on both	$y = 3$	Subtract on both
$2y = 6$	Subtract on both		
$y = 3$	Divide on both		

TEACHER: Mandy and Erica solved the problem differently, but they got the same answer. Why? Would you choose to use Mandy's way or Erica's way? Why?

Ask questions such as:

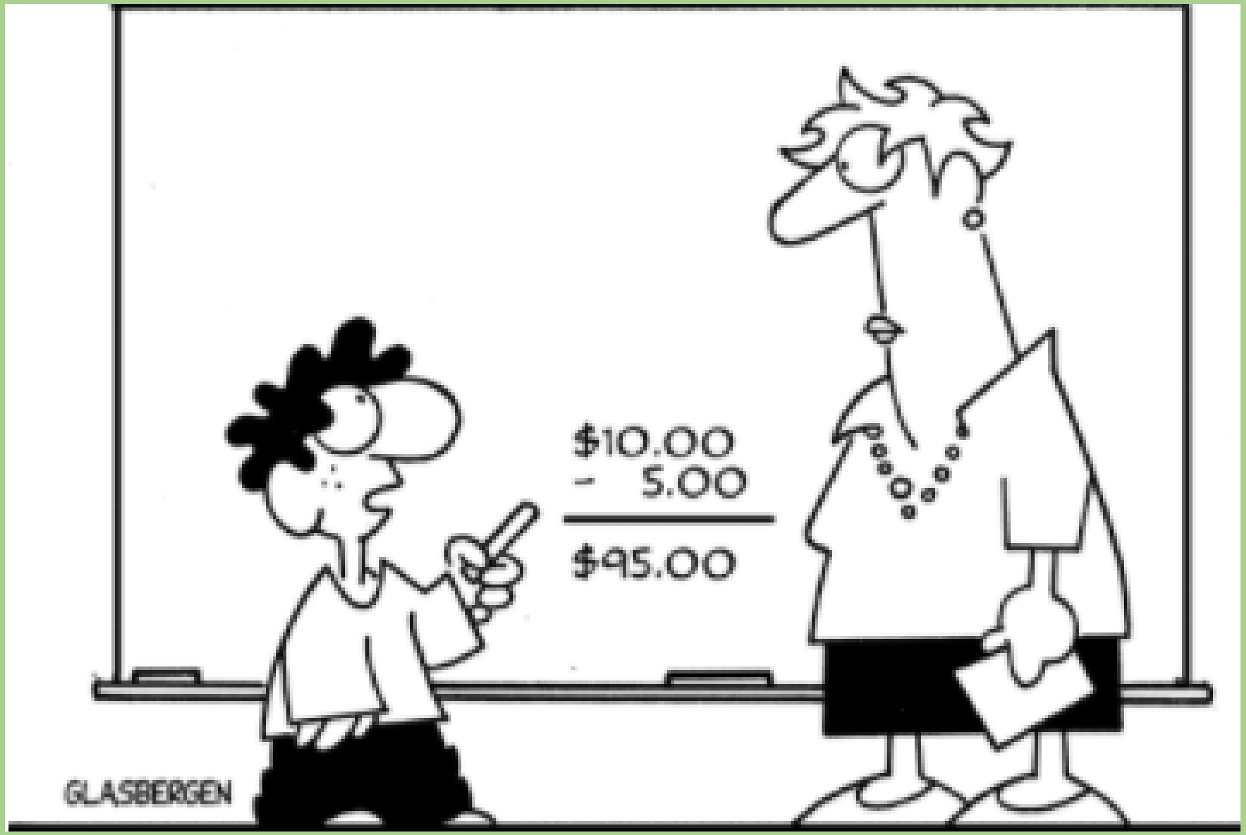
- How are the strategies similar? How are they different?
- Which method would you use to solve the problem? Why would you choose this approach?
- The problem was solved differently, but the answer is the same. How is that possible?

Recommendation 5

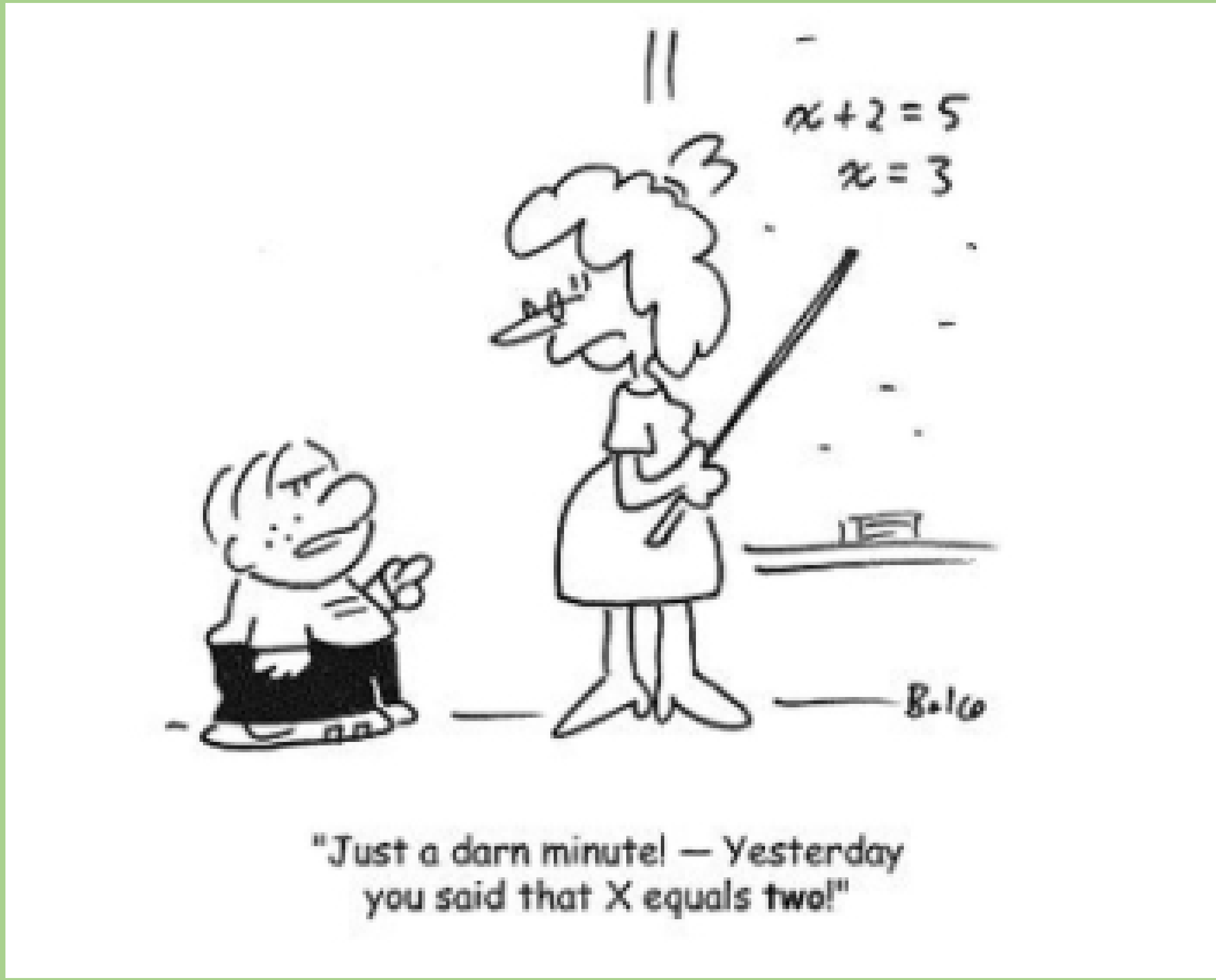
Help students recognize and articulate mathematical concepts and notation.

1. Describe relevant mathematical concepts and notation, and relate them to the problem-solving activity.

- Turn problem-solving activities into learning opportunities by connecting students' intuitive understanding to formal mathematical concepts and notation



2. Ask students to explain each step used to solve a problem in a worked example.



How mathematically valid are they?

An explanation that is not mathematically valid

STUDENT: To find an equivalent fraction, whatever we do to the top of ⅔, we must do to the bottom.

This description is not mathematically valid because, using this rule, we might think we could add the same number to the numerator and denominator of a fraction and obtain an equivalent fraction. However, that is not true. For example, if we add 1 to both the numerator and denominator of ⅔, we get (2 + 1)/(3 + 1), which is ¾. ¾ and ⅔ are not equivalent. Below is an explanation of how teacher questioning can clarify students' explanations and reasoning.

A correct description, but still not a complete explanation

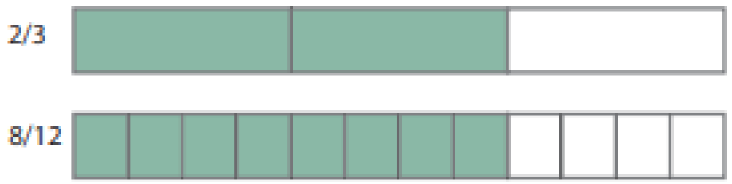
STUDENT: Whatever we multiply the top of ⅔ by, we must also multiply the bottom by.

This rule is correct, but it doesn't explain why we get an equivalent fraction this way.

A mathematically valid explanation

STUDENT: You can get an equivalent fraction by multiplying the numerator and denominator of ⅔ by the same number. If we multiply the numerator and denominator by 4, we get 8⁄12.

If I divide each of the third pieces in the first fraction strip into 4 equal parts, then that makes 4 times as many parts that are shaded and 4 times as many parts in all. The 2 shaded parts become $2 \times 4 = 8$ smaller parts and the 3 total parts become $3 \times 4 = 12$ total smaller parts. So the shaded amount is ⅔ of the strip, but it is also 8⁄12 of the strip.



This explanation is correct, complete, and logical.

3. Help students make sense of algebraic notation.

- Introduce similar arithmetic problems before algebraic problems to revisit students' earlier mathematical understanding.

- Help students explain how the algebraic notation represents each component in the problem.

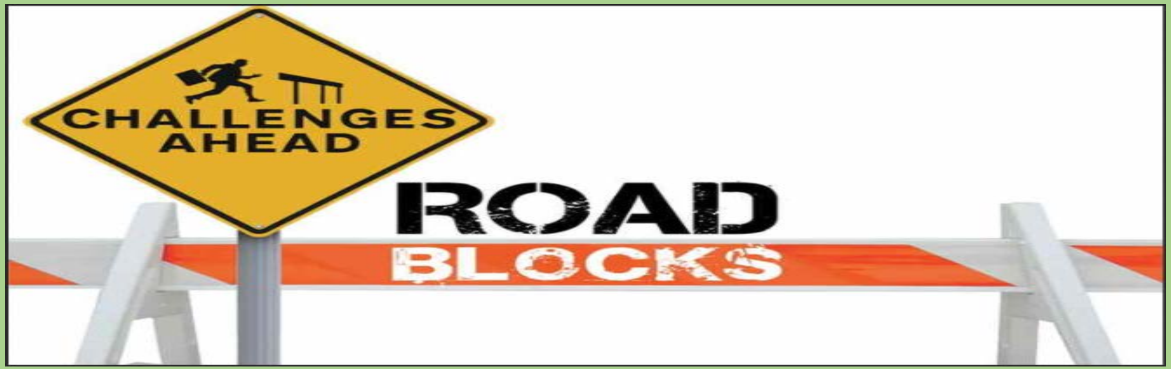
Potential Roadblocks and Solutions

Roadblock: Students' explanations are too short and lack clarity and detail. It is difficult for teachers to identify which mathematical concepts they are using.

Solution: Teacher can ask "how it was solved" and "what they thought about that problem".
Solution: Create "reason sheet" of mathematical rules.

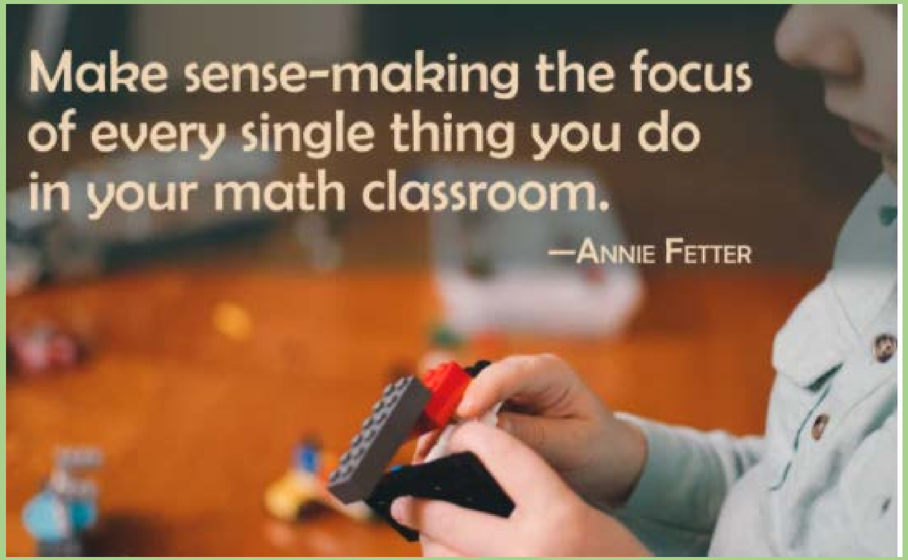
Roadblock: Students may be confused by mathematical notations used in algebraic equations.

Solution: Encourage students to use arbitrary variables, such as (x and y). Arbitrary variables can facilitate student understanding of the abstract role that variables play in representing quantities.



Engage Student Reasoning

Instead of dismissing the context of word problems, teachers should take time with students to make sense of word problems and their supporting context.



https://www.cde.state.co.us/comath/word-problems-guide

Tip: Provide students with a list of prompts to help them monitor and reflect during the problem-solving process.

- **Provide** a list of prompts. For example:
  1. *Task lists* that help students complete steps in the problem-solving process
  2. *Questions* that students should ask themselves and answer as they solve problems
- **Select** a reasonable number of prompts (tasks or questions) rather than an exhaustive list, as too many prompts may slow down the problem-solving process.
- **List prompts** on the board or on index cards and include them on worksheets so students can easily access them.
- **Use prompts** that help students evaluate their work at each stage of the problem-solving process.
- **Encourage** students to explain and justify their response to each prompt either orally or in writing.

Sample Task List

Subject: Math

- Identify the information provided and goals of the problem.
- Identify the problem type.
- Recall similar problems to help solve the current problem.
- Use a visual to represent and solve the problem.
- Solve the problem.
- Check the solution.

Sample Questions

Subject: Math

- What is the problem asking?
- What do I know about the problem so far? What information is given to me? How can this help me?
- Which information in the problem is relevant?
- In what way is this problem similar to problems I have previously solved?
- What are the various ways I might approach the problem?
- Is my approach working? If I am stuck, is there another way I can think about solving this problem?
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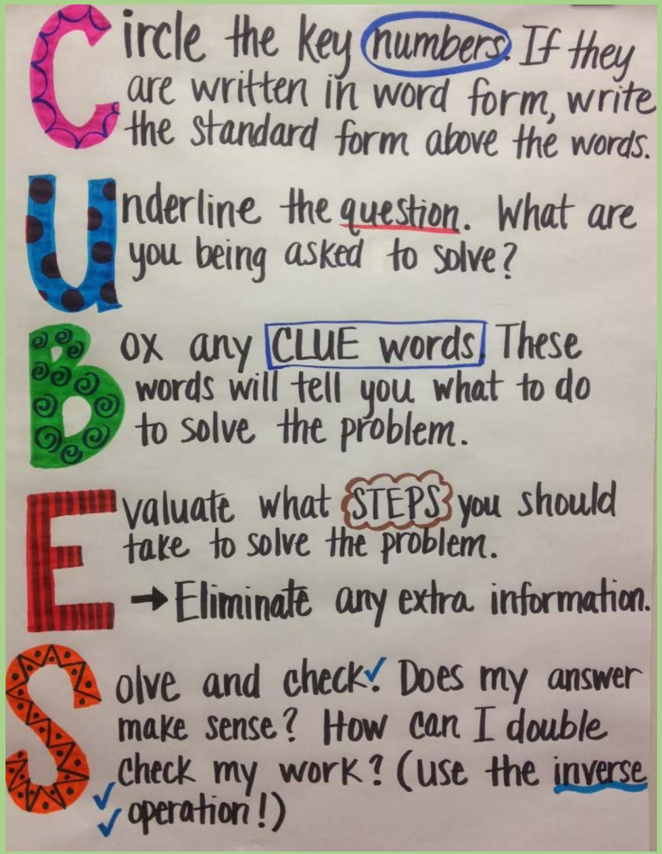
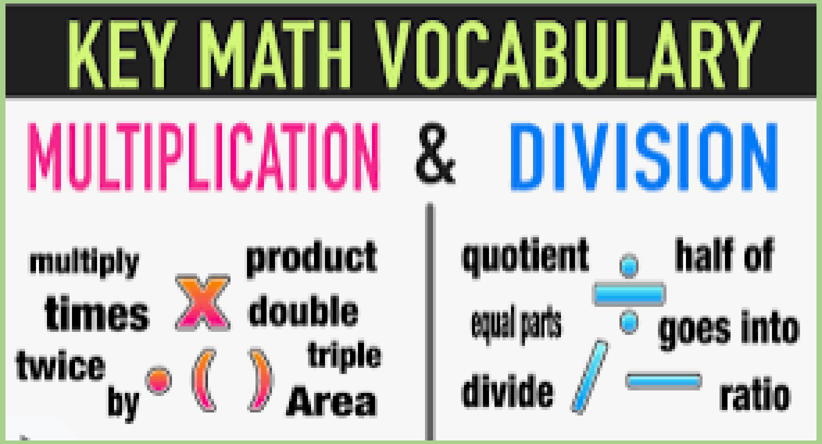
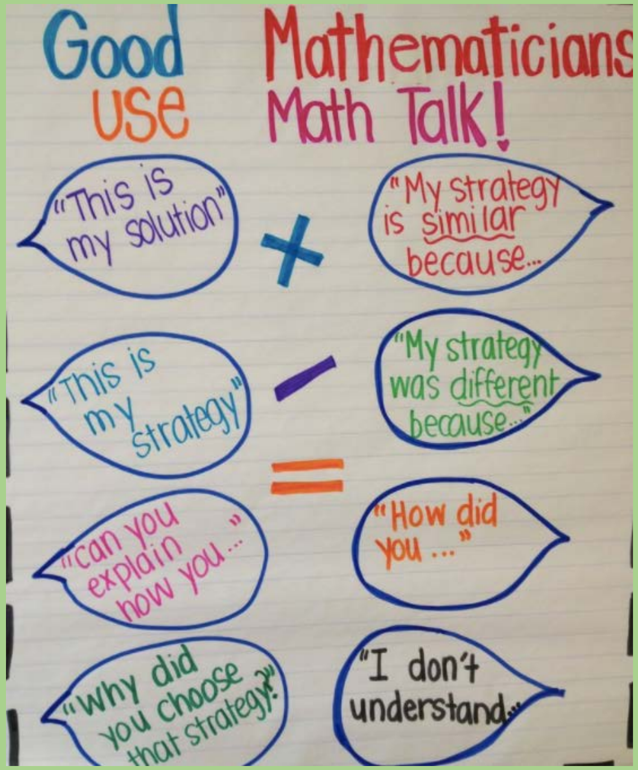
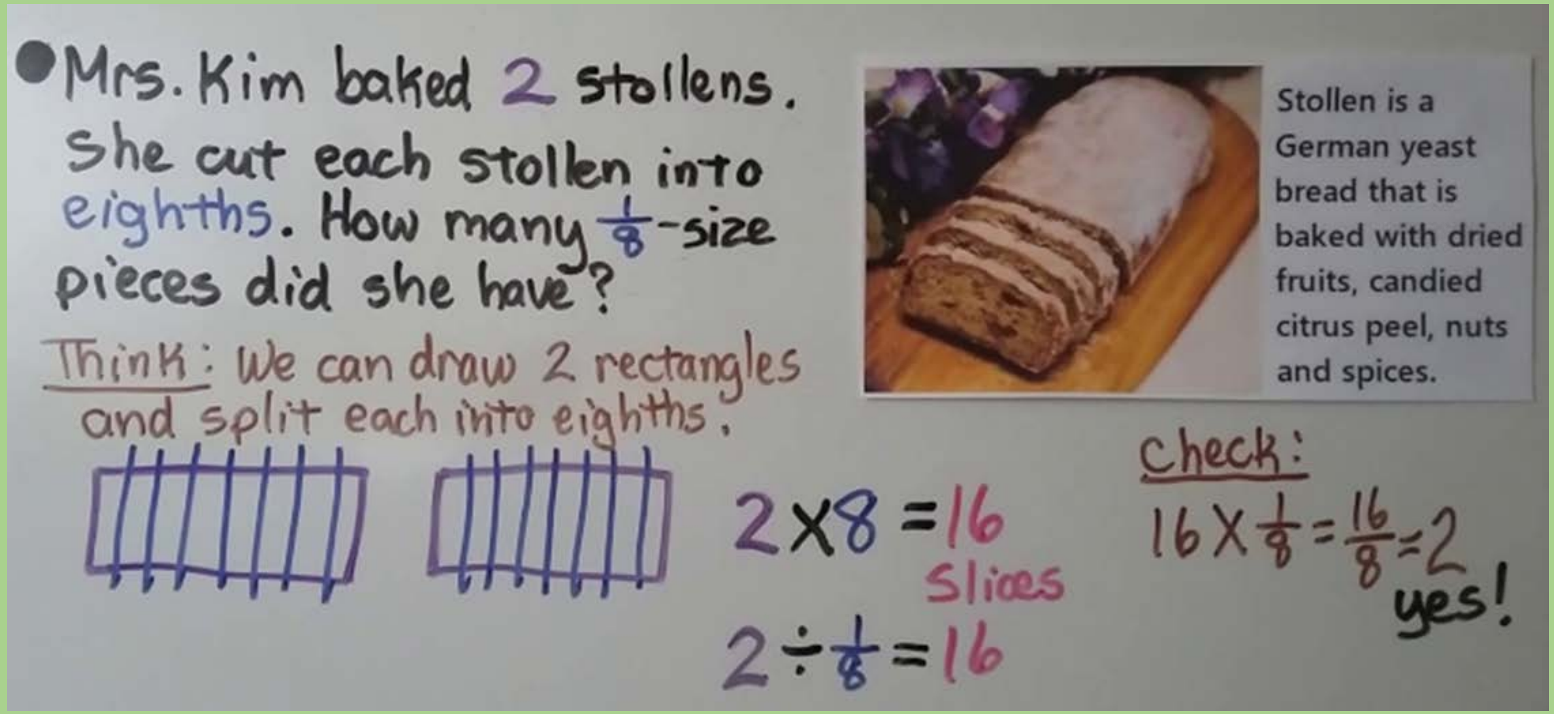
- Why did these steps work or not work?
- What would I do differently next time?




KEY MATH VOCABULARY
MULTIPLICATION & DIVISION
multiply
times
twice
by
product
double
triple
Area
quotient
equal parts
divide
half of
goes into
ratio


Good Mathematicians use Math Talk!
"This is my solution"
"My strategy is similar because..."
"This is my strategy"
"My strategy was different because..."
"Can you explain how you..."
"How did you ..."
"Why did you choose that strategy?"
"I don't understand..."


Mrs. Kim baked 2 stollens. She cut each stollen into eighths. How many 1/8-size pieces did she have?
Stollen is a German yeast bread that is baked with dried fruits, candied citrus peel, nuts and spices.
Think: We can draw 2 rectangles and split each into eighths.
2 X 8 = 16 slices
2 ÷ 1/8 = 16
Check:
16 X 1/8 = 16/8 = 2 yes!

Thank you!