SFDR 7th Grade General Math

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CHAPTER **1**

Relationships with Real Number System

Chapter Outline

7.2a: Extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers

1.1 Rational Number Relationships (7.2a and 8.2a)

Real Numbers

All numbers belong to the set of numbers known as the **real number system**. The real number system consists of every number you have ever dealt with since you were old enough to count. The numbers in the real number system are divided into two main groups. One group is called the **rational numbers** and the other is called the **irrational numbers**. The set of irrational numbers consists of all numbers that are not rational. This set of **irrational numbers** includes those numbers that cannot be written as the ratio of two integers, decimal numbers that are non-terminating and decimals that do not have a repeating pattern of digits. For example, $pi(\pi)$, $\sqrt{2}$, $-2.345876921\ldots$ are irrational numbers.

The set of **rational numbers** includes natural numbers, whole numbers, integers, numbers that can be expressed as the ratio of two integers, decimal number that terminate and decimal numbers that have a repeating pattern of digits. The **natural numbers** are the set of positive integers. For example, 1, 2, 3... are all natural numbers. The **whole numbers** are the natural numbers and zero. For example, 0, 1, 2, 3... are all whole numbers. The **integers** are the whole numbers and their opposites. For example, -2, -1, 0, 1, 2... are all integers. A **rational number** is any number that can be expressed in the form $\frac{a}{b}$ where $b \neq 0$. When a rational number is expressed as a decimal then the decimal will terminate (end) or it will have a repeating pattern of digits. For example, $\frac{1}{2}$, $\sqrt{81}$, $-7.456\,545\,654$ are all rational numbers.

When you classify numbers remember that they can often belong to more than one set of numbers. If you think of the number 3, you can call it a natural number, a whole number, an integer and a rational number.

The following table may help you to better understand the real number system.

REAL NUMBER SYSTEM

RATIONAL NUMBERS

- Can be expressed as the ratio of two integers

  $\dfrac{a}{b}$ where $b \neq 0$

- Decimal numbers that are terminating
- Decimals that are non-terminating and have a repeating pattern of digits

Examples:

$\dfrac{2}{3}, -\dfrac{6}{1}, 1.75, -1.56, 1.121\,212\,121$

IRRATIONAL NUMBERS

- Cannot be expressed as the ratio of two integers
- Decimal numbers that are non-terminating
- Decimal numbers that do not have a repeating pattern of digits

Examples:

$\pi, \sqrt{2}, 0.461\,538\,465\,821, \dfrac{\pi}{2}$

INTEGERS

Whole numbers and their opposites

Examples: $\ldots, -2, -1, 0, 1, 2, \ldots$

WHOLE NUMBERS

Positive integers and zero

Examples: $0, 1, 2, 3, 4\ldots$

NATURAL NUMBERS

Positive integers

Examples: $1, 2, 3, 4, 5, \ldots$

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Faith and her circular flower bed. She wants to compare the new circular flower bed to the original square one using exact measurements. First, Faith should compare the perimeter of the square to the circumference of the circle. Then, she should compare the area of the square to the area of the circle.

First, determine the diameter of the circle which will be the side length of the square.

The diameter of a circle is two times the length of the radius. The radius of the circle is 3 feet.

$$
\begin{aligned}
d &= 2r \\
d &= 2(3) \\
d &= 6
\end{aligned}
$$

The answer is 6.

The diameter of the circle is 6 feet. Each side of the square is also 6 feet in length.

The answer of 6 is a natural number, a whole number, an integer and a rational number.

Next, determine the perimeter of the square. The perimeter of the square is the distance around its outer edges and can be found by adding the lengths of each side or by simply multiplying the side length by four.

$$
\begin{aligned}
P_{\text{square}} &= 4s \\
P_{\text{square}} &= 4(6) \\
P_{\text{square}} &= 24
\end{aligned}
$$

The answer is 24.

The perimeter of the square is 24 feet.

The answer of 24 is a natural number, a whole number, an integer and a rational number.

Next, determine the circumference of the circle. The perimeter of a circle is known as its circumference and is the distance around the outer edge of the circle. The perimeter of a circle can be found by multiplying the diameter by π.

$$
\begin{aligned}
C &= \pi d \\
C &= (3.141592654)(6) \\
C &= 18.84955592
\end{aligned}
$$

Using the value for π from the calculator (3.141592654) gives the answer 18.84955592.

The circumference of the circle is 18.84955592 feet.

The answer of 18.84955592 is an irrational number. It is a non-terminating, non-repeating decimal.

Next, determine the area of the square. The area of the square can be found using the formula:

$A = s^2$ where 's' is the side length of the square.

$$
\begin{aligned}
A &= s^2 \\
A &= (6)^2 \\
A &= 36
\end{aligned}
$$

The answer is 36.

The area of the square flower bed is 36 ft^2.

The answer of 36 is a natural number, a whole number, an integer and a rational number.

Next, determine the area of the circle. The area of the circle can be found using the formula:

$A = \pi r^2$ where 'r' is the radius of the circle.

$$
\begin{aligned}
A &= \pi r^2 \\
A &= (3.141592654)(3)^2 \\
A &= (3.141592654)(9) \\
A &= 28.27433388
\end{aligned}
$$

The answer is 28.27433388.

The area of the circular flower bed is 28.27433388 ft^2.

Using the value for π from the calculator (3.141592654) gives the answer 28.27433388.

The answer of 28.27433388 is an irrational number. It is a non-terminating, non-repeating decimal.

Next, subtract the area of the circular flower bed from the area of the square flower bed. Round the area of the circular flower bed to the nearest tenth and then perform the subtraction.

The difference in the area of the two flower beds is 36.0 ft^2 − 28.3 ft^2 = 7.7 ft^2

The answer of 7.7 is a rational number since it is a terminating decimal.

Example 2

Let's look at classifying some numbers.

For each of the following numbers displayed in the table, indicate the number set or sets to which they belong.

TABLE 1.1: Classifying Numbers Table

Number	Natural	Whole	Integer	Rational	Irrational
$\frac{22}{7}$				⋆	
1.141 141 114…					⋆
$\frac{17}{4}$				⋆	
-6			⋆	⋆	
$\frac{\pi}{2}$					⋆
0		⋆	⋆	⋆	
9	⋆	⋆	⋆	⋆	

$\frac{22}{7}$ is a rational number because it is written as the ratio of two integers.

1.141 141 114… is an irrational number because it is a non-terminating decimal.

$\frac{17}{4}$ is a rational number because it is written as the ratio of two integers.

-6 is an integer because it is the opposite of the whole number 6. It is a rational number because it can be expressed as the ratio of two integers as $-\frac{6}{1}$.

$\frac{\pi}{2}$ is an irrational number because pi is a non-terminating decimal and is an irrational number.

0 is a whole number, an integer and a rational number. Zero is included in each of these number sets.

9 is a natural number, a whole number, an integer and a rational number. It is included in each of these sets and can be expressed as the ratio of two integers as $\frac{9}{1}$.

The following table may help you to understand the real number system.

Example 3

For each of the following statements state if they are sometimes true, always true, or never true.

1. Whole numbers are integers.
2. Integers are whole numbers.
3. If a number is an integer, then it is also a rational number.
4. The number 3π is a rational number.

5. A number can be both rational and irrational
6. If a number is rational, then it is also an integer.

First, read each statement and review what numbers are included in each of the number sets.

Next, decide your answer based on what you have reviewed.

Then, justify your answer.

a) The numbers that belong to the set of integers are the whole numbers and their opposites.

Therefore it is always true that the whole numbers are integers.

b) Integers are only whole numbers if the numbers are positive whole numbers or zero. Integers that are negative do not belong to the set of whole numbers.

Therefore it is sometimes true that integers are whole numbers.

c) The set of rational numbers includes any number that can be expressed as the ratio of two integers.

Therefore it is always true that if a number is an integer, then it is also a rational number.

d) The value of π expressed as a decimal is a non-terminating decimal with no repeated pattern of digits. π belongs to the set of irrational numbers.

Therefore it is never true that 3π is a rational number.

e) A number that belongs to the set of rational numbers cannot belong to the set of irrational numbers. Any number that cannot be written as a rational number is an irrational number.

Therefore it is never true that a number can be both rational and irrational.

f) If a number is rational, then it is also a natural number or a whole number or an integer.

Therefore, it is sometimes true that if a number is rational, then it is an integer.

Example 4

For each of the following numbers displayed in the table, indicate the number set or sets to which they belong. Justify your answer.

First, review what numbers are included in each of the number sets.

Next, decide your answer based on what you have reviewed.

Then, justify your answer.

Remember a number can belong to one or more than one number set.

TABLE 1.2: Classifying Numbers Table

Number	Natural	Whole	Integer	Rational	Irrational	Justification
6.18						Terminating decimal
0.89563...					⋆	Non-terminating decimal
12	⋆	⋆	⋆	⋆		Can be expressed as $+12, 12, \frac{12}{1}$

TABLE 1.2: (continued)

-7			$\star$	$\star$		Can be expressed as $-7, -\frac{7}{1}$
$-\frac{5}{9}$				$\star$		fraction
$\sqrt{36}$	$\star$	$\star$	$\star$	$\star$		Value is 6
$-\sqrt{5.5}$					$\star$	Non-repeating, non-terminating decimal
$\frac{1}{3}$				$\star$		fraction

PRACTICE

Classify each of the following numbers as real, whole, integer, rational or irrational. Some numbers will have more than one classification.

1. 3.45

2. -9

3. 1270

4. 1.232323

5. $\frac{4}{5}$

6. -232 323

7. -98

8. 1.98

9. $\sqrt{16}$

10. $\sqrt{2}$

State whether the following statements are true or false.

11. An irrational number can also be a real number.

12. An irrational number is a real number and an integer.

13. A whole number is also an integer.

14. A decimal is considered a real number and a rational number.

15. A negative decimal can still be considered an integer.

16. An irrational number is a terminating decimal.

17. A radical is always an irrational number.

18. Negative whole numbers are integers and are also rational numbers.

19. Pi is an example of an irrational number.

20. A repeating decimal is also a rational number.

Faith has been planting a square flower bed for many years and has decided that the bedding plants are getting too expensive to fill such an area. This year she is going to plant a circular flower bed within the original square area as shown below:
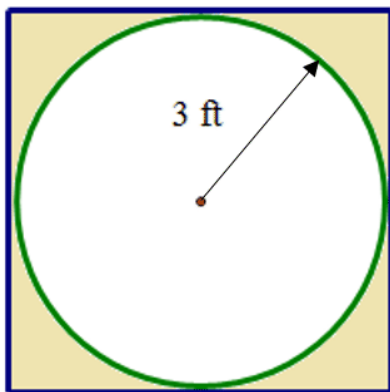
As Faith reviews her plan, she has decided to find some exact measurements so she can figure out just how much smaller the new circular flower bed will be compared to the original square flower bed. However, Faith is unsure of what measurements she has to figure out - "I hope I at least come out with some rational numbers!"

In this concept, you will learn to classify real numbers.

Vocabulary:

- Natural Numbers
- Counting Numbers
- Whole Numbers
- Integers
- Rational Numbers
- Irrational Numbers

CHAPTER **2**

Rational Number Operations

Chapter Outline

7.3a: Add, subtract, multiply, and divide rational numbers fluently

7.3b: apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers

2.1 Rational Numbers (7.3a and 7.3b)

Boxes of Erasers

Kelly and Mallory are comparing the number of boxes of erasers that each one of them has sold. Kelly works in the store one day and Mallory works in the store the next day. They decide to write a riddle about their sales and give it to Trevor to figure out. When Trevor comes in the store on the day after Mallory has worked he finds this riddle.

Mallory sold 4 more boxes of erasers than the three-fourths that Kelly sold. If Mallory sold 13 boxes, how many boxes did Kelly sell?

Trevor is puzzled. He knows that there has to be a variable because the number of boxes that Kelly sold is unknown. He also knows that he needs to write an equation and that this equation will have a rational number, three-fourths in it.

He starts to work.

Using rational numbers is a skill that you will need as you approach higher levels of mathematics. To solve this problem, you will need to understand rational numbers. Let's begin with this lesson and at the end, you can help Trevor with this problem.

What You Will Learn

In this lesson you will learn how to complete the following skills.

- Find equivalent forms of rational numbers given in assorted forms.
- Compare and order rational numbers on a number line and with inequality symbols.

- Apply properties and use order of operations to evaluate numerical and variable expressions involving rational numbers.
- Model and solve real-world problems using simple equations involving rational numbers.

Teaching Time

I. **Find Equivalent Forms of Rational Numbers Given in Assorted Forms**

You have learned about different kinds of numbers. You have learned about decimals, fractions, integers and whole numbers, now we are going to investigate rational numbers.

What is a rational number?

A *rational number* **is a number that can be written in fraction form.**

That is a good question. It means that the number can be written as a fraction, not that it necessarily is a fraction right away.

We can think of fractions first. A fraction is a rational number because it is written in fraction form. Rational numbers can also be positive or negative. Think about if you had lost one-half. Then the number would be negative.

$$\frac{-1}{2} \ and \ \frac{1}{2}$$

Both of these are rational numbers.

What other numbers can be written in fraction form and are therefore rational numbers?

Well, you can think of integers. Remember that an integer is the set of whole numbers and their opposites. We can write any integer as a fraction over 1. This makes all integers rational numbers.

$$-4 = \frac{-4}{1}$$
$$13 = \frac{13}{1}$$

These integers are rational numbers too.

What about decimals?

A decimal is related to a fraction. Most decimals can be written as fractions. When you understand how to convert a decimal to a fraction and a fraction to a decimal, you will be able to determine whether or not the decimal is a rational number. Let's look at an example.

Example

Is .34 a rational number?

If you look at this example, what is the value of the decimal? It is 34 hundredths. We can write this decimal as a fraction with a denominator of 100.

$.34 = \frac{34}{100}$

This is a rational number. Regular decimals are also rational numbers.

Example

Is .3434343434 a rational number?

This is a tricky one because we have a *repeating decimal*. However, because it has an end to it, we can find its fraction equivalent. All repeating decimals are also rational numbers.

Example

Is .35678921 a rational number?

This is not a repeating decimal, but it is a terminating decimal. A *terminating decimal* has an end to it. As long as it has an end, it is a rational number.

Which decimals are not rational numbers?

Decimals that do not have an end are not rational numbers. We can think of an irrational number in this way. An example of an irrational number is pi. We say pi is equal to 3.14, but really it goes on and on and on.

Pi or 3.14... . These values are not rational numbers.

Examples of rational numbers

1. Fractions – both positive and negative

2. Decimals- positive, negative, terminating or repeating too.

3. Integers

Take a few minutes and write these examples in your notebook.

We can also convert rational numbers into different forms.

Example

What decimal is equivalent to the fraction $\frac{7}{8}$?

Divide to find the equivalent decimal.

$7 \div 8 = 0.875$

The decimal 0.875 is equivalent to $\frac{7}{8}$.

To convert from a decimal to a fraction, place the numbers in the decimal over the appropriate place value. For example, to convert 0.875 back to a fraction, count the decimal places. There are three decimal places, so there will be three zeros in the denominator.

$0.875 = \frac{875}{1000}$

Now simplify the fraction to lowest terms.

$\frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$

Example

What fraction is equivalent to the decimal 0.3125?

Place the numbers in the decimal over the appropriate place value. Count the decimal places. There are four decimal places, so there will be four zeros in the denominator.

$0.3125 = \frac{3125}{10000}$

Now simplify the fraction to lowest terms.

$\frac{3125}{10000} = \frac{125}{400} = \frac{5}{16}$

The fraction $\frac{5}{16}$ is equivalent to 0.3125.

That is a great question. Because a percent also represents a part of a whole, percents can also be rational numbers. We can convert a percent to a decimal and to a fraction too. Remember that a *percent* means out of 100.

Example

What is 30% as a fraction? As a decimal?

We can start by knowing that 30% means 30 out of 100. Now we can write it as a fraction and as a decimal.

$30\% = \frac{30}{100} = .30$

Percents that have been converted to decimals and fractions can also be considered rational numbers.

II. <u>**Compare and Order Rational Numbers on a Number Line and with Inequality Symbols**</u>

To compare and order rational numbers, you should first convert each number to the same form so that they are easier to compare. Usually it will be easier to convert each number to a decimal. Then you can use a number line to help you order the numbers.

Example

Place the following number on a number line in their approximate locations: $8\%, \frac{1}{8}, 0.8$

Convert each number to a decimal.

$$8\% = 0.08$$
$$\frac{1}{8} = 1 \div 8 = 0.125$$
$$0.8$$

All of the numbers are between 0 and 1. You can use place value to find the correct order of the numbers. Since 0.08 has a 0 in the tenths place, 8% is the least number. Since 0.125 has a 1 in the tenths place, $\frac{1}{8}$ is the next greatest number. Since 0.8 has an 8 in the tenths place, it is the greatest number.

We wrote these three values on a number line. This is one way to show the different values. We can also use inequality symbols.

Inequality symbols **are <less than, >greater than, $\leq$ less than or equal to, and $\geq$ greater than or equal to.**

Example

Which inequality symbol correctly compares 0.29% to 0.029?

Change the percent to a decimal. Then use place value to compare the numbers.

Move the decimal point two places to the left.

$0.29\% = 0.0029$

Now compare the place value of each number. Both numbers have a 0 in the tenths place. 0.029 has a 2 in the hundredths place, while 0.0029 has a 0 in the hundredths place. So 0.0029 is less than 0.029.

$0.29\% < 0.029$

Remember, the key to comparing and ordering rational numbers is to be sure that they are all in the same form. You want to have all fractions, all decimals or all percentages so that your comparisons are accurate. You may need to convert before you compare!!

III. **Apply Properties and Use Order of Operations to Evaluate Numerical and Variable Expressions Involving Rational Numbers**

Let's start by thinking back to the order of operations. The order of operations can help you simplify numerical and variable expressions. Here is a reminder of the order of operations.

- First evaluate expressions in parentheses.
- Then evaluate exponents.
- Then multiply and divide in order from left to right.
- Finally, add and subtract in order from left to right.

Now let's look at an example.

Example

If $x = 4.5$, what is the value of $4x(x - 10)$?

Substitute the value for *x* into the expression.

$4(4.5)(4.5 - 10)$

Evaluate the expression in parentheses: $4(4.5)(4.5 - 10) = 4(4.5)(-5.5)$

There are no exponents, so multiply in order from left to right: $4(4.5)(-5.5) = 18(-5.5) = -99$

The answer is -99.

Notice that in this example, we used the Distributive Property to work with the values outside of the parentheses. When we multiplied those values, we were able to get rid of the parentheses.

Also, you can see that we could use the Commutative Property of Multiplication at the end of the problem. It does not matter which order we multiply those values, the product is still the same.

Now let's look at an example with fractions.

Example

If $x = \frac{1}{3}$ and $y = \frac{1}{2}$, what is $2xy - 6x$?

Look at the expression. Multiplying fractions several times can be difficult and time consuming. The Distributive Property can help you simplify the expression before substituting the values for x and y.

$2xy - 6x = 2x(y - 3)$

Now substitute the values for x and y into the expression.

$2\left(\frac{1}{3}\right)\left(\frac{1}{2} - 3\right)$

Now use the order of operations to simplify the expression.

Evaluate the expression in parentheses: $2\left(\frac{1}{3}\right)\left(\frac{1}{2} - 3\right) = 2\left(\frac{1}{3}\right)\left(-\frac{5}{2}\right)$

Multiply in order from left to right: $2\left(\frac{1}{3}\right)\left(-\frac{5}{2}\right) = \frac{2}{3}\left(-\frac{5}{2}\right) = -\frac{5}{3} = -1\frac{2}{3}$

The answer is $-1\frac{2}{3}$.

IV. **Model and Solve Real - World Problems Using Simple Equations Involving Rational Numbers**

You have seen how fractions, decimals, and integers can be used to solve real-world problems. You can use rational numbers to write and solve equations to represent real-world problems.

Example

Candy is 5 years older than one-third Liam's age. If Candy is 16, how old is Liam?

Choose a variable to represent Liam's age. Let l represent Liam's age.

Write an equation using the information in the problem.

The phrase "5 years older" translates to "+5." The phrase "one-third Liam's age" translates to "$\frac{1}{3} \cdot l$." Since the problem tells you that Candy is 16, set the expression that shows Candy's age equal to 16.

Put the parts together to form an equation.

$\frac{1}{3}l + 5 = 16$

Now solve the equation for l. Remember that any operation you perform on one side of the equation must also be performed on the other side.

First, subtract 5 from both sides to isolate the variable term on one side of the equation.

$$\frac{1}{3}l + 5 - 5 = 16 - 5$$

$$\frac{1}{3}l = 11$$

Now you need to remove the $\frac{1}{3}$ from the variable term. In order to eliminate a fractional coefficient, multiply by the

reciprocal.

$$\frac{3}{1} \cdot \frac{1}{3} l = 11 \cdot \frac{3}{1}$$
$$l = 33$$

Liam is 33 years old.

Now let's go back to the problem in the introduction and use what we have learned to solve it.

Real-Life Example Completed

Boxes of Erasers

Here is the original problem once again. Use what you have learned to solve it. You will need to write an equation, solve it for the number of boxes that Kelly sold, and write an inequality comparing the two quantities. There are three parts to your answer.

Kelly and Mallory are comparing the number of boxes of erasers that each one of them has sold. Kelly works in the store one day and Mallory works in the store the next day. They decide to write a riddle about their sales and give it to Trevor to figure out. When Trevor comes in the store on the day after Mallory has worked he finds this riddle.

Mallory sold 4 more boxes of erasers than the three-fourths that Kelly sold. If Mallory sold 13 boxes, how many boxes did Kelly sell?

Trevor is puzzled. He knows that there has to be a variable because the number of boxes that Kelly sold is unknown. He also knows that he needs to write an equation and that this equation will have a rational number, three-fourths in it.

He starts to work.

Now work the problem through in your notebook. Remember that there are three parts to the answer.

Solution to the Real - Life Example

First, you need to write an equation. Here is how each part of the equation breaks down.

$x =$**Kelly's boxes**

$+4 =$**the four more boxes that Mallory sold**

$\frac{3}{4} =$**three-fourths of Kelly's boxes**

$13 =$**the number that Mallory sold**

Here is the equation.

$\frac{3}{4}x + 4 = 13$

Next, we can solve it.

Start by subtracting four from both sides. This makes perfect sense in simplifying.

$\frac{3}{4}x = 9$

Next, we want to get the variable alone, so we can multiply by the reciprocal. This is the Inverse Property of Multiplication. Any number multiplied by its inverse or reciprocal is equal to 1.

$$\frac{4}{3} \cdot \frac{3}{4}x = 9 \cdot \frac{4}{3}$$
$$x = \frac{36}{3} = 12$$

Now we know that Kelly sold 12 boxes and Mallory sold 13 boxes.

We can write an inequality the compares Kelly's boxes to Mallory's boxes.

$12 < 13$

Vocabulary

Here are the vocabulary words that are found in this lesson.

Rational Number

number that can be written in fraction form.

Integer

the set of whole numbers and their opposites.

Percent

number representing a part out of 100.

Terminating Decimal

a decimal that has an ending even though many digits may be present.

Repeating Decimal

a decimal that has an ending even though many digits may repeat.

Irrational Number

a decimal that has no ending, pi or 3.14... is an example.

Inequality Symbols

symbols used to compare numbers using <or >.

PRACTICE

Directions: Identify whether each is a rational number or not. Write yes or no for your answer. Then identify the form of the number: integer, decimal, repeating decimal, fraction, terminating decimal, irrational number

1. .456
2. $\frac{2}{3}$
3. -45
4. 567
5. $-8,970$
6. .3434343434
7. .234...
8. .234567
9. $-.876$
10. $-\frac{2}{7}$

Directions: Compare each pair of rational numbers using <or >.

11. .34 _____ .87
12. -8 _____ -11
13. $\frac{1}{6}$ _____ $\frac{7}{8}$
14. .45 _____ 50%
15. 66% _____ $\frac{3}{4}$
16. .78 _____ 77%
17. $\frac{4}{9}$ _____ 25%
18. .989898 _____ .35
19. .67 _____ 32%
20. .123000 _____ .87

Directions: Use the order of operations to evaluate the following expressions.

21. 3x, when x is .50
22. 4y, when y is $\frac{3}{4}$
23. 5x + 1, when x is −12
24. 6y − 7, when y is $\frac{1}{2}$
25. 3x − 4x, when x is −5
26. 6x + 8y, when x is 2 and y is −4

2.2 Adding and Subtracting Decimals (7.3a and 7.3b)

GUIDANCE

Lunch Menu

Side Caesar Salad $1.99
Pepperoni Pizza.........$5.99
Chicken Wrap...........$6.99
Juice.................... $0.99
Milk.................... $1.25
Bottled Water $2.00

Hannah and Allison are going out for lunch. They need to estimate their bill before they go to lunch so they can see if they have enough money or need to go to the ATM. Hannah wants to get a Caesar salad and a pepperoni pizza with a juice. Allison wants a Caesar salad and chicken wrap along with her milk. Based on the menu above, if each of the girls has $10.00, will they need to go to the ATM?

In this concept, you will learn to add and subtract decimals with and without rounding.

Adding and Subtracting Decimals

A **decimal** is a number that uses a decimal point and place value to show tenths, hundredths, thousandths, and so on. The **decimal point** divides the whole number portion from the fractional portion of the number.

Looking at the number 35.492, the whole number portion is 35, or 3 tens and 5 ones. The fractional portion is 0.492, or 4 tenths, 9 hundredths, and 2 thousandths. Sometimes there are decimals with both wholes and parts, and sometimes, there are decimals with only parts.

You can add and subtract decimals by adding according to place value or by rounding the values before adding them.

Decimals can be added the same way whole numbers can: by lining up the place values. For decimals, this means lining up the decimal points and adding each place value with its common place value. For instance, don't add hundredths and tens, add hundredths and hundredths, and add tenths and tenths.

Let's look at an example.

Add $48.08 + 6.215$.

First, line up the addition problem by lining up the place values.

$$
\begin{array}{r}
48.08 \\
+\ \ 6.215 \\
\hline
\end{array}
$$

Next, add each place value, remembering to carry when necessary.

$$
\begin{array}{r}
48.080 \\
+\ \ 6.215 \\
\hline
54.295
\end{array}
$$

The answer is 54.295.

You can also find an approximate sum by estimating. Remember that when you **estimate** you will find an approximate answer, which means it will not be exact. One way of estimating is by rounding.

You can round each value to the nearest whole number. To determine which whole number to round a number to, look at the decimal portion of the number. If the decimal part is less than .5, then round down. If the decimal part is .5 or greater, then round up.

Let's look at an example.

Round 4.56 to the nearest whole number.

The decimal part (.56) is greater than .5, so 4.56 rounds up to 5.

You can also subtract decimals by using place value or by rounding.

Let's look at an example.

Complete the subtraction problem $56.93 - 10.14$.

First, line up the values according to place value so that one can be subtracted from the other.

$$
\begin{array}{r}
56.93 \\
-\ \ 10.14 \\
\hline
\end{array}
$$

Next, perform the subtraction.

$$
\begin{array}{r}
56.93 \\
-\ \ 10.14 \\
\hline
46.79
\end{array}
$$

The answer is 46.79.

You can also find the difference by rounding to the nearest whole number. You need to round each number to the nearest whole number and then find the difference between the two values.

56.93 rounds up to 57

10.14 rounds down to 10

$57 [U+0088][U+0092] 10 = 47$

The answer is 47.

Notice that the estimated and actual answers are close.

Examples

Example 1

Earlier, you were given a problem about Hannah and Allison going to lunch.
The girls need to know if they need to stop at the ATM first. Let's add up the bills for both girls to see if they have enough money for lunch.

Lunch Menu
Side Caesar Salad ……. $1.99
Pepperoni Pizza……….$5.99
Chicken Wrap………….$6.99
Juice……………………. $0.99
Milk……………………….$1.25
Bottled Water …………...$2.00

Hannah's order:

Caesar salad….…...$1.99

Pepperoni pizza….$5.99

Juice…………….$0.99

Total……….….....$8.97

Allison's order:

Caesar salad….....$1.99

Chicken wrap…..$6.99

Milk. $1.25

Total. $10.23

Since each of the girls has only $10.00, Hannah will have enough for her lunch but Allison will have to stop at the ATM first.

Subtract the following decimals with and without estimation: $5.678 - 0.82$

Let's estimate the subtraction.

First, round 5.678 up to 6.

Next, round .82 up to 1.

Then, subtract: $6 - 1 = 5$

The estimated answer is 5.

Now let's calculate the subtraction.

First, line up the values according to place value so that you can subtract one from the other.

$$
\begin{array}{r}
5.678 \\
-\ 0.82 \\
\hline
\end{array}
$$

Next, perform the subtraction.

$$
\begin{array}{r}
5.678 \\
-\ 0.82 \\
\hline
4.858
\end{array}
$$

The accurate answer is 4.858.

The estimate of 5 is very close to the accurate answer of 4.858.

Example 2

Round 2.3 to the nearest whole number.

Remember, with rounding, if the decimal is greater than 0.5, you round up and if less than 0.5, you round down.

0.3 is less than 0.5 so you round down.

The answer is 2

Example 3

Estimate by rounding $48.08 + 7.578$

Remember, with rounding, if the decimal is greater than 0.5, you round up and if less than 0.5, you round down.

First, round 48.08 to the nearest whole number. Note that 0.08 is less than 0.5 so you round down.

48.08 now will be 48.

Next, round 7.578 to the nearest whole number. Note that 0.0578 is greater than 0.5 so you round up.

7.578 now will be 8.

Then, complete the addition problem. $48 + 7 = 55$

The answer is 55.

Example 4

Subtract $49.45 - 6.234$

First, line up the values according to place value so that you can subtract one from the other.

$$
\begin{array}{r}
49.45 \\
- \ \ 6.234 \\
\hline
\end{array}
$$

Next, perform the subtraction.

$$
\begin{array}{r}
49.45 \\
- \ \ 6.234 \\
\hline
43.216
\end{array}
$$

The answer is 43.216.

Review

Find the exact sum or difference by adding or subtracting the following decimals according to place value.

1. $16.27 + 3.45 =$ _____
2. $22.34 + 9.21 =$ _____
3. $34.5 + 1.234 =$ _____
4. $5.6 + 8.9 =$ _____
5. $1.02 + 12.34 =$ _____
6. $67.89 + 23.45 =$ _____
7. $123.4 + 7.89 =$ _____
8. $34.05 + 102.10 =$ _____
9. $34.56 - 11.23 =$ _____
10. $67.09 - 2.34 =$ _____
11. $88.9 - 13.24 =$ _____

12. $234.5 - 16.7 = $ _____
13. $708.90 - 45.67 = $ _____
14. $27.56 - 1.20 = $ _____
15. $327.66 - 301.20 = $ _____
16. $540.26 - 18.50 = $ _____

Answers for Review Problems

To see the Review answers, open this PDF file and look for section 2.1.

Resources

DECIMALS:
SUBTRACTING (EX.1)

KHANACADEMY

MEDIA

Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/167085
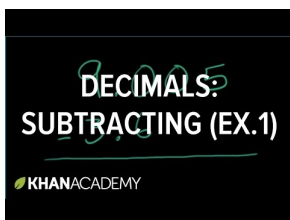
2.3 Combining Rational Numbers (7.3a and 7.3b)

GUIDANCE

Adding and Subtracting Integers

Integers include the set of whole numbers and their opposites: {... -3, -2, -1, 0, 1, 2, 3 ... }. As you work with integers you are working with both positive and negative numbers.

You work with integers when adding, subtracting, multiplying and dividing. When working with integers and operations, it's important to keep track of whether the integer is positive or negative.

Let's look at an example.

$$-5 + -6 = -11$$

Notice that the number increases but in the negative direction. If you think about this on a number line, you can see how your number becomes more negative.

First, draw your number line and place your dot on -5 as this is where your problem starts.
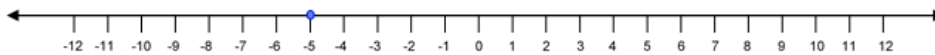
Next, move the blue dot six places to the left (left because it is negative).
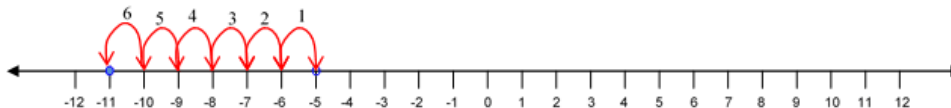
The answer is -11.

You can't always use a number line as your numbers are sometimes just too big for a number line.

Let's look at an example.

$$-623 + 215$$

To add two numbers with opposite signs, you are really subtracting and adding the sign that occurs with the larger number.

First, find the difference between the two numbers.

$$623 - 215 = 408$$

Next, since 623 is larger and has a negative sign, the answer is negative.

The answer is -408.

Let's look at another example using subtraction.

$$-412 - 244$$

First, use the subtraction rule in which you know that to subtract b from a is the same as adding the opposite of b to a. In other words, $a - b = a + (-b)$.

$$-412 - 244 = -412 + (-244)$$

Next, since both of the numbers are negative, your answer will be larger but in the negative direction.

$$-412 - 244 = -656$$

The answer is -656.

Let's look at another example.

$$54 - (-789)$$

First, add the opposite of -789.

Next, use the rules of addition to evaluate.

$$54 + 789 = 843$$

The answer is 843.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about total earnings at a school store.

The workers need to determine their total earnings for the first few days of sales at the school store.

First, list all of the sales and debits from the store.

Monday: sales = $87.00

Tuesday: sales = $45.00, debits = $20.00

Wednesday: sales = $25.00, debits = $3.00

Next, write an equation so you can find the total sales.

$$87.00 + 45.00 - 20.00 + 25.00 - 3.00 = \underline{}$$

Then, add to find the total earnings.

$$87.00 + 45.00 + (-20.00) + 25.00(-3.00) = 134$$

The answer is 134.

The total sales is $134.

Example 2

$$-229 + 563$$

First, find the difference between the two numbers.

$$563 - 229 = 334$$

Next, since 563 is larger and has a positive sign, the answer is positive.

The answer is 334.

Example 3

$-7 + 11$

First, draw your number line and place your dot on -7 as this is where your problem starts.

Next, move the blue dot eleven places to the right (right because it is positive).
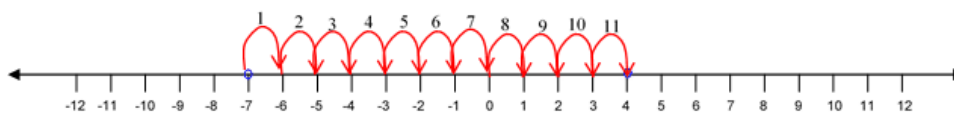
The answer is 4.

Example 4

$45 + -19$

First, find the difference between the two numbers.

$$45 - 49 = 26$$

Next, since 45 is larger and has a positive sign, the answer is positive.

The answer is 26.

Example 5

$23 - (-12)$

First, add the opposite of -12.

$$23 - (-12) = 23 + 12$$

Next, use the rules of addition to solve.

$$23 + 12 = 35$$

The answer is 35.

PRACTICE

Add the following integers.

1. $6 + 7 = $ ___
2. $5 + (-8) = $ ___
3. $8 + (-8) = $ ___
4. $6 + (-10) = $ ___
5. $8 + (-2) = $ ___
6. $9 + (-4) = $ ___
7. $-14 + (-7) = $ ___
8. $-12 + (-14) = $ ___
9. $-13 + (-10) = $ ___
10. $-18 + (-30) = $ ___

Subtract the following integers.

11. $-9 - 5 =$ ___
12. $-8 - 7 =$ ___
13. $-12 - 8 =$ ___
14. $6 - 9 =$ ___
15. $10 - 15 =$ ___
16. $18 - (-5) =$ ___
17. $12 - (-4) =$ ___
18. $23 - (-9) =$ ___
19. $-5 - (-2) =$ ___
20. $-8 - (-5) =$ ___

Resources

MEDIA

Click image to the left or use the URL below.

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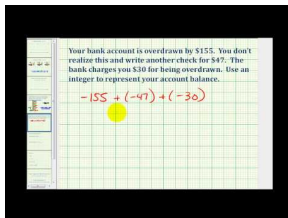
2.4 Multiply and Divide Rational Numbers (7.3a and 7.3b)

GUIDANCE

Multiplying and Dividing Integers

Integers are the values in the set of whole numbers and their opposites, $\{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$. When you multiply and divide integers, there are some rules:

1. When multiplying or dividing a positive integer by a positive integer, the product or quotient is positive.
2. When multiplying or dividing a positive integer by a negative integer, or a negative integer by a positive integer, the product or quotient is negative.
3. When multiplying or dividing a negative integer by a negative integer, the product or quotient is positive.

Another way to remember multiplying and dividing integers is to remember if the signs are the same, the answer is positive. If the signs are different, the answer is negative.

Let's look at an example where you can apply these rules.

$$12(-5)$$

Notice that this is a multiplication problem. Parentheses are used around a single value to show multiplication.

First, multiply the two values.

$$12 \times 5 = 60$$

Next, add the sign. Remember rule 2, when you multiply a positive integer by a negative integer, the answer is negative.

The answer is -60.

Here is another example.

$$\frac{-150}{-50}$$

First, divide the two values.

$$\frac{150}{50} = 3$$

Next, add the sign. Remember rule 3, when you dividing a negative integer by a negative integer, the answer is positive.

The answer is 3.

GUIDED PRACTICE

Example 1

Remember the bald eagle? You know the distance (d), speed (v), and time (t). When you use these three variables together, you use the formula: $v = \frac{d}{t}$.

Now, look at the two questions in the original problem.

How far would the eagle be able to go at this speed in 2 seconds?

$v = 110 \frac{ft}{sec}, t = 2 \ sec, d =?$

First, you need to find the distance so rearrange the formula to solve for the distance.

$d = v(t)$

Next, put the values for speed and time into the formula to solve for the distance.

$d = 110(2)$

Then, solve for the distance. Remember rule 1, when you multiply a positive integer by a positive integer, the answer is positive.

$d = 220$

The answer is 220.

The distance is 220 feet.

If the eagle sees prey 750 feet below, how much farther must it dive to reach its prey?

$v = 110 \frac{ft}{sec}, d = 750 \ ft, t =?$

First, you need to find the time so rearrange the formula to solve for the time.

$t = \frac{d}{v}$

Next, put the values for distance and speed into the formula to solve for the time.

$t = \frac{750}{110}$

Then, solve for the time. Remember rule 1, when you divide a positive integer by a positive integer, the answer is positive.

$t = 6.82$

The answer is 6.82.

The time is 6.82 seconds.

Example 2

Multiply.

$$(8)(-5)(10)$$

First, multiply the three values.

$$8 \times 5 \times 10 = 400$$

Next, add the sign. Remember rule 2, when you multiply a positive integer(s) by a negative integer, the answer is negative.

The answer is -400.

Example 3

Multiply.

$$-9(7)$$

First, multiply the two values.

$$9 \times 7 = 63$$

Next, add the sign. Remember rule 2, when you multiply a positive integer by a negative integer, the answer is negative.

The answer is -63.

Example 4

Multiply.

$$-3(-12)$$

First, multiply the two values.

$$3 \times 12 = 36$$

Next, add the sign. Remember rule 3, when you multiplying a negative integer by a negative integer, the answer is positive.

The answer is 36.

Example 5

Divide.

$$\frac{-169}{13}$$

First, divide the two values.

$$\frac{169}{13} = 13$$

Next, add the sign. Remember rule 2, when you divide a negative integer by a positive integer, the answer is negative.

The answer is -13.

PRACTICE

Multiply the following integers.

1. $-6(-8) = $ _____
2. $5(-10) = $ _____
3. $3(-4) = $ _____
4. $-3(4) = $ _____
5. $8(-9) = $ _____
6. $-9(12) = $ _____
7. $8(-11) = $ _____
8. $(-5)(-9) = $ _____
9. $-7(-8) = $ _____
10. $(-12)(12) = $ _____

Divide the following integers.

11. $-12 \div 2 = $ _____
12. $-18 \div -6 = $ _____
13. $-24 \div 12 = $ _____
14. $-80 \div -4 = $ _____
15. $-60 \div -30 = $ _____
16. $\frac{28}{4} = $ _____
17. $\frac{-36}{4} = $ _____
18. $\frac{-45}{9} = $ _____
19. $-75 \div 25 = $ _____
20. $-68 \div -2 = $ _____

Resources

MEDIA
Click image to the left or use the URL below.
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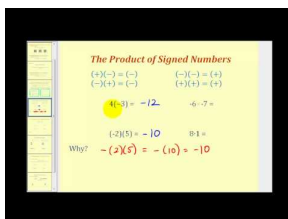
2.5 Approximating Irrational Numbers (8.2b)

GUIDANCE

Suppose an elementary school has a square playground with an area of 3000 square feet. Could you find the width of the playground? Would the width be a rational or irrational number? In this Concept, you'll learn how to take the square root of a number and decide whether the result is rational or irrational so that you can answer questions such as these.

Human chess is a variation of chess, often played at Renaissance fairs, in which people take on the roles of the various pieces on a chessboard. The chessboard is played on a square plot of land that measures 324 square meters with the chess squares marked on the grass. How long is each side of the chessboard?

To answer this question, you will need to know how to find the square root of a number.

The **square root** of a number n is any number s such that $s^2 = n$.

Every positive number has two square roots, the positive and the negative. The symbol used to represent the square root is $\sqrt{x}$. It is assumed that this is the positive square root of x. To show both the positive and negative values, you can use the symbol $\pm$, read "plus or minus."

For example:

$\sqrt{81} = 9$ means the positive square root of 81.

$-\sqrt{81} = -9$ means the negative square root of 81.

$\pm\sqrt{81} = \pm 9$ means the positive or negative square root of 81.

Example A

The human chessboard measures 324 square meters. How long is one side of the square?

Solution: The area of a square is $s^2 = Area$. The value of *Area* can be replaced with 324.

$$s^2 = 324$$

The value of *s* represents the square root of 324.

$$s = \sqrt{324} = 18$$

The chessboard is 18 meters long by 18 meters wide.

Approximating Square Roots

When the square root of a number is a whole number, this number is called a **perfect square.** 9 is a perfect square because $\sqrt{9} = 3$.

Not all square roots are whole numbers. Many square roots are irrational numbers, meaning there is no rational number equivalent. For example, 2 is the square root of 4 because $2 \times 2 = 4$. The number 7 is the square root of 49 because $7 \times 7 = 49$. What is the square root of 5?

There is no whole number multiplied by itself that equals five, so $\sqrt{5}$ is not a whole number. To find the value of $\sqrt{5}$, we can use estimation.

To estimate the square root of a number, look for the perfect integers **less than** and **greater than** the value, and then estimate the decimal.

Example B

Estimate $\sqrt{5}$.

Solution: The perfect square below 5 is 4 and the perfect square above 5 is 9. Therefore, $4 < 5 < 9$. Therefore, $\sqrt{5}$ is between $\sqrt{4}$ and $\sqrt{9}$, or $2 < \sqrt{5} < 3$. Because 5 is closer to 4 than 9, the decimal is a low value: $\sqrt{5} \approx 2.2$.

Identifying Irrational Numbers

Recall the number hierarchy from a previous Concept. Real numbers have two categories: rational and irrational. If a value is not a perfect square, then it is considered an **irrational number.** These numbers cannot be written as a fraction because the decimal does not end **(non-terminating)** and does not repeat a pattern **(non-repeating)**. Although irrational square roots cannot be written as fractions, we can still write them **exactly**, without typing the value into a calculator.

For example, suppose you do not have a calculator and you need to find $\sqrt{18}$. You know there is no whole number squared that equals 18, so $\sqrt{18}$ is an irrational number. The value is between $\sqrt{16} = 4$ and $\sqrt{25} = 5$. However, we need to find the exact value of $\sqrt{18}$.

Begin by writing the **prime factorization** of $\sqrt{18}$. $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2}$. $\sqrt{9} = 3$ but $\sqrt{2}$ does not have a whole number value. Therefore, the exact value of $\sqrt{18}$ is $3\sqrt{2}$.

You can check your answer on a calculator by finding the decimal approximation for each square root.

Example C

Find the exact value of $\sqrt{75}$.

Solution:

$$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$$

Video Review

	MEDIA
Multimedia	Click image to the left or use the URL below. URL: https://www.ck12.org/flx/render/embeddedobject/79450

–>

GUIDED PRACTICE

The area of a square is 50 square feet. What are the lengths of its sides?

Solution:

We know that the formula for the area of a square is $a = s^2$. Using this formula:

$$a = s^2$$
$$50 = s^2$$
$$\sqrt{50} = \sqrt{s^2}$$
$$\sqrt{50} = s$$

Now we will simplify:

$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}.$

The length of each side of the square is $5\sqrt{2}$ feet.

PRACTICE

Find the following square roots **exactly without using a calculator**. Give your answer in the simplest form.

1. $\sqrt{25}$
2. $\sqrt{24}$
3. $\sqrt{20}$
4. $\sqrt{200}$
5. $\sqrt{2000}$
6. $\sqrt{\dfrac{1}{4}}$

7. $\sqrt{\dfrac{9}{4}}$
8. $\sqrt{0.16}$
9. $\sqrt{0.1}$
10. $\sqrt{0.01}$

Use a calculator to find the following square roots. Round to two decimal places.

11. $\sqrt{13}$
12. $\sqrt{99}$
13. $\sqrt{123}$
14. $\sqrt{2}$
15. $\sqrt{2000}$
16. $\sqrt{0.25}$
17. $\sqrt{1.35}$
18. $\sqrt{0.37}$
19. $\sqrt{0.7}$
20. $\sqrt{0.01}$

Vocabulary:

- Decimal
- Fractions
- Integers
- Per (unit rate)
- Percents
- Rational numbers
- Irrational numbers

CHAPTER **3** # Equations and Inequalities

Chapter Outline

7.11a: model and solve one-variable, two-step equations and inequalities

7.11c: write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships

8.8c: model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants

7.10a: write one-variable, two-step equations and inequalities to represent constraints or conditions within problems

7.10b: represent solutions for one-variable, two-step equations and inequalities on number lines

7.10c: write a corresponding real-world problem given a one-variable, two-step equation or inequality

3.1 Model and Solve One-Variable, Two-Step Equations (7.11a)

Introduction

In this concept you will begin your study of mathematical equations by learning how to solve equations with only one variable. What this means is that the mathematical expression will have a letter and possibly some number on one side of the equal sign and numbers on the other side of the equal sign. It is your job to find a value for the letter that makes the expression true. Once you have mastered this process, you can solve for two variables and also add more complex numbers.

It is interesting to note that the variables are known as literal coefficients. Literal means "letter". So the literal coefficient is the unknown quantity in the mathematical expression represented by a letter (also called a **variable**). What do you think a numerical coefficient is?

Right! A numerical coefficient is a number in the mathematical expression. In the algebraic term $5abc$, the numerical coefficient is 5.

Watch This

Khan Academy Slightly More Complicated Equations

MEDIA

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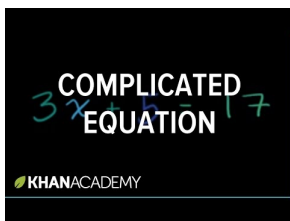
GUIDANCE

Erin, Jillian, Stephanie and Jacob went to the movies. The total bill for the tickets and snacks came to $72.00. How much should each teen pay to split the bill evenly.

There are four teens going to the movies (Erin, Jillian, Stephanie, and Jacob). The total bill was $72.00. Therefore our equation is $4x = 72$. We divide by 4 to find our answer.

$$x = \frac{72}{4}$$
$$x = 18$$

Therefore each teen will have to pay $18.00 for their movie ticket and snack.

Example A

$5a + 2 = 17$

The problem can be solved if we think about it in terms of a balance (or a seesaw). We know that the two sides are equal so the balance has to stay horizontal. We can place each side of the equation on each side of the balance.

In order to solve the equation, we have to get the literal coefficient (or the *a*) all by itself. Always remember that we need to keep the balance horizontal. This means that whatever we do to one side of the equation, we **have** to do to the other side.

Let's first subtract 2 from both sides to get rid of the 2 on the left.
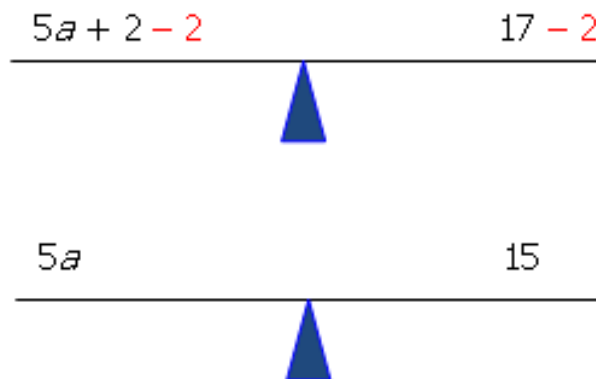
Since 5 is multiplied by *a*, we can get *a* by itself (or isolate it) by dividing by 5. Remember that whatever we do to one side, we have to do to the other.

$$\frac{5a}{5} \qquad\qquad \frac{15}{5}$$

If we simplify this expression, we get:

$$a \qquad\qquad 3$$

Therefore $a = 3$.

We can check our answer to see if we are correct.

$$5a + 2 = 17$$
$$5(3) + 2 = 17$$
$$15 + 2 = 17$$
$$17 = 17 \ Y$$

Example B

$7b - 7 = 42$

Again, we can solve the problem if we think about it in terms of a balance (or a seesaw). We know that the two sides are equal so the balance has to stay horizontal. We can place each side of the equation on each side of the balance.

$$7b - 7 \qquad\qquad 42$$

In order to solve the equation, we have to get the literal coefficient (or the b) all by itself. Always remember that we need to keep the balance horizontal. This means that whatever we do to one side of the equation, we **have** to do to the other side.

Let's first add 7 from both sides to get rid of the 7 on the left.

$$7b - 7 + 7 \qquad\qquad 42 + 7$$

Since 7 is multiplied by *b*, we can get *a* by itself (or isolate it) by dividing by 7. Remember that whatever we do to one side, we have to do to the other.

If we simplify this expression, we get:

Therefore $b = 7$.

We can check our answer to see if we are correct.

$$7b - 7 = 42$$
$$7(7) - 7 = 42$$
$$49 - 7 = 42$$
$$42 = 42 \ Y$$

Example C

This same method can be extended by using algebra tiles. If we let green rectangles represent the variable, green square tiles represent the positive numbers and white tiles represent the negative numbers, we can solve the equations using an alternate method.

$$3c + 2 = 11$$
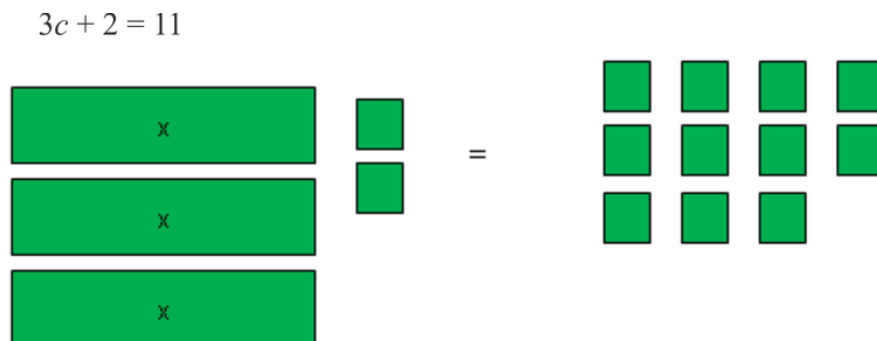
The green algebra $x-$tiles represent variables (or the literal coefficients). Therefore there are 3 c blocks for the equation. The other green blocks represent the numbers or constants. There is a 2 on the left side of the equation so there are 2 green blocks. There is an 11 on the right side of the equation so there are 11 green blocks on the right side of the equation.

To solve, add two negative tiles to the right and left hand sides. The same rule applies to this problem as to all of the previous problems. Whatever we do to one side we have to do to the other.

You can cancel the same number of white tiles as there are green tiles.
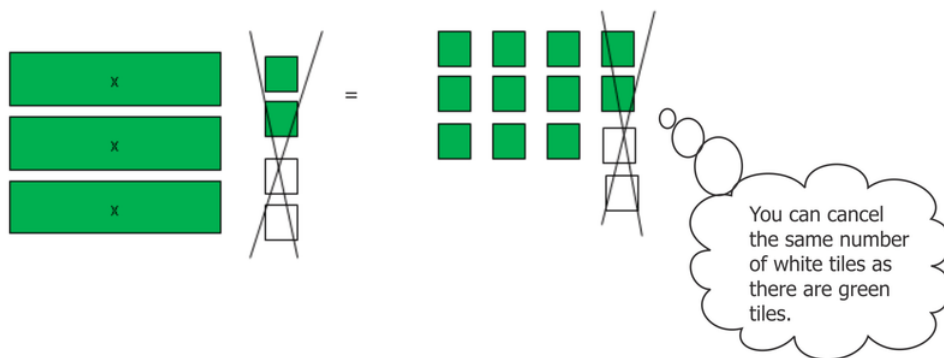
This leaves us with the following:
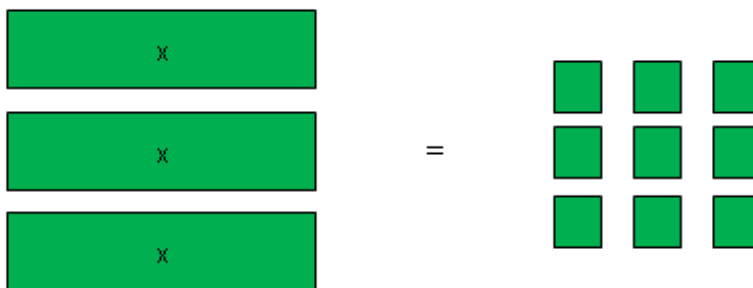
We can reorganize these to look like the following:
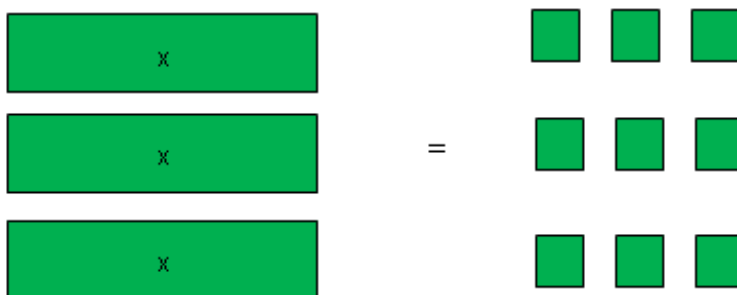
Organizing the remaining algebra tiles allows us to realize the answer to be $x = 3$ or for our example $c = 3$.

Let's do our check as with the previous problem.

$$3c + 2 = 11$$
$$3(3) + 2 = 11$$
$$9 + 2 = 11$$
$$11 = 11 \ Y$$

Vocabulary

Constant

A *constant* is a numerical coefficient but does not contain a variable. For example in the equation $4x + 72 = 0$, the 72 is a constant.

Equation

An *equation* is a pair of mathematical expressions separated by an equal sign.

Numerical Coefficient

In mathematical equations, the *numerical coefficients* are the numbers associated with the variable. For example, with the expression $4x$, 4 is the numerical coefficient and x is the literal coefficient.

Variable

A *variable* is an unknown quantity in a mathematical expression. It is represented by a letter. It is often referred to as the literal coefficient.

GUIDED PRACTICE

1. Use a model to solve for the variable in the problem $x - 5 = 12$.
2. Use a different model than used in question (1) to solve for the variable in the problem $3y + 9 = 12$.
3. Using one of the models from the lesson, solve for x in the equation $3x - 2x + 16 = -3$.

Answers

1. $x - 5 = 12$
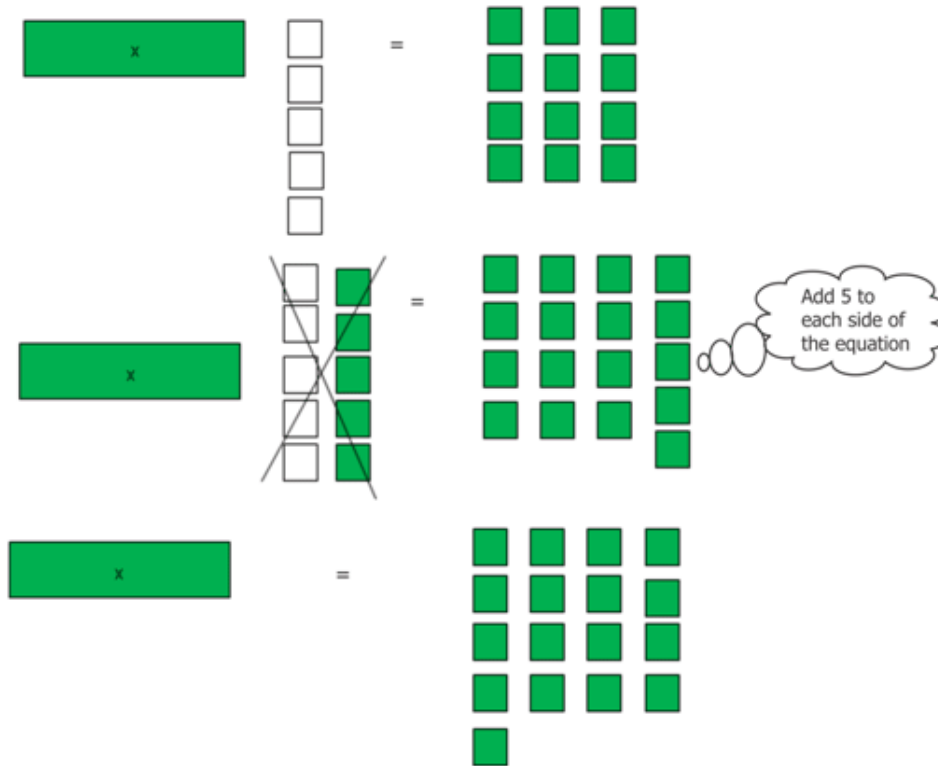
Therefore, $x = 17$. Let's do a check to make sure.

$$x - 5 = 12$$
$$(17) - 5 = 12$$
$$17 - 5 = 12$$
$$12 = 12 \ Y$$

2. $3y + 9 = 12$

First you have to subtract 9 from both sides of the equation in order to start to isolate the variable.

$$3y \qquad\qquad\qquad 3$$

Now, in order to get y all by itself, you have to divide both sides by 3. This will isolate the variable y.

$$\frac{3y}{3} \qquad\qquad\qquad \frac{3}{3}$$

$$y \qquad\qquad\qquad 1$$
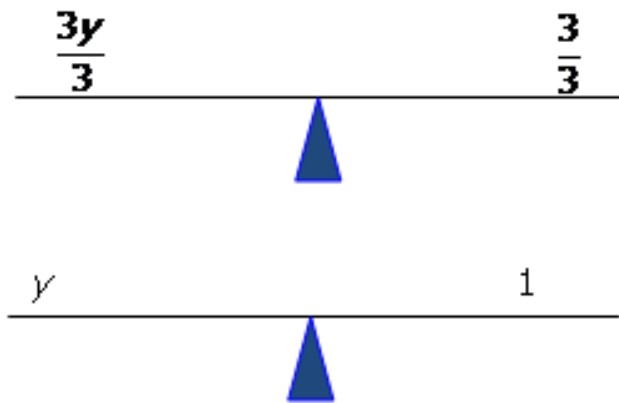
Therefore, $y = 1$. Let's do a check to make sure.

$$3y + 9 = 12$$
$$3(1) + 9 = 12$$
$$3 + 9 = 12$$
$$12 = 12 \ Y$$

3. $3x - 2x + 16 = -3$

You can use any method to solve this equation. Remember to isolate the x variable. You will notice here that there are two x values on the left. First let's combine these terms.

$$3x - 2x + 16 = -3$$
$$x + 16 = -3$$

Now you can use any method to solve the equation. You now should just have to subtract 16 from both sides to isolate the x variable.

$$x + 16 - 16 = -3 - 16$$
$$x = -19$$

Let's do a check to make sure.

$$3x - 2x + 16 = -3 \quad \text{(original problem)}$$
$$x + 16 = -3 \quad \text{(simplified problem)}$$
$$-19 + 16 = -3$$
$$-3 = -3 \ Y$$

Summary

Solving equations with variables on one side can be done using models such as a balance (or a seesaw) or by using algebra tiles.

Remember that when solving equations with variables on one side of the equation there is one main rule to follow: whatever you do to one side of the equal sign you must do the same to the other side of the equal sign. For example, if you add a number to the left side of an equal sign, you must add the same number to the right side of the equal sign.

PRACTICE

Solve for each of the following variables.

1. $a + 3 = -5$
2. $2b - 1 = 5$
3. $4c - 3 = 9$
4. $2 - d = 3$
5. $4 - 3e = -2$

Use algebra tiles to solve for each of the following variables.

1. $x + 3 = 14$
2. $2y - 7 = 5$
3. $3z + 6 = 9$
4. $5 + 3x = -3$
5. $2x + 2 = -4$

Use the models that you have learned to solve for the variables in the following problems.

1. $-4x + 13 = 5$
2. $3x - 5 = 22$
3. $11 - 2x = 5$
4. $2x - 4 = 4$
5. $5x + 3 = 28$

For each of the following models, write a problem involving an equation with a variable on one side of the equation expressed by the model and then solve for the variable.

1.

2.

3.

4.
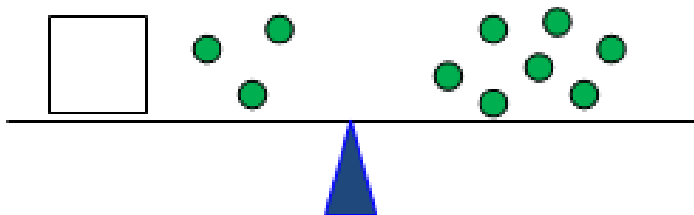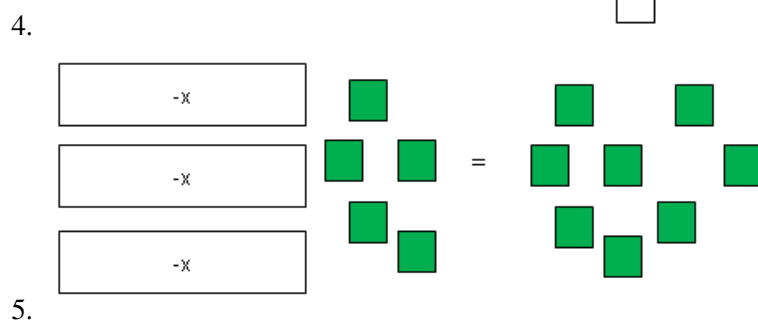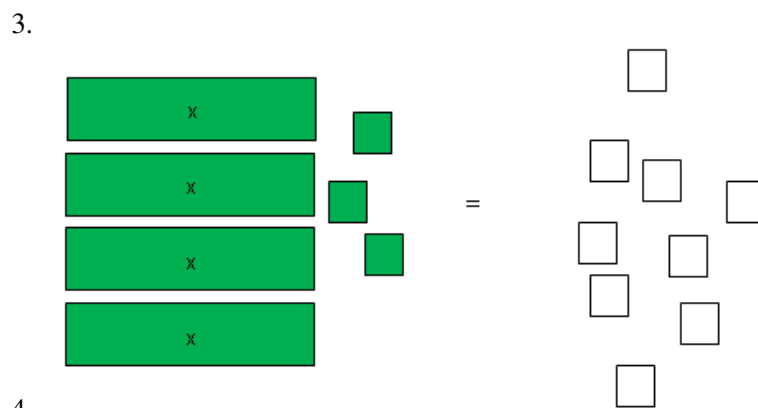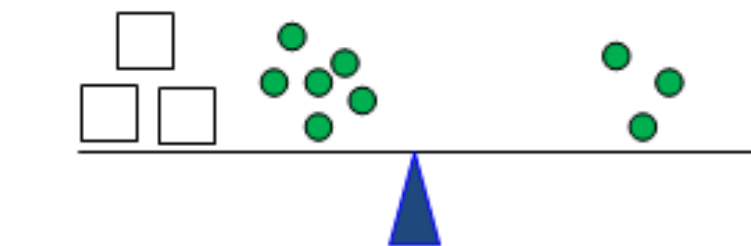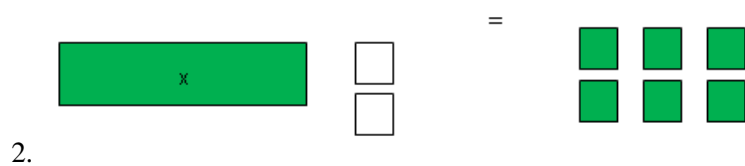
5.

3.2 Model and Solve One-Variable, One-Step Inequalities (7.11a)

Could you solve an inequality like $x - 12 > -5$? After completing this Concept, you'll be able to solve one-step inequalities like this one.

Watch This

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/133223

CK-12 Foundation: 0602S Solving One-Step Inequalities (H264)
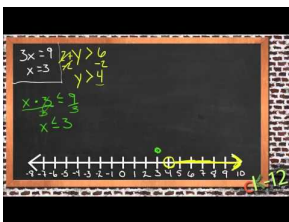
GUIDANCE

To solve an inequality we must **isolate** the variable on one side of the inequality sign. To isolate the variable, we use the same basic techniques used in solving equations.

We can solve some inequalities by adding or subtracting a constant from one side of the inequality.

Example A

Solve the inequality and graph the solution set.

$x - 3 < 10$

Solution

Starting inequality: $x - 3 < 10$

STEP 1: Model the inequality use a < between each expression.

X − 3 < 10
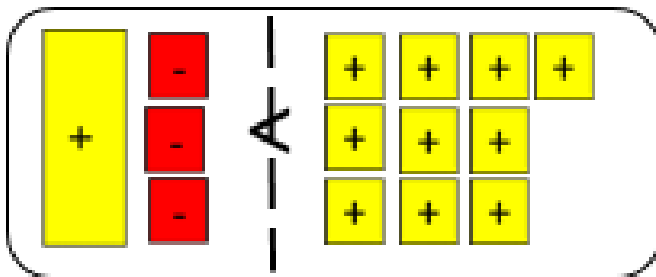
FIGURE 3.1

Add **3** to both sides of the inequality: $x - 3 + 3 < 10 + 3$

STEP 2: Add three ⊕ to both sides.
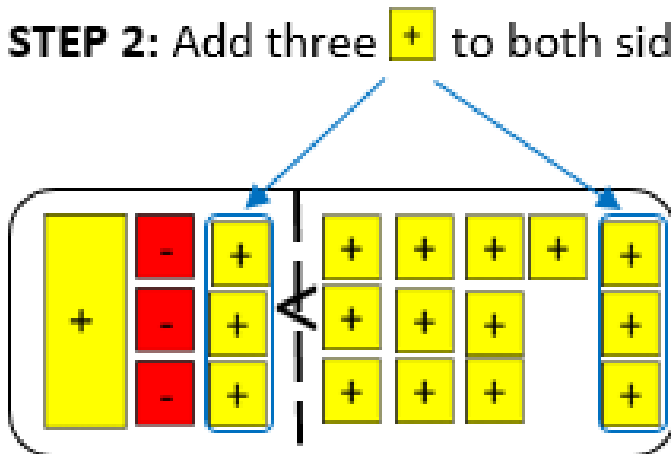
FIGURE 3.2

STEP 3: Remove **zero pairs** from both sides if needed.
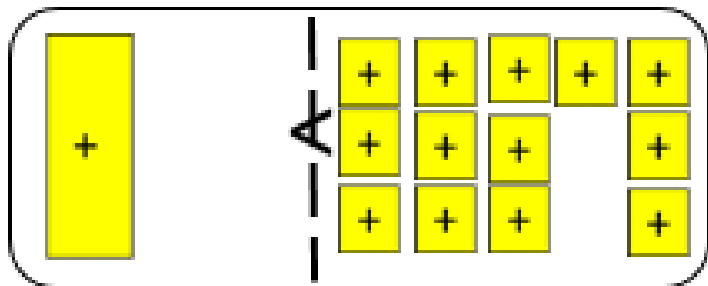
Simplify: $x < 13$

STEP 4: Write the solution

$X < 13$

Example B

Solve the inequality and graph the solution set.

$x - 20 \leq 14$

Solution:

Starting inequality: $x - 20 \leq 14$

Add **20** to both sides of the inequality: $x - 20 + 20 \leq 14 + 20$

Simplify: $x \leq 34$

Solving Inequalities Using Multiplication and Division

We can also solve inequalities by multiplying or dividing both sides by a constant. For example, to solve the inequality $5x < 3$, we would divide both sides by 5 to get $x < \frac{3}{5}$.

However, something different happens when we multiply or divide by a negative number. We know, for example, that 5 is greater than 3. But if we multiply both sides of the inequality $5 > 3$ by -2, we get $-10 > -6$. And we know that's not true; -10 is less than -6.

This happens whenever we multiply or divide an inequality by a negative number, and so we have to flip the sign around to make the inequality true. For example, to multiply $2 < 4$ by -3, first we multiply the 2 and the 4 each by -3, and then we change the <sign to a >sign, so we end up with $-6 > -12$.

The same principle applies when the inequality contains variables.

Example C

Solve the inequality.

$4x < 24$

Solution:

Original problem: $4x < 24$

Divide both sides by 4: $\frac{4x}{4} < \frac{24}{4}$

Simplify: $x < 6$

Example D

Solve the inequality.

$-5x \leq 21$

Solution:

Original problem: $-5x \leq 21$

Divide both sides by -5 : $\frac{-5x}{-5} \geq \frac{21}{-5}$ ***Flip the inequality sign.***

Simplify: $x \geq -\frac{21}{5}$

Watch this video for help with the Examples above.

MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/133224

CK-12 Foundation: Solving One-Step Inequalities

GUIDED PRACTICE

Solve each inequality.

a) $x + 8 \leq -7$

b) $x + 4 > 13$

c) $\frac{x}{25} < \frac{3}{2}$

d) $\frac{x}{-7} \geq 9$

Solutions:

a) Starting inequality: $x + 8 \leq -7$

Subtract **8** from both sides of the inequality: $x + 8 - 8 \leq -7 - 8$

Simplify: $x \leq -15$
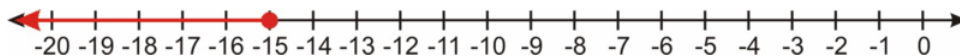
b) Starting inequality: $x + 4 > 13$

Subtract **4** from both sides of the inequality: $x + 4 - 4 > 13 - 4$

Simplify: $x > 9$
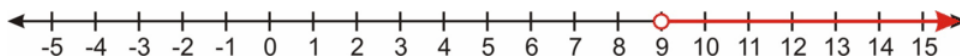
c) Original problem: $\frac{x}{25} < \frac{3}{2}$

Multiply both sides by 25: $25 \cdot \frac{x}{25} < \frac{3}{2} \cdot 25$

Simplify: $x < \frac{75}{2}$ or $x < 37.5$

d) Original problem: $\frac{x}{-7} \geq 9$

Multiply both sides by -7: $-7 \cdot \frac{x}{-7} \leq 9 \cdot (-7)$ **Flip the inequality sign.**

Simplify: $x \leq -63$

PRACTICE

For 1-8, solve each inequality and graph the solution on the number line.

1. $x - 5 < 35$
2. $x + 15 \geq -60$
3. $x - 2 \leq 1$
4. $x - 8 > -20$
5. $x + 11 > 13$
6. $x + 65 < 100$
7. $x - 32 \leq 0$
8. $x + 68 \geq 75$

For 9-12, solve each inequality. Write the solution as an inequality and graph it.

9. $3x \leq 6$
10. $\frac{x}{5} > -\frac{3}{10}$
11. $-10x > 250$
12. $\frac{x}{-7} \geq -5$

Answers for Explore More Problems

To view the Explore More answers, open this PDF file and look for section 6.2.

3.3 Model and Solve One-Variable, Two-Step Inequalities (7.11a)

Your community center is selling boxes of cards. Your goal is to sell at least $500 worth of cards. Each box sells for $5 but you also need to pay the company $50 to ship all of the cards to your house to begin selling. Determine the least number of boxes of cards you must sell in order to meet your goal.

In this concept, you will learn to solve inequalities involving combining like terms.

GUIDANCE

You can solve inequalities in many ways. Some inequalities can be solved in a single step. You could solve $b + 4 < 10$ in one step—by subtracting 4 from each side.

However, two or more steps may be required to solve some inequalities. Inequalities that need more than one inverse operation to solve them can be called **multi-step inequalities**.

Let's start by looking at combining like terms when you solve an inequality.

$$4x + 3x < 14$$
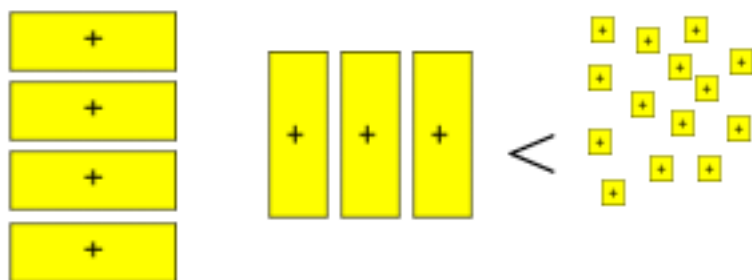
FIGURE 3.5

First, you can see that you have two terms that have the same variable. These are **like terms**. To solve an inequality with like terms, you will need to combine the like terms and then you can solve the inequality.

$$7x < 14$$

FIGURE 3.6

Here you divide(separate) both sides of the inequality by 7(7 groups). Division is the inverse operation of Multiplication.
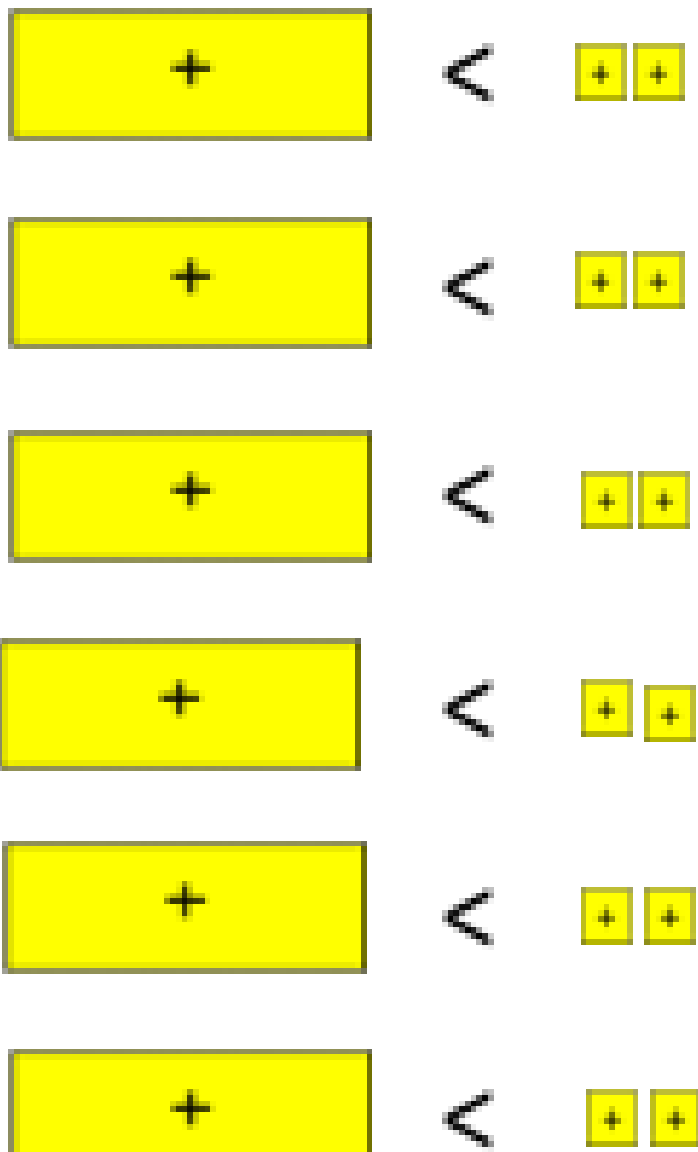
$$\frac{7x}{7} < \frac{14}{7}$$

The answer is $x < 2$.

FIGURE 3.8

This means that "x" can be any value less than 2.

Let's look at another example.

Solve for *b*:

$$2b + 4 < 10$$
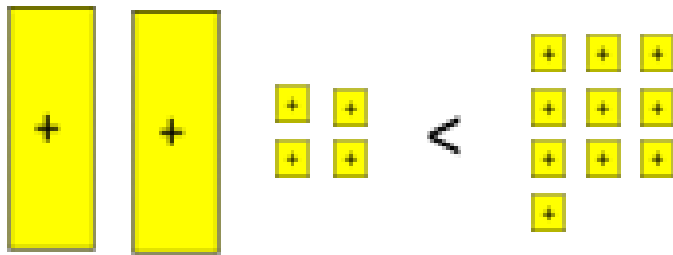
FIGURE 3.9

First, notice that there are two terms on the left side of the inequality, 2*b* and 4. Therefore, use inverse operations to get the term that includes a variable, 2*b*, by itself on one side of the inequality. In the inequality, 4 is *added* to 2*b*. So, you can use the inverse of addition is subtraction. Therefore, subtract 4 from both sides of the inequality.

$$2b + 4 - 4 < 10 - 4$$

FIGURE 3.10
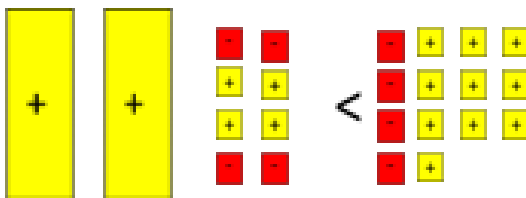
$$2b + 4 - 4 < 10 - 4$$

FIGURE 3.11
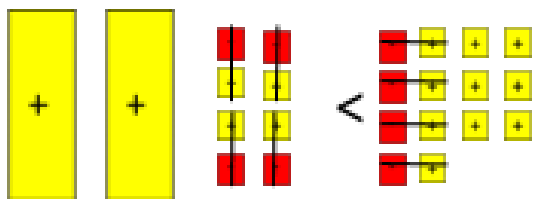
$$2b \qquad < 6$$

FIGURE 3.12
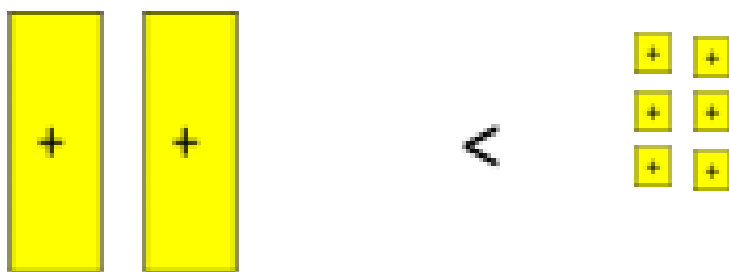
Next, use inverse operations to get the 'b' by itself. Since '2b' means $2 \times b$, divide(separate) both sides of the inequality by 3(3 groups) to solve for the variable.

$$\frac{3b}{3} < \frac{6}{3}$$

The answer is $b < 2$.

This means that 'b' can any value less than 2.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about boxes of cards being sold.

First, write an inequality using the given information. You want to raise at least \$500 by selling boxes of cards worth \$5 each. You also need to pay the supplier \$50 for shipping. Let 'b' represent the number of boxes of cards.

$5b - 50 \geq 500$

Next, isolate the term with the variable, $5b$, on one side of the inequality. Since 50 is subtracted from $5b$, you should add 50 to both sides of the inequality.

$$
\begin{aligned}
5b - 50 &\geq 500 \\
5b - 50 + 50 &\geq 500 + 50 \\
5b &\geq 550
\end{aligned}
$$

Next, since $5b$ means $5 \times b$, you can divide each side of the inequality by 5 to get 'b' by itself on one side of the inequality.

$$
\begin{aligned}
5b &\geq 550 \\
\frac{5b}{5} &\geq \frac{550}{5} \\
b &\geq 110
\end{aligned}
$$

The answer is $b \geq 110$.

You need to sell at least 110 boxes of cards.

Example 2

Solve for n:

$7n$[U+0088][U+0092]$8n$[U+0088][U+0092]$3 > 23$.

First, subtract $7n$[U+0088][U+0092]$8n$ because $7n$ and $8n$ are like terms.

$$
\begin{aligned}
7n\text{[U+0088][U+0092]}8n\text{[U+0088][U+0092]}3 &> 23 \\
7n + (\text{[U+0088][U+0092]}8n)\text{[U+0088][U+0092]}3 &< 23 \\
-n - 3 &< 23
\end{aligned}
$$

Next, isolate the term with the variable, $-n$, on one side of the inequality. Since 3 is subtracted from $-n$, you should add 3 to both sides of the inequality.

$$
\begin{aligned}
-n - 3 &< 23 \\
-n - 3 + 3 &< 23 + 3 \\
-n &< 26
\end{aligned}
$$

Then, since $-n$ means $-1n$ or $-1 \times n$, you can divide each side of the inequality by -1 to get a positive n by itself on one side of the equation. Since that involves dividing both sides of the inequality by a negative number, you must reverse the inequality symbol.

$$\begin{aligned} -n &< 26 \\ \tfrac{-n}{-1} &< \tfrac{26}{-1} \\ n &> -26 \end{aligned}$$

The answer is $n > -26$.

Example 3

Solve for x:

$4x + 5 < 21$

First, isolate the term with the variable, $4x$, on one side of the inequality. Since 5 is added to $4x$, you should subtract 5 from both sides of the inequality.

$$\begin{aligned} 4x + 5 &< 21 \\ 4x + 5 - 5 &< 21 - 5 \\ 4x &< 16 \end{aligned}$$

Next, since $4x$ means $4 \times x$, you can divide each side of the inequality by 4 to get 'x' by itself on one side of the inequality.

$$\begin{aligned} 4x &< 16 \\ \tfrac{4x}{4} &< \tfrac{16}{4} \\ x &< 4 \end{aligned}$$

The answer is $x < 4$.

Example 4

Solve for x:

$3x - 6 > 30$

First, isolate the term with the variable, $3x$, on one side of the inequality. Since 6 is subtracted from $3x$, you should add 6 to both sides of the inequality.

$$\begin{aligned} 3x - 6 &> 30 \\ 3x - 6 + 6 &> 30 + 6 \\ 3x &> 36 \end{aligned}$$

Next, since $3x$ means $3 \times x$, you can divide each side of the inequality by 3 to get 'x' by itself on one side of the inequality.

$$\begin{aligned} 3x &> 36 \\ \tfrac{3x}{3} &> \tfrac{36}{3} \\ x &> 12 \end{aligned}$$

$.x > 12$The answer is

Example 5

Solve for a:

$-3a + 2 < 14$

First, isolate the term with the variable, $-3a$, on one side of the inequality. Since 2 is added to $-3a$, you should subtract 2 from both sides of the inequality.

$$
\begin{aligned}
-3a + 2 &< 14 \\
-3a + 2 - 2 &< 14 - 2 \\
-3a &< 12
\end{aligned}
$$

Next, since $-3a$ means $-3 \cdot a$, you can divide each side of the inequality by -3 to get 'a' by itself on one side of the equation. Remember to reverse the inequality sign since you are dividing by a negative number.

$$
\begin{aligned}
-3a &< 12 \\
\frac{-3a}{-3} &< \frac{12}{-3} \\
a &> -4
\end{aligned}
$$

The answer is $a > -4$.

PRACTICE

Solve each inequality.

1. $2x + 5 > 13$

2. $4x - 2 < 10$

3. $6y + 9 > 69$

4. $2x - 3 \leq -4$

5. $5x + 2 \geq -8$

6. $2x - 9 \leq -5$

7. $\frac{x}{3} + 1 > 5$

8. $\frac{x}{2} - 1 < 3$

9. $\frac{x}{5} + 3 > -9$

10. $\frac{x}{2} - 5 > -10$

11. $6k - 3 > 15$

12. $11 + \frac{x}{4} \leq 12$

13. $12 + 9j + j < 72$

14. $12b - 3b + 5 \geq -31$

15. $18 + 7n + 3 + 6n \leq 86$

16. $3z - 15z - 30 > 54$

Review (Answers)

To see the Review answers, open this PDF file and look for section 3.16.

3.4 Model and Solve One-Variable Equations with Variables on Both Sides (7.11a, 8.8a)

GUIDANCE

Thomas has $50 and Jack has $100. Thomas is saving $10 per week for his new bike. Jack is saving $5 a week for his new bike. Can you represent this situation with an equation? How long will it be before the two boys have the same amount of money?

Watch This

SOLVING EQUATION WITH
VARIABLES ON
BOTH SIDES
KHANACADEMY

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/58475
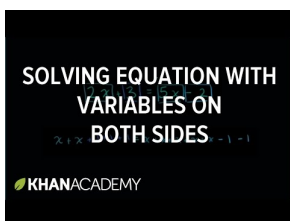
The methods used for solving equations with variables on both sides of the equation are the same as the methods used to solve equations with variables on one side of the equation. What differs is that there is the added step of combining like terms with the variables before isolating the variable to find the solution.

Combining like terms means that you are putting all of the variables that match on the same side of the equation. A like term is one in which the degrees match and the variables match. So, for example, $3x$ and $4x$ are like terms, $3x$ and $4z$ are not. Three apples and four apples are like terms, three apples and four oranges are not.

Remember that your goal for solving any equation is to get the variables on one side and the constants on the other side. You do this by adding and subtracting terms from both sides of the equals sign. Then you isolate the variables by multiplying or dividing. You must remember in these problems, as with any equation, whatever operation (addition, subtraction, multiplication, or division) you do to one side of the equals sign, you must do to the other side. This is a big rule to remember in order for equations to remain equal or to remain in balance.

Example A

$x + 4 = 2x - 6$

We will solve this problem using the balance method.

$$x + 4 \qquad\qquad 2x - 6$$

You could first try to get the variables all on one side of the equation. You do this by subtracting x from both sides of the equation.

$$x - x + 4 \qquad\qquad 2x - x - 6$$

Next, isolate the x variable by adding 6 to both sides.

$$4 + 6 \qquad\qquad x - 6 + 6$$

$$10 \qquad\qquad x$$

Therefore $x = 10$.

Check:
$$x + 4 = 2x - 6$$
$$(10) + 4 = 2(10) - 6$$
$$14 = 20 - 6$$
$$14 = 14$$

Example B

$14 - 3y = 4y$

We will solve this problem using algebra tiles.
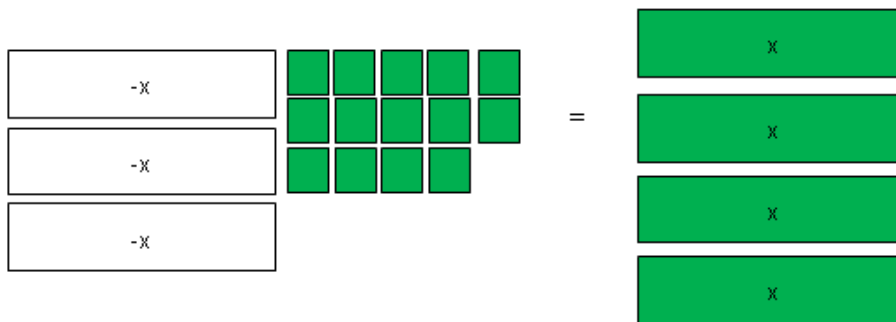
We first have to combine our variables (x) tiles onto the same side of the equation. We do this by adding $3x$ tiles to both sides of the equals sign. In this way the $-3y$ will be eliminated from the left hand side of the equation.
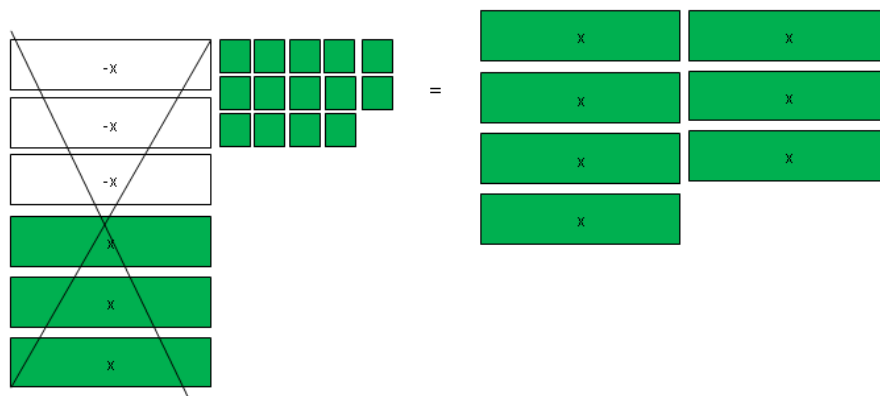
By isolating the variable (y) we are left with these algebra tiles.
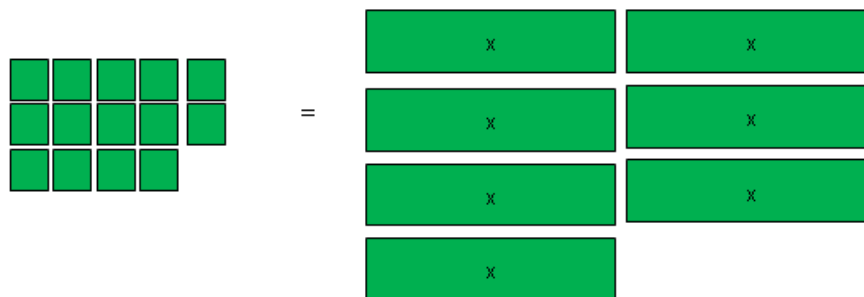
Rearranging we will get the following. {Note: Remember that rearranging is not necessary, it simply allows you to quickly see what the value for the variable is.}
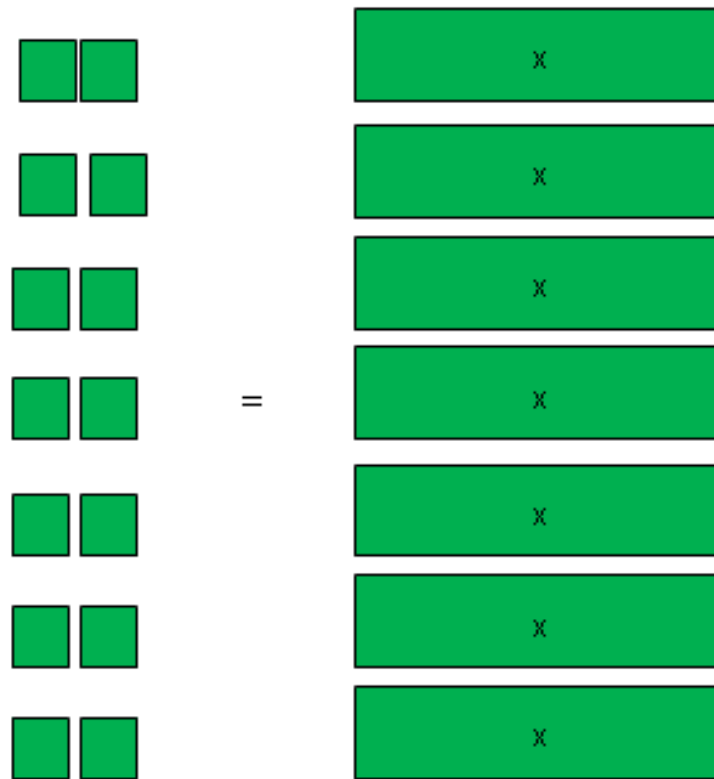
Check:

$$14 - 3y = 4y$$
$$14 - 3(2) = 4(2)$$
$$14 - 6 = 8$$
$$8 = 8$$

Therefore $y = 2$.

Example C

We can use these same methods for any of the equations involving variables. Sometimes, however, numbers are so large that one method is more valuable than the other. Let's look at the following problem.

$53a - 99 = 42a$

To solve this problem, we would need to have a large number of algebra tiles! It might be more efficient to use the balance method to solve this problem.

$$53a - 99 \qquad\qquad 42a$$

$$53a - 42a - 99 \qquad\qquad 42a - 42a$$

$$11a - 99 + 99 \qquad\qquad 0 + 99$$

$$11a \qquad\qquad 99$$

$$\frac{11a}{11} \qquad\qquad \frac{99}{11}$$

$$a \qquad\qquad 9$$

Check:

$$53a - 99 = 42a$$
$$53(9) = 42(9)$$
$$477 - 99 = 378$$
$$378 = 378$$

Therefore, $a = 9$.

Concept Problem Revisited

Thomas has $50 and Jack has $100. Thomas is saving $10 per week for his new bike. Jack is saving $5 a week for his new bike.

If we let w be the number of weeks, we can write the following equation.

$$\underbrace{10x + 50}_{\text{Thomas's money: \$10 per week+\$50}} = \underbrace{5x + 100}_{\text{Jack's money: \$5 per week+\$100}}$$

We can solve the equation now by first combining like terms.

$$10x + 50 = 5x + 100$$
$$10x - 5x + 50 = 5x - 5x + 100 \quad \text{-combining the } x \text{ variables to left side of the equation}$$
$$5x + 50 - 50 = 100 - 50 \quad \text{-combining the constants to left side of the equation}$$
$$5x = 50$$

We can now solve for x to find the number of weeks until the boys have the same amount of money.

$$5x = 50$$
$$\frac{5x}{5} = \frac{50}{5}$$
$$x = 10$$

Therefore, in 10 weeks Jack and Thomas will each have the same amount of money.

GUIDED PRACTICE

1. Use algebra tiles to solve for the variable in the problem $6x + 4 = 5x - 5$.

2. Use the balance (seesaw) method to solve for the variable in the problem $7r - 4 = 3 + 8r$.

3. Determine the most efficient method to solve for the variable in the problem $10b - 22 = 29 - 7b$. Explain your choice of method for solving this problem.

Answers:

1. $6x + 4 = 5x - 5$

x
x
x
x
x
x
-x
-x
-x
-x
-x

=

x
x
x
x
x

Here you are subtracting 5x from both sides of the equation

x

=

Here you are subtracting 4 from both sides of the equation

x

=
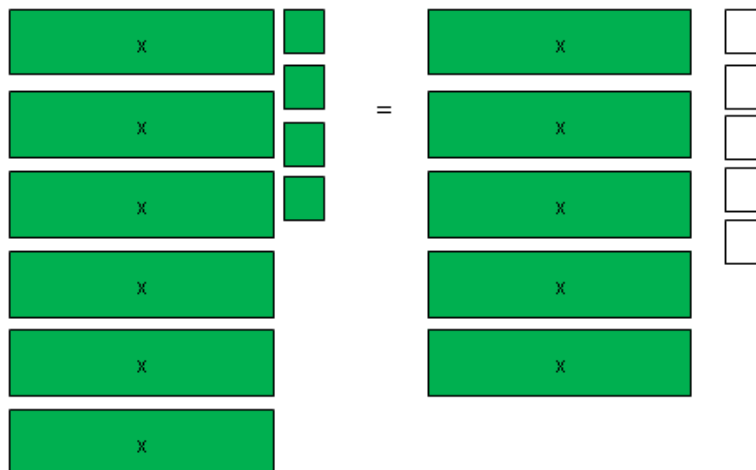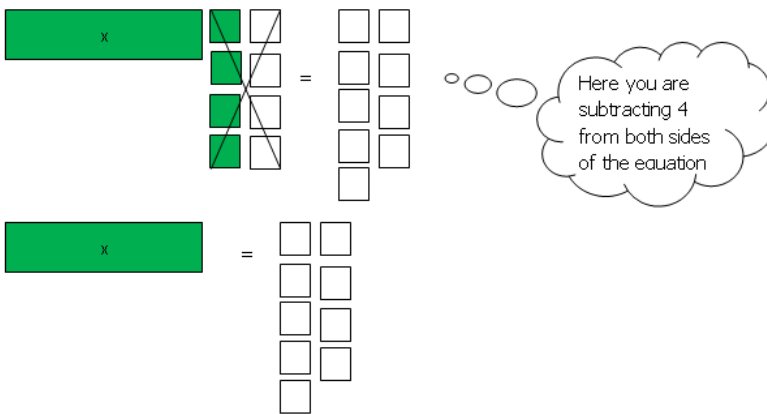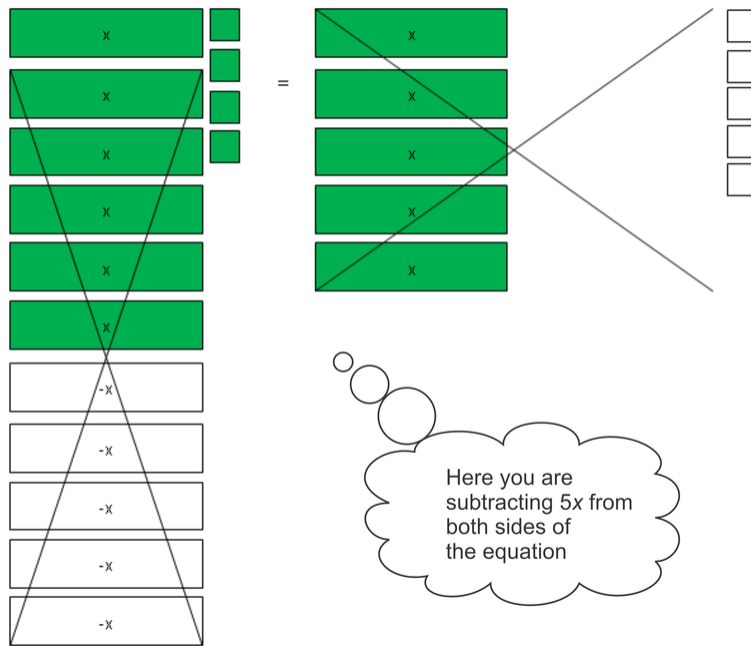
Therefore $x = -9$.

Check:
$$6x + 4 = 5x - 5$$
$$6(-9) + 4 = 5(-9) - 5$$
$$-54 + 4 = -45 - 5$$
$$-50 = -50$$

2. $7r - 4 = 3 + 8r$

$7r - 4$ $3 + 8r$

You can begin by combining the *r* terms. Subtract 8*r* from both sides of the equation.

$$7r - 8r - 4 \qquad\qquad 3 + 8r - 8r$$

$$-r - 4 \qquad\qquad 3$$

You next have to isolate the variable. To do this, add 4 to both sides of the equation.

$$-r - 4 + 4 \qquad\qquad 3 + 4$$

$$-r \qquad\qquad 7$$

But there is still a negative sign with the *r* term. You now have to divide both sides by -1 to finally isolate the variable.

$$\frac{-r}{-1} \qquad\qquad \frac{7}{-1}$$

$$r \qquad\qquad -7$$

Therefore $r = -7$.

$$\text{Check:}$$
$$7r - 4 = 3 + 8r$$
$$7(-7) - 4 = 3 + 8(-7)$$
$$-49 - 4 = 3 - 56$$
$$-53 = -53$$

3. $10b - 22 = 29 - 7b$

You could choose either method but there are larger numbers in this equation. With larger numbers, the use of algebra tiles is not an efficient manipulative. You should solve the problem using the balance (or seesaw) method. Work through the steps to see if you can follow them.

$$10b - 22 \qquad\qquad 29 - 7b$$

$$10b + 7b - 22 \qquad\qquad 29 - 7b + 7b$$

$$17b - 22 \qquad\qquad 29$$

$$17b - 22 + 22 \qquad\qquad 29 + 22$$

$$17b \qquad\qquad 51$$

$$\frac{17b}{17} \qquad\qquad \frac{51}{17}$$

$$b \qquad\qquad 3$$
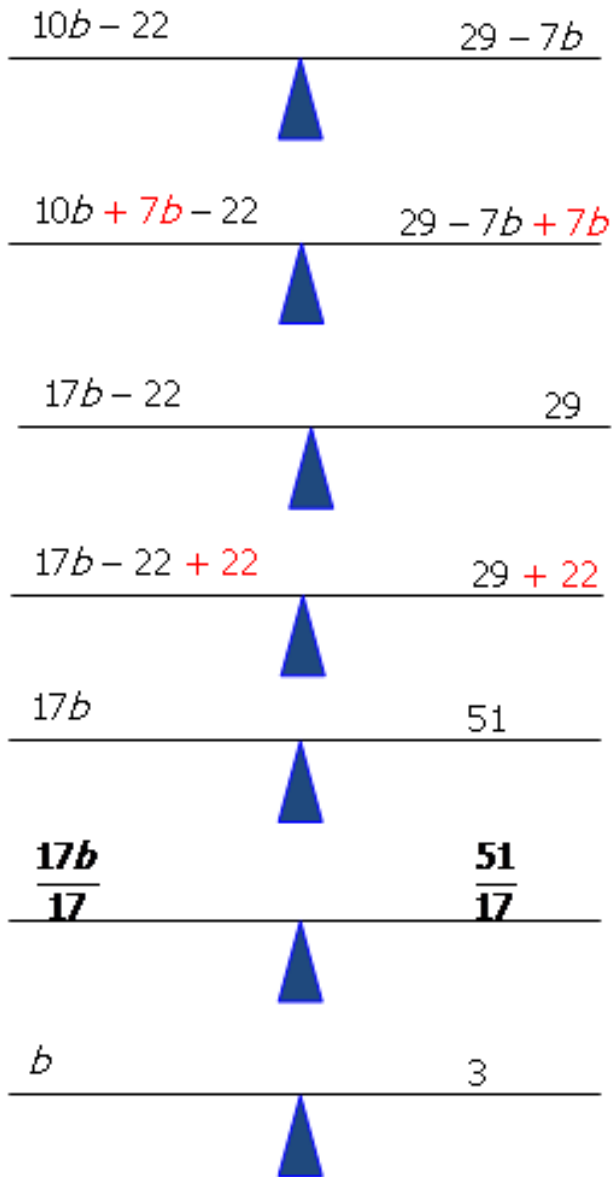
Therefore $b = 17$.

Check:

$$10b - 22 = 29 - 7b$$
$$10(3) - 22 = 29 - 7(3)$$
$$30 - 22 = 29 - 21$$
$$8 = 8$$

Practice

Use the balance method to find the solution for the variable in each of the following problems.

1. $5p + 3 = -3p - 5$
2. $6b - 13 = 2b + 3$
3. $2x - 5 = x + 6$
4. $3x - 2x = -4x + 4$
5. $4t - 5t + 9 = 5t - 9$

Use algebra tiles to find the solution for the variable in each of the following problems.

6. $6 - 2d = 15 - d$
7. $8 - s = s - 6$
8. $5x + 5 = 2x - 7$
9. $3x - 2x = -4x + 4$
10. $8 + t = 2t + 2$

Use the methods that you have learned for solving equations with variables on both sides to solve for the variables in each of the following problems. Remember to choose an efficient method to solve for the variable.

11. $4p - 7 = 21 - 3p$
12. $75 - 6x = 4x - 15$
13. $3t + 7 = 15 - t$
14. $5 + h = 11 - 2x$
15. $9 - 2e = 3 - e$

For each of the following models, write a problem to represent the model and then find the variable for the problem.

16.

17.

18.

19.

20.
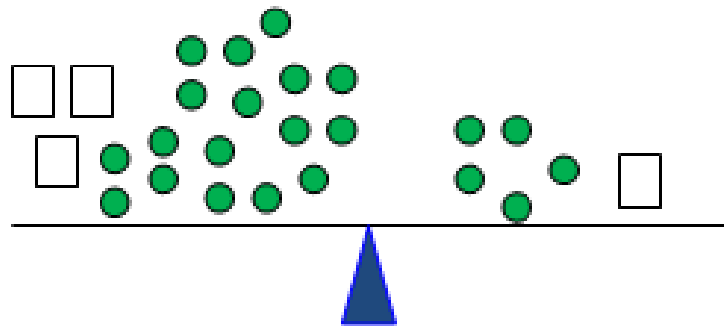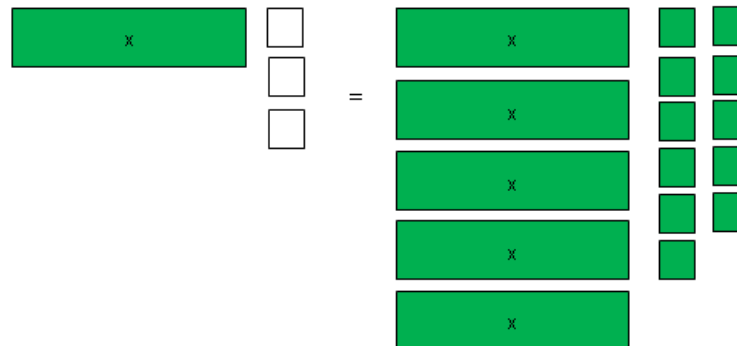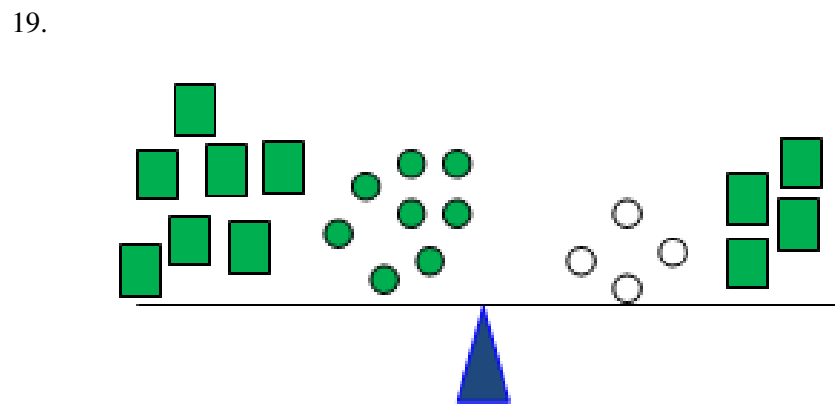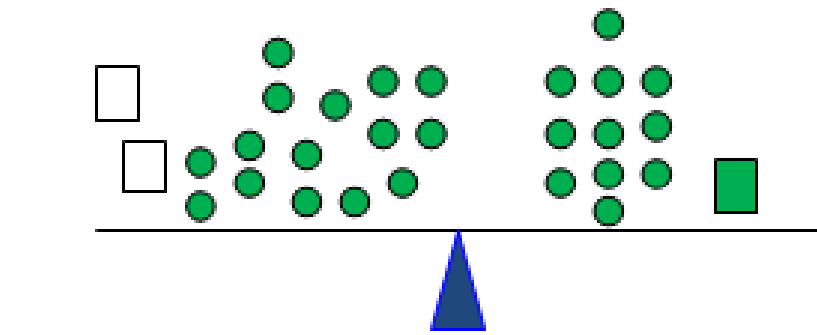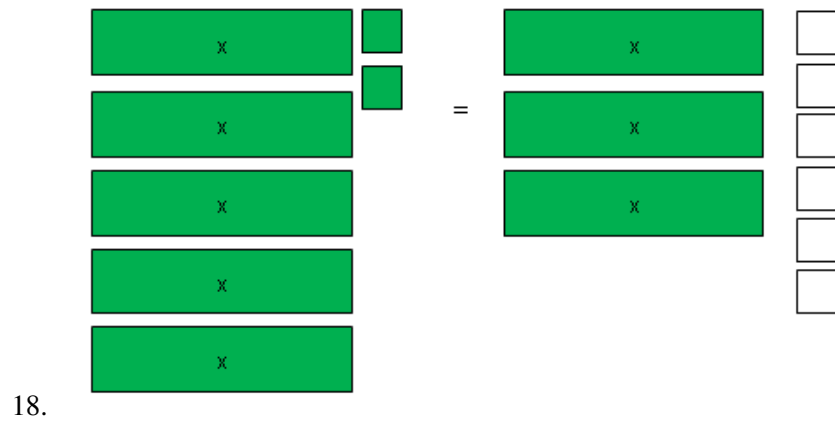
3.5 Model and Solve Equations with Geometry Concepts (7.11c)

GUIDANCE

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Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/187728
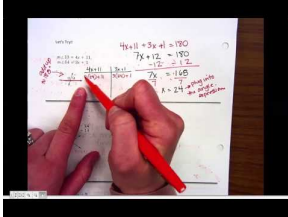
When solving equations recall that what action you do to one side of an equation must be done to the other side also. This process uses the Properties of Equality. The Addition Property of Equality states we can add any number we choose if we add it to both sides. The Subtraction Property of Equality states that we can subtract any number we choose if we subtract it from both sides. The Multiplication Property of Equality states we can multiply both sides by any non-zero number we choose if we multiply both sides. The Division Property of Equality states we can divide both sides by any non-zero number. The goal in solving an equation is to determine the value of the variable that will make the statement true. The variable must be isolated to do this. These Properties of Equality will help you accomplish that goal.

When a person looks around the room they are in, angles are everywhere. Angles are formed when two rays have a common endpoint. The angle below is angle *ABC*. *B* is the vertex of the angle. An angle can be named by three letters. The middle letter must be the vertex letter. The other two letters are letters from each ray. The angle can also be named by the vertex letter only IF no other angle shares that vertex. Also the angle can be named by a number that is indicated in the interior of the angle, such as angle 1.
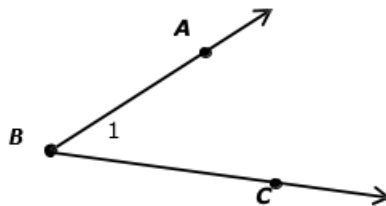
FIGURE 3.15

Many of the angles used in construction are right angles. A **right angle** has a measure of 90°. When two segments or lines are perpendicular, they form right angles when they intersect. Do you see any right angles in the classroom?

Angles that measure less than 90° are called **acute angles**. Do you see any acute angles in the classroom?

Angles that measure more than 90° are called **obtuse angles**. Do you see any obtuse angles in the classroom?

Bend your arm at the elbow to show an acute angle, then a right angle, and finally an obtuse angle.

Angles that measure the same are congruent angles. All right angles are congruent because they always measure 90°.   Are all acute angles congruent?   Why?   Are all obtuse angles congruent? Why?

When two angles share a side but do not overlap, they are called **adjacent angles**.   Angle 1 and 2 below are adjacent angles.
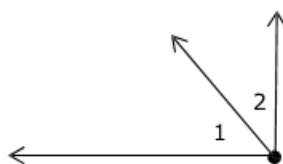
FIGURE 3.16

GUIDED PRACTICE

PRACTICE

If the two adjacent angles form a right angle, then the two angles are complementary angles. **Complementary angles** are two angles whose measures have a sum of 90°. The sides they do not share will be perpendicular to each other. Angles 1 and 2 below are complementary angles.
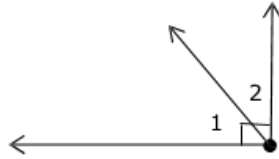
If the two adjacent angles form a line, then the two angles are supplementary angles. They are called a linear pair. **Supplementary angles** are two angles whose measures have a sum of 180°. Angles 1 and 2 in the sketch below are supplementary angles. They are a linear pair because they are adjacent and they form a line.
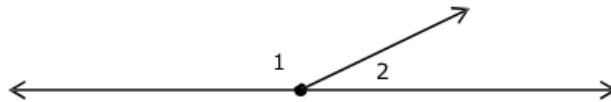
A linear pair will either be two right angles or one obtuse angle and one acute angle. Two acute angles cannot form a linear pair and two obtuse angles cannot form a linear pair. Why?

Supplementary angles do not have to be a linear pair. Any two angles whose measures have a sum of 180° are supplementary angles. For example, an angle that measures 145° and an angle that measures 35° are supplementary angles.

When two lines intersect, they form 4 angles. There are two pairs of angles that are not adjacent angles. These angles are called **vertical angles**. For example, in the sketch, angle 1 is adjacent to angle 2 and angle 4 but it is not adjacent to angle 3. The only thing angle 1 and angle 3 share is a vertex. Their sides form two lines. They are vertical angles.

FIGURE 3.17

Vertical angles always have equal measures. Angles 2 and 4 are also a vertical pair. They will measure the same. Any pair of adjacent angles in the sketch below is a linear pair. There are 4 pairs of linear pairs in the sketch. Can you name all 4 pairs?
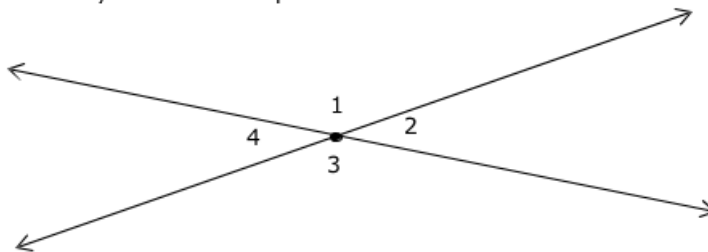
FIGURE 3.18
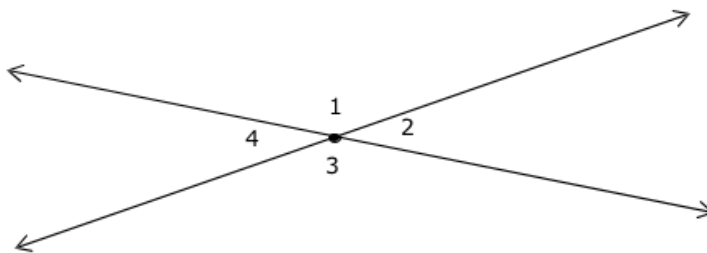
Example: If the measure of angle 1 is represented by $2x + 10$ and the measure of angle 2 is 40°, what is the value of x?

Angle 1 and angle 2 form a linear pair.   They are formed by intersecting lines and are adjacent.
$2x + 10 + 40 = 180$ Linear pairs of angles are supplementary and thus measures have a sum of 180.
$2x + 50 = 180$ Combined constants
$2x = 130$ Subtraction Property of Equality (subtracted 50 from both sides)
$x = 65$ Division Property of Equality (divided both sides by 2)

FIGURE 3.19

Angles of a Triangle

A triangle is a three-sided polygon. It also has three angles. A triangle can be classified by the type of angles it contains. An **acute triangle** contains three acute angles. A **right triangle** has one right angle and two acute angles. An **obtuse triangle** has one obtuse angle and 2 acute angles. An equiangular triangle has three congruent angles.

An **isosceles triangle** has two congruent sides and two congruent angles called the base angles. The third angle of an isosceles angle is called the vertex angle.

FIGURE 3.20

Look at the triangle below.
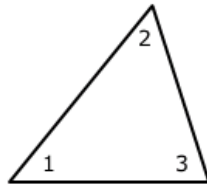
If we use scissors and cut out the triangle, then we can tear off the three angles and rearrange them so that their vertices lie at a point on a line.
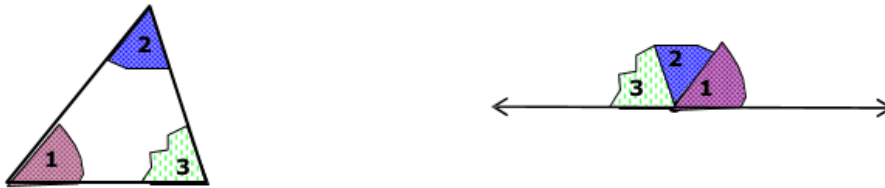
The three angles form a straight angle. This means they have a sum of 180°. For three angles to be the angles of a triangle, their measures must have a sum of 180°. This can be written as an equation.

$$m\angle 1 + m\angle 2 + m\angle 3 = 180°$$

FIGURE 3.21

Example: Find the measures of $\angle A$ and $\angle C$ in triangle ABC shown below.
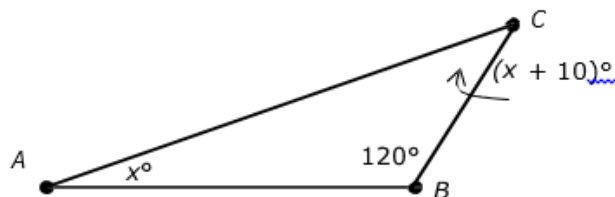
The sum of the measures of the angles of a triangle must have a sum of 180°.

Statement	Justification
$x + (x + 10) + 120 = 180$	Sum of the measures of the angles of a triangle
$x + x + 130 = 180$	Combined constants
$2x + 130 = 180$	Combined variables
$2x = 50$	Subtraction Property of Equality (subtracted 130)
$x = 25$	Division Property of Equality (divided by 2)

The measure of $\angle A$ is x° so 25°.   The measure of $\angle C$ is $(x + 10)$° so 35°.

Do the measures of the three angles of the triangles have a sum of 180°?

$25 + 35 + 120 = 60 + 120 = 180$.   Yes, the sum of the measures is 180°.

FIGURE 3.22

Example: Can the measures of the angles of a triangle be 40°, 70°, and 60°?

Check to see if the sum of the measures is 180°.

$40° + 70° + 60° = 110° + 60° = 170°$

No, they cannot be the measures of the angles of a triangle because they do not have a sum of 180°.

FIGURE 3.23

Perimeter of a Polygon

The perimeter of a figure is the length of the boundary of the figure. For a triangle it is the sum of the lengths of the three sides. For a rectangle, the perimeter is the sum of the two lengths and two widths.

Example: The perimeter of a triangle is 28 inches. The length of the three sides are 8 inches, $(x + 4)$ inches, and $(x + 2)$ inches. What is the value of x?

Write an equation representing the relationship among the length of the sides and the perimeter.

$8 + (x + 4) + (x + 2) = 28$

Solve the equation.

$8 + (x + 4) + (x + 2) = 28$

$14 + 2x = 28$	Combine like terms
$2x = 14$	Subtraction Property of Equality
$x = 7$	Division Property of Equality

The side lengths are 8 inches, 11 inches, and 9 inches.

FIGURE 3.24

Example: The perimeter of a rectangle is 30 inches. The length is $(x + 4)$ inches and the width is $(x + 2)$ inches. What is the value of x?

Write an equation representing the relationship among the length of the sides and the perimeter.

$(x + 4) + (x + 2) + (x + 4) + (x + 2) = 30$

Solve the equation.

$(x + 4) + (x + 2) + (x + 4) + (x + 2) = 30$

$12 + 4x = 30$	Combine Like Terms
$4x = 18$	Subtraction Property of Equality
$x = 4.5$	Division Property of Equality

The length is 8.5 inches and the width is 6.5 inches.

FIGURE 3.25

Use the sketch below to answer questions 1-5.
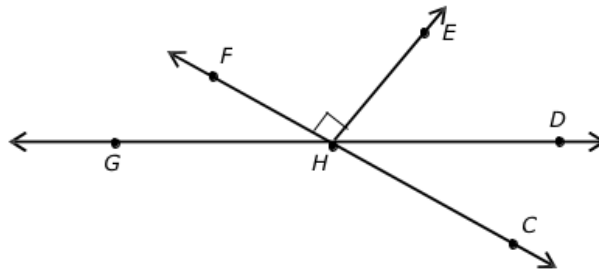
1. Name the angle that is the vertical angle with ∠ FHG. _____

    Name an angle that is adjacent to but does not form a linear pair with ∠FHG. _____

2. If the measure of ∠ FHG is 25°, what is the measure of ∠ EHD? Show work to support your answer.

FIGURE 3.26

---

3. Name a linear pair of angles in the sketch.   _____ and _____.

    What will be sum of their measures? _____

4. Name a right angle in the sketch other than ∠ FHE.

5. ∠ EHD and ∠_____complementary angles. The sum of their measures must be_____°.

    If the measure of ∠ EHD is 48°, what will be the measure of ∠ CHD

FIGURE 3.27

The measures of the angles of a triangle are given below. Write an equation to show the relationship among the measures. Solve the equation, showing all your work and justification of the steps. List the measure of the three angles of the triangle.

1. 35°, x°, and $(x + 15)$°

2. 135°, x°, and $(2x + 15)$°

3. 50°, $(2x - 10)$°, and $(3x + 15)$°

FIGURE 3.28

4. 80°, $(2x + 10)$°, and $(3x - 10)$°

5. 90°, $(x + 23)$°, and $(3x + 27)$°

FIGURE 3.29

The lengths of the sides of a triangle and its perimeter are given below. Write an equation to show the relationship among the measures. Solve the equation, showing all your work and justification of the steps. List the lengths of the three sides of the triangle.

1. 35 in., x in., and $(x + 15)$ in.; Perimeter: 86 in.

2. 14 cm, $3x$ cm, and $(2x + 15)$ cm; Perimeter: 54 cm

FIGURE 3.30

3. The opposite sides of a parallelogram are $(x + 12)$ units and 3x units. Write an equation to represent the relationship of these lengths. Solve the equation and determine the lengths of these two sides of the parallelogram,

    If the other two sides of the parallelogram are 8 units each, what is the perimeter of the parallelogram?

4. Opposite angles of a parallelogram are equal in measure. The figure shows a parallelogram and the measures of two opposite angles.

$(2x + 40)°$

$(3x - 10)°$
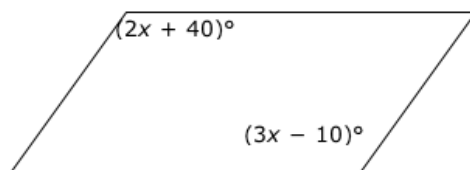
FIGURE 3.31

3.6 Solving Equations using Substitution (7.11b)

GUIDANCE

Your summer landscaping job pays a fixed rate of $20 per job plus $4 an hour. How much total money would you would make if it takes you 3 hours to complete a single job?

Evaluating Algebraic Expressions and Equations

You have probably seen letters in a mathematical expression, such as $3x - 8$. These letters, also called **variables,** represent an unknown number. One of the goals of algebra is to solve various equations for a variable. Typically, x is used to represent the unknown number, but any letter can be used.

To evaluate an expression or equation, we would need to substitute in a given value for the variable and test it. In order for the given value to be true for an equation, the two sides of the equation must simplify to the same number.

Evaluate the following expressions

Evaluate $2x^2 - 9$ for $x = -3$.

We know that $2x^2 - 9$ is an expression because it does not have an equals sign. Therefore, to evaluate this expression, plug in -3 for x and simplify using the Order of Operations.

$$2(-3)^2 - 9 \rightarrow (-3)^2 = -3 \cdot -3 = 9$$
$$2(9) - 9$$
$$18 - 9$$
$$9$$

You will need to remember that when squaring a negative number, the answer will always be positive. There are three different ways to write multiplication: $2 \times 9, 2 \cdot 9$, and $2(9)$.

Determine if $x = 5$ is a solution to $3x - 11 = 14$.

Even though the directions are different, this problem is almost identical to Example A. However, this is an equation because of the equals sign. Both sides of an equation must be equal to each other in order for it to be true. Plug in 5 everywhere there is an x. Then, determine if both sides are the same.

$$
\begin{aligned}
3(5) - 11 &\overset{?}{=} 14 \\
15 - 11 &\neq 14 \\
4 &\neq 14
\end{aligned}
$$

Because $4 \neq 14$, this is not a true equation. Therefore, 5 is not a solution.

Determine if $t = -2$ is a solution to $7t^2 - 9t - 10 = 36$.

Here, t is the variable and it is listed twice in this equation. Plug in -2 everywhere there is a t and simplify.

$$
\begin{aligned}
7(-2)^2 - 9(-2) - 10 &\overset{?}{=} 36 \\
7(4) + 18 - 10 &\overset{?}{=} 36 \\
28 + 18 - 10 &= 36 \\
36 &= 36
\end{aligned}
$$

-2 is a solution to this equation.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about a summer landscaping job.

Rewrite the sentence as an algebraic expression. \$20 plus \$4 an hour, would be $20 + 4h$, where h equals the number of hours you work. Then, evaluate the expression for $h = 3$.

$20 + 4(3) = 20 + 12 = 32$

You will make a total of \$32 for this particular job.

Example 2

Evaluate $s^3 - 5s + 6$ for $s = 4$.

Plug in 4 everywhere there is an s.

$$
\begin{aligned}
4^3 &- 5(4) + 6 \\
64 &- 20 + 6 \\
&50
\end{aligned}
$$

Example 3

Determine if $a = -1$ is a solution to $4a - a^2 + 11 = -2 - 2a$.

Plug in -1 for a and see if both sides of the equation are the same.

$$4(-1) - (-1)^2 + 11 \overset{?}{=} -2 - 2(-1)$$
$$-4 - 1 + 11 \overset{?}{=} -2 + 2$$
$$6 \neq 0$$

Because the two sides are not equal, -1 is not a solution to this equation.

PRACTICE

Evaluate the following expressions for $x = 5$.

1. $4x - 11$
2. $x^2 + 8$
3. $\frac{1}{2}x + 1$

Evaluate the following expressions for the given value.

4. $-2a + 7; a = -1$
5. $3t^2 - 4t + 5; t = 4$
6. $\frac{2}{3}c - 7; c = -9$
7. $x^2 - 5x + 6; x = 3$
8. $8p^2 - 3p - 15; p = -2$
9. $m^3 - 1; m = 1$

Determine if the given values are solutions to the equations below.

10. $x^2 - 5x + 4 = 0; x = 4$
11. $y^3 - 7 = y + 3; x = 2$
12. $7x - 3 = 4; x = 1$
13. $6z + z - 5 = 2z + 12; z = -3$
14. $2b - 5b^2 + 1 = b^2; b = 6$
15. $-\frac{1}{4}g + 9 = g + 15; g = -8$

Find the value of each expression, given that $a = -1, b = 2, c = -4$, and $d = 0$.

16. $ab - c$
17. $b^2 + 2d$
18. $c + \frac{1}{2}b - a$
19. $b(a + c) - d^2$

For problems 20-25, use the equation $y^2 + y - 12 = 0$.

20. Is $y = 4$ a solution to this equation?
21. Is $y = -4$ a solution to this equation?
22. Is $y = 3$ a solution to this equation?
23. Is $y = -3$ a solution to this equation?
24. Do you think there are any other solutions to this equation, other than the ones found above?
25. **Challenge** Using the solutions you found from problems 20-23, find the sum of these solutions and their product. What do you notice?

3.7 Solving Inequalities Using Substitution (7.11b)

GUIDANCE

What if you were given an inequality like $-3x^3 < -81$ and told that one of its solutions was $x > 3$? How could you determine if that solution were correct? After completing this Concept, you'll be able to check the solutions to inequations like this one.

Watch This
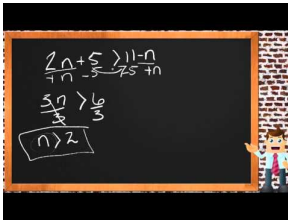
To check the solution to an inequality, replace the variable in the inequality with the value of the solution. If the solution is correct, the simplified inequality will produce a true statement.

Example A

Check that the given number is a solution to the inequality: $a = 10$; $20a \leq 250$

Solution

Replace the variable in the inequality with the given value.

$$\leq 25020(10)$$

$200 \leq 250$ **is a true statement**. This means that $a = 10$ is a solution to the inequality $20a \leq 250$.

Note that $a = 10$ is not the only solution to this inequality. Recall that inequalities may be simplified using essentially the same processes as equations. If we divide both sides of the inequality by 20, we can write it as $a \le 12.5$. This means that actually any number less than or equal to 12.5 is a solution to the inequality.

Example B

Check that the given number is a solution to the inequality: $b = -0.5;\ \frac{3-b}{b} > -4$

Solution:

$$> -4 \frac{3-(-0.5)}{(-0.5)}$$

This statement is false. This means that $b = -0.5$ is not a solution to the inequality $\frac{3-b}{b} > -4$.

Example C

To organize a picnic, Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs. What is the possible number of hamburgers Peter has?

Solution:

Define

Let x represent the number of hamburgers

Translate

Peter needs at least two times as many hamburgers as hot dogs. He has 24 hot dogs.

This means that twice the number of hot dogs is less than or equal to the number of hamburgers.

$$2 \times 24 \le x,\ \text{or}\ 48 \le x$$

Answer

Peter needs at least 48 hamburgers.

Check

48 hamburgers is twice the number of hot dogs. More than 48 hamburgers is more than twice the number of hot dogs. **The answer checks out**.

Watch this video for help with the Examples above.

MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/133261
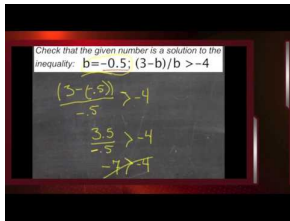
GUIDED PRACTICE

Check that the given number is a solution to the inequality: $x = \frac{3}{4}$; $4x + 5 \leq 8$

Solution:

$$\leq 84\left(\frac{3}{4}\right) + 5$$

The statement $8 \geq 8$ **is true** because this inequality includes an equals sign, and since 8 is equal to itself it is also "greater than or equal to" itself. This means that $x = \frac{3}{4}$ is a solution to the inequality $4x + 5 \leq 8$.

PRACTICE

For 1-4, check whether the given number is a solution to the corresponding inequality.

1. $x = 12$; $2(x + 6) \leq 8x$
2. $z = -9$; $1.4z + 5.2 > 0.4z$
3. $y = 40$; $-\frac{5}{2}y + \frac{1}{2} < -18$
4. $t = 0.4$; $80 \geq 10(3t + 2)$
5. On your new job you can be paid in one of two ways. You can either be paid $1000 per month plus 6% commission of total sales or be paid $1200 per month plus 5% commission on sales over $2000. For what amount of sales is the first option better than the second option? Assume there are always sales over $2000.

For 6-14, suppose a phone company offers a choice of three text-messaging plans. Plan A gives you unlimited text messages for $10 a month; Plan B gives you 60 text messages for $5 a month and then charges you $0.05 for each additional message; and Plan C has no monthly fee but charges you $0.10 per message.

6. If *m* is the number of messages you send per month, write an expression for the monthly cost of each of the three plans.
7. For what values of *m* is Plan A cheaper than Plan B?
8. For what values of *m* is Plan A cheaper than Plan C?
9. For what values of *m* is Plan B cheaper than Plan C?

10. For what values of *m* is Plan A the cheapest of all? (Hint: for what values is A both cheaper than B and cheaper than C?)
11. For what values of *m* is Plan B the cheapest of all? (Careful—for what values is B cheaper than A?)
12. For what values of *m* is Plan C the cheapest of all?
13. If you send 30 messages per month, which plan is cheapest?
14. What is the cost of each of the three plans if you send 30 messages per month?

3.8 Writing Equations Given Situations – addition and subtraction (7.10a)

ID

GUID

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Here you'll learn to write addition and subtraction phrases as single variable equations.

GUIDANCE

Have you ever been to Boston? Have you ever spent the summer with your grandparents?

Kara and her twin brother Marc are going to be spending one month in Boston with their grandparents. They are very excited! Not only is it summer in Boston and the weather will be terrific, but Boston has a fantastic subway system called the T and they will be able to ride it to get all around.

Kara is very excited about visiting the museums, but Marc who is a huge baseball fan is hoping for a trip to Fenway Park before the month is up. Seeing the Red Sox play would be a huge bonus!

Kara has earned $125.00 babysitting and has some birthday money too.

Before the trip, she counted her savings and realized that she has $54.25.

You can write an equation to show the unknown in this dilemma.

This Concept will teach you how to write addition and subtraction phrases as single variable equations.

An *expression* shows how numbers and/or *variables* are connected by operations, such as addition, subtraction, multiplication, and division. Notice that an expression does not have an equal sign. This is because the value of the variable in each expression can change, or you could say that we can evaluate an expression using different given values for the variable.

$$50 - 2 \qquad\qquad 4 - a \qquad\qquad 12z \qquad\qquad \frac{4x}{3}$$

Three of the expressions above include variables, such as a, z, and x.

An expression that includes one or more variables is called an *algebraic expression*. Each variable in an algebraic expression can have a different value. Once again, you will not see an equal sign with an algebraic expression. We can use algebraic expressions to represent words or phrases.

Often in word problems or in other situations in math, you will be given a set of words or a phrase that you will need to rewrite as an expression. When you do this, you will be looking for words that mean different operations or things in math. This way you can write an expression that correctly represents the words or phrase.

We are going to start with addition and subtraction phrases. Take a look at this chart.

TABLE 3.1:

Addition Expressions		Subtraction Expressions	
1 **plus** a	$1 + a$	4 **less** d	$4 - d$
2 **and** b	$2 + b$	6 **less than** g	$g - 6$
3 **more than** c	$3 + c$	h**fewer than** 7	$7 - h$

The bolded key words in the phrases above provide clues about whether or not you should write an addition or a subtraction expression. While key words can be a helpful guide, it is important not to rely on them totally. It is always most important to think about which operation makes the most sense for a particular situation.
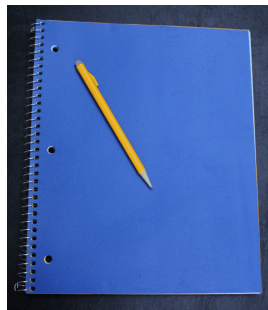
Write these key words in your notebook and then continue with the Concept.

Abdul has \$5 more than Xavier has. Let x stand for the number of dollars Xavier has. Write an algebraic expression to show the number of dollars Abdul has.

The phrase is "\$5 more than Xavier." Use a number, an operation sign, or a variable to represent each part of that phrase.

$$\$\underline{5} \ \underline{more \ than} \ \underline{Xavier}$$
$$\downarrow \qquad \downarrow \qquad \downarrow$$
$$5 \qquad + \qquad x$$

Notice that in this phrase, the key words "more than" means you should write an addition expression.

So, the expression $5 + x$ represents the number of dollars Abdul has. We also could have written this as. $x + 5$ because addition is commutative. That means, the order in which numbers are added does not matter.

Our answer is $x+5$**.**

Here is another one.

6 less than a number

Notice the key words "less than" which means subtraction.

A number means a variable.

Because it says "6 less than a number," the 6 will follow the variable.

$x-6$

Our answer is $x-6$**.**

Remember, sometimes writing an expression is not as simple as relying on key words.

Lian is x inches shorter than Hannah. Hannah is 65 inches tall. Write an algebraic expression to show Lian's height in inches.

The phrase is "x **inches shorter than Hannah."** You also know that Hannah's height is 65 inches.

There are no key words, so you need to think about what makes sense. If Lian is shorter than Hannah, her height will be *less than* **65 inches.** So, write a subtraction expression. Use 65 to represent Hannah's height. Since Lian's height is less than Hannah's height, you will need to subtract x inches from Hannah's height to represent Lian's height.

<div align="center">

x inches <u>shorter than</u> <u>Hannah</u>.

☐   ↓   ☐

☐   ↓   ☐

☐   ↓   ☐

65   −   *x*

</div>

The answer is $65-x$**.**

If you knew what any of these expressions were equal to, then you could use an equals sign and write an equation. Take a look at these examples.

Write an equation for each phrase.

Example A

A number plus five is ten

Solution: $x+5=10$

Example B

Six more than a number is eighteen.

Solution: $x+6=18$

Example C

Fifteen less than a number is twenty

Solution: $x - 15 = 20$

Here is the original problem once again.

Kara and her twin brother Marc are going to be spending one month in Boston with their grandparents. They are very excited! Not only is it summer in Boston and the weather will be terrific, but Boston has a fantastic subway system called the T and they will be able to ride it to get all around.

Kara is very excited about visiting the museums, but Marc who is a huge baseball fan is hoping for a trip to Fenway Park before the month is up. Seeing the Red Sox play would be a huge bonus!

Kara has earned $125.00 babysitting and has some birthday money too.

Before the trip, she counted her savings and realized that she has $54.25.

You can write an equation to show the unknown in this dilemma.

To write an equation for this dilemma, we can include what we know and what we don't know.

We know that Kara earned $125.00 babysitting.

We don't know how much money she received from her birthday. This is our unknown.

We do know the total amount of money.

Here is our equation.

$125 + x = 54.25$

This is our answer.

Vocabulary

Expression

a number sentence with variables, numbers and operations.

Variable

a letter used to represent an unknown quantity.

Algebraic Expression

a combination of multiple variables, numbers and operations.

Equation

a number sentence with an equal sign where the quantity on one side of the equals is the same as the quantity on the other side.

GUIDED PRACTICE

Here is one for you to try on your own.

Write an equation to represent: Four less than an unknown number is eighteen.

Answer

To figure this out, we can first identify the operation as subtraction because of the key words less than.

Next, notice that the statement is four less than, so the four is being taken away from the unknown number.

Now we can write the equation.

$x - 4 = 18$

This is our answer.

Video Review
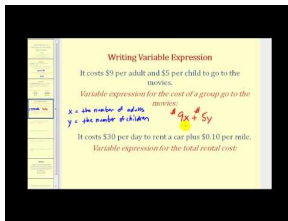
PRACTICE

Directions: Write an expression for each phrase.

1. 5 more than a number

2. A number plus six

3. 8 and a number

4. Seven less than a number

5. Eight take a way four

6. Nine more than a number

Directions: Write a simple equation for each phrase.

7. Five less than a number is ten.

8. Eight take away four is a number.

9. Five and a number is twelve.

10. Sixteen less than an unknown number is eighty.

11. Twenty and a number is fifty - five.

12. A number and fifteen is forty.

13. A number and twelve is sixty.

14. Fifteen less than a number is ninety.

15. Sixty less than a number is eighty.

3.9 Writing Equations Given Situations – Multiplication and Division (7.10a)

In this concept, you will learn how to write multiplication and division phrases as single variable equations.

There is a bunny population behind Ishmael's house. Ishmael is helping the local park ranger track the population numbers. At the start of July there are 127 females. By the end of the month, Ishmael and the ranger count 415 new bunnies. What is the average number of bunnies that each female had during that time? How can Ishmael write an equation that shows this situation?

In this concept, you will learn how to write multiplication and division phrases as single variable equations.

GUIDANCE

Just as you can write addition and subtraction expressions from words or phrases, you can also write multiplication and division expressions. You can use key words to help you with this. The more familiar you become with the key words that identify a multiplication or division expression, the better you will become at writing expressions.

Here are some phrases that can be translated into multiplication or division expressions.

TABLE 3.2:

Multiplication Phrases		Division Phrases	
9 **times** k	$9 \times k$ or $9k$	8 **divided into** n groups	$8 \div n$ or $\frac{8}{n}$
twice as much as m	$2 \times m$ or $2m$	q **shared equally** by 3 people	$q \div 3$ or $\frac{q}{3}$
		half of r	$r \div 2$ or $\frac{r}{2}$
		one-third of p	$p \div 3$ or $\frac{p}{3}$

Remember, these words are only a helpful guide. You should always think about which operation makes sense for a

particular situation.

Let's look at an example.

Write an algebraic expression to represent the phrase "three times a number, t."

Use a number, an operation sign, or a variable to represent each part of the phrase.

First, consider the phrase and look for key terms.

three times a number, t

Next, rewrite the phrase using numbers and operations.

$3 \times t$

Then, simplify the expression.

$3t$

The phrase is represented by the expression $3t$.

Here is another example.

Mr. Warren separated 30 students into n equal groups. Write an algebraic expression to represent the number of students in each group.

First, consider the phrase and look for key terms.

30 students into n equal groups

Separating 30 students into n equal groups means dividing 30 students into n equal groups.

Next, write a division expression.

$\frac{30}{n}$

The phrase is represented by the expression $\frac{30}{n}$.

When an expression equals something it is an equation. If the number of students in each group needed to be five, for instance, you could write the following equation.

$n = 5$

Then, you could evaluate the expression $\frac{30}{n}$ using the given variable 5.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Ishmael's local bunnies.

There are 127 females and 415 bunnies. Ishmael needs to write an equation that represents the average number of bunnies each female had.

First, let n be the average number of bunnies per female.

$127n$

This is the total number of bunnies in the population.

Then, because you know there are 415 bunnies, you can write an equation.

$127n = 415$

Example 2

Write a single variable equation for the following phrase.

Keith bought tickets to the movies. The tickets were $8.50 each. Keith spent a total of $34.00. How many tickets did Keith buy?

First, let t represent the number of tickets that Keith bought.

The cost of each ticket is $8.50. The total price of the tickets will be the number of tickets, represented by t, times the price of each ticket.

Next, write an expression that represents this.

$8.5t$

Then, because you know the total cost of the tickets was $34.00, write an equation.

$8.5t = 34.00$

Write a multiplication or division equation for the following phrases.

Example 3

Four times a number is eight.

Let x be "a number."

"Times" means multiplication and "is" means equals.

The answer is $4x = 8$.

Example 4

Sixteen candles divided into a number of piles is two candles in each pile. Let x be the number of piles.

"Divided into" means division but be careful about the order, and "is" means equals.

The answer is $\frac{16}{2} = 2$.

Example 5

The product of five and a number is fifteen.

Let x be "a number."

"Product" means you are multiplying, and "is" means equals.

The answer is $5x = 15$.

PRACTICE

Write an equation for each phrase.

1. The product of four and a number is twelve.
2. Six times a number is thirty.
3. Twelve times a number is forty-eight.
4. Fourteen times a number is twenty-eight.
5. The product of five and a number is thirty.

6. Eight times a number is sixty-four.
7. Twenty divided by a number is four.
8. Eighty divided by a number is four.
9. Nineteen times an unknown number is ninety-five.
10. Thirteen times an unknown number is thirty-nine.
11. Twelve divided into groups is six.
12. An unknown number divided by two is eight.
13. An unknown number divided by seven is fourteen.
14. An unknown number times five is thirty-five.
15. An unknown number divided by twelve is twelve.

Review (Answers)

To see the Review answers, open this PDF file and look for section 7.2.

Resources

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/182118
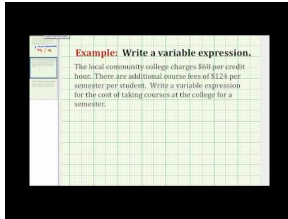
3.10 Writing Inequalities Given a Situation (7.10a)

GUIDANCE

Marc and Kara have made several friends while swimming at the town pool. One rainy day, they decide to invite the group to go to a movie. Everyone is very excited. Grandma is so pleased that Kara and Marc have been making friends that she offers to treat.

"How many of you are going?" she asks Kara.

"Well, there are eight of us altogether," Kara said.

"Alright, let me see," Grandma says while reaching into her wallet. "I have forty-eight dollars to contribute. Here it is."

"Thank you Grandma," the twins say, smiling.

Later at the movie theater, Kara takes out the money. They have $48.00 to spend on tickets. Last minute, one of the friends has brought her brother along. Kara isn't sure that they have enough money for the brother too. If each ticket is $6.00, how many tickets can she buy without going over the $48.00?

This problem requires an inequality. The total must be less than or equal to $48.00 without going over. The movie is going to start soon. Pay attention to the work in this lesson and you will know how to help Kara purchase the tickets when you complete it.

What You Will Learn

By the end of this lesson, you will have learned how to complete following:

- Graph inequalities on a number line.
- Recognize equivalent inequalities.
- Solve inequalities and graph solutions.
- Write and solve real-world problems involving inequalities.

Teaching Time

I. **Graph Inequalities on a Number Line**

In the last few lessons you have been learning about equations and about balancing an equation. Let's review a short summary of solving equations. **An *equation* is a number statement with an equal sign.** The equal sign tells us that the quantity on one side of the equation is equal the quantity on the other side of the equation. We can solve an equation by figuring out the quantity that will make the equation a true statement.

Example

$x + 5 = 12$

If we think about this equation, we can use mental math and calculate that the unknown quantity is equal to 7. If we substitute 7 in for *x*, we will have a true statement.

$$7 + 5 = 12$$
$$12 = 12$$

Our equation is balanced because one side is equal to the other side. Notice that there is only one answer for *x* **that makes this a true statement.**

What is an inequality?

An *inequality* **is a mathematical statement that can be unequal.**

We use the following symbols to show that we are working with an inequality.

>means greater than

<means less than

$\geq$ means greater than or equal to

$\leq$ means less than or equal to

How do we apply these symbols?

Well, if you think about it, we use the symbols the same way we used the equals sign before: to make a true statement. Let's look at an example.

Example

$x + 3 > 5$

There are many possible answers that will make this a true statement. We need the quantity on the left side of the inequality to create a sum that is greater than five. Notice that the sign does not have a line under it. We want a quantity that is greater than five, not greater than or equal to five, on the left side of the inequality.

To make this true, we can choose a set of numbers that has a 3 or greater in it.

$x = \{3, 4, 5 \ldots\}$

We don't need to worry about solving inequalities yet, the key thing to notice is that there are many possible answers that will make an inequality a true statement. In fact, it is kind of cool to realize that there are an infinite number of possible answers that work.

We can use graphs to help us understand inequalities in a visual way.

Graphing inequalities on a number line can help us understand which numbers are solutions for the inequality and which numbers are not solutions.

Here are a couple of rules for graphing inequalities on a number line.

- **Use an open circle to show that a value is not a solution for the inequality. You will use open circles to graph inequalities that include the symbols >or <.**
- **Use a closed circle to show that a value is a solution for the inequality. You will use closed circles to graph inequalities that include the symbols $\geq$ or $\leq$.**
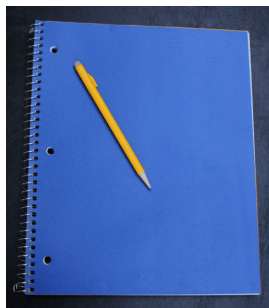
Write these hints in your notebook and then continue with the lesson.

Example

Graph the inequality $x > 3$.

To complete this task, first draw a number line from -5 to 5.

The inequality $x > 3$ is read as "x is greater than 3." So, the solutions of this inequality include all numbers greater than 3. It does not include 3, so draw an open circle at 3 to show that 3 is not a solution for this inequality. Then draw an arrow showing all numbers greater than 3. The arrow should face to the right because the greater numbers are to the right on a number line.
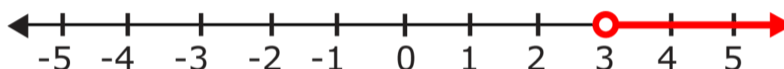
The graph above shows the solutions for the inequality $x > 3$.

The graph above can help you see which numbers are solutions for $x > 3$ and which are not. For example, the arrow includes the numbers 3.5, 4, and 5. If you continued the number line, the arrow would also include the numbers 10 and 100. So, all those numbers: 3.5, 4, 5, 10, and 100–are possible values for x.

Example

Graph the inequality $x < -1$.

First, draw a number line from -5 to 5.

The inequality $x < -1$ is read as "x is less than -1." So, the solutions of this inequality include all numbers less than -1. It does not include -1, so draw an open circle at -1 to show that -1 is not a solution for this inequality. Then draw an arrow showing all numbers less than -1. The arrow should face left because the lesser numbers are to the left on a number line.
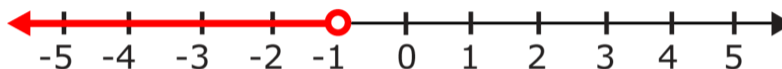
The graph above shows the solutions for the inequality $x < -1$.

Example

Graph the inequality $x \geq 0$.

First, draw a number line from -5 to 5.

The inequality $x \geq 0$ is read as "x is greater than or equal to 0." So the solutions of this inequality include zero and all numbers that are greater than 0. Draw a closed circle at 0 to show that 0 is a solution for this inequality. Then draw an arrow showing all numbers greater than 0.
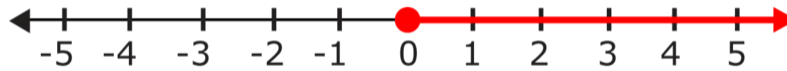
The graph above shows the solutions for the inequality $x \geq 0$.

<u>Lesson Exercises</u>

Answer the following true/false questions.

1. **An open circle on a graph means that the number is not included in the solution set.**
2. **An inequality can never be equal.**
3. **A closed circle on a graph means that the number is included in the solution set.**

Take a few minutes to check your answers with a friend.

II. <u>Recognize Equivalent Inequalities</u>

Sometimes, you may need to rewrite an inequality as an ***equivalent inequality*** in order to better understand it. This means that an inequality can be written in two different ways so that we can understand it. Look at the example below.

Example

Graph the inequality $4 \geq x$.

Draw a number line including several numbers larger and smaller than the 'starting value', 4. A good choice might be from -5 to 5.

The inequality $4 \geq x$ is read as "4 is greater than or equal to x." This inequality will be easier to understand if we rewrite it so that the variable is listed first. If we list the x first, we will be describing the relationship in reverse, so we must reverse the inequality symbol. That means changing the "greater than or equal to" symbol ($\geq$) to a "less than or equal to symbol" $\leq$.

$4 \geq x$ is equivalent to $x \leq 4$.

Logically, if 4 is greater than or equal to x, then x must be less than or equal to 4.

The inequality $x \leq 4$ is read as "x is less than or equal to 4." So the solutions of this inequality include 4 and all numbers that are less than 4. Draw a closed circle at 4 to show that 4 is a solution for this inequality. Then draw an arrow showing all numbers less than 4.
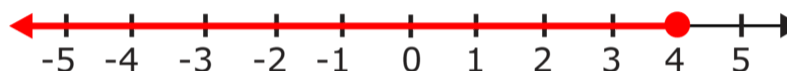
The graph above shows the solutions for the inequality $x \leq 4$.

Example

Write an equivalent inequality for $x \geq 5$.

To write an equivalent inequality, we reverse the terms. If x is greater than or equal to five, then five is less than or equal to x. Let's rewrite this.

The equivalent inequalities are $x \geq 5$ *and* $5 \leq x$.

7N. Lesson Exercises

Write an equivalent inequality for each example.

1. $12 \leq y$
2. $x > 7$
3. $a < 4$

Check your answers with a partner.

III. **Solve Inequalities and Graph Solutions**

Now that you have developed an understanding of inequalities, we can solve them and graph the solution sets.

We can solve an inequality using all of the skills we would use to solve an equation with only one exception: if we multiply or divide by a negative number, we need to switch the direction of the >or <symbol. Once we solve the inequality, we can graph the solution. Let's look at an example.

Example

Solve this inequality and graph the solution: $n - 4 \leq 3$.

Solve the inequality just as you would solve an equation, by using inverse operations. Since the 4 is subtracted from n, add 4 to both sides of the inequality to solve it. You do not need to multiply or divide both sides by a negative number, so you do not need to reverse the inequality symbol. The symbol should stay the same.

$$n - 4 \leq 3$$
$$n - 4 + 4 \leq 3 + 4$$
$$n + (-4 + 4) \leq 7$$
$$n + 0 \leq 7$$
$$n \leq 7$$

Now, graph the solution. The inequality $n \leq 7$ is read as "n is less than or equal to 7." So, the solutions of this inequality include 7 and all numbers that are less than 7.

Draw a number line from 0 to 10. Add a closed circle at 7 to show that 7 is a solution for this inequality. Then draw an arrow showing all numbers less than 7.

The solution for this inequality is $n \leq 7$, and its graph is shown above.

Example

Solve this inequality and graph its solution: $-2n < 14$.

Solve this inequality as you would solve an equation, by using inverse operations. Since the -2 is multiplied by the n, divide both sides of the inequality by -2 to solve it. Since this involves multiplying both sides of the inequality by a negative number, the sense of the inequality will change and you will need to reverse the inequality symbol. This means changing the inequality symbol from a "less than" symbol (<) to a "greater than" symbol (>).

$$-2n < 14$$
$$\frac{-2n}{-2} > \frac{14}{-2}$$
$$1n > -7$$
$$n > -7$$

Now, graph the solution. The inequality $n > -7$ is read as "n is greater than -7." So, the solutions of this inequality include all numbers that are greater than -7.

Draw a number line from -10 to 0. Add an open circle at -7 to show that -7 is not a solution for this inequality. Then draw an arrow showing all numbers greater than -7.
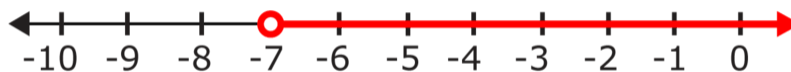
The solution for this inequality is $n > -7$, and its graph is shown above.

Sometimes, you will need to take more than one step to solve an inequality. You can think of these problems in the same way that you thought about two-step equations.

Example

$\frac{n}{3} + 9 \geq -9$.

Solve this inequality as you would solve an equation, by using inverse operations. First, try to get the term with the variable, $\frac{n}{3}$, by itself on one side of the inequality. Since the 9 is being added to $\frac{n}{3}$, subtract 9 from both sides of the inequality. You do not need to multiply or divide both sides by a negative number, so you do not need to reverse the inequality symbol during this step.

$$\frac{n}{3} + 9 \geq -9$$
$$\frac{n}{3} + 9 - 9 \geq -9 - 9$$
$$\frac{n}{3} + 0 \geq (-9 + -9)$$
$$\frac{n}{3} \geq -18$$

There is a second step you must take to find the solution. Since n is divided by 3, you must multiply both sides of the inequality by 3 to find its solution. This involves multiplying by a positive number, 3, so you do not need to reverse the inequality symbol. Be careful! It is true that you will need to multiply 3 by -18 to find the solution.

However, since you are not multiplying both sides of the inequality by a negative number, you do not reverse the inequality symbol.

$$\frac{n}{3} \geq -18$$

$$\frac{n}{3} \times 3 \geq -18 \times 3$$

$$\frac{n}{3} \times \frac{3}{1} \geq -54$$

$$\frac{n}{\cancel{3}} \times \frac{\cancel{3}}{1} \geq -54$$

$$\frac{n}{1} \geq -54$$

$$n \geq -54$$

The solution for this inequality is $n \geq -54$.

Lesson Exercises

Solve each inequality.

1. $x - 4 < 10$
2. $2y + 4 \geq 12$
3. $-4x \leq 16$

Check your answers with a friend. Then continue with the next section.

GUIDED PRACTICE

Write and Solve Real-World Problems Involving Inequalities

Now that you know how to graph and solve inequalities, let's take a look at some ways we can use inequalities to solve problems. We can use inequalities to represent some real-world problem situations, too.

Let's take a look at some key words that can help us write inequalities to represent real-world problems.

TABLE 3.3:

>	<	$\geq$	$\leq$
greater than more than	less than fewer than	greater than or equal to at least	less than or equal to at most

The key words above provide clues about which inequality symbol you should use to represent a problem situation. While key words can be a helpful guide, it is important not to rely on them totally. It is always most important to think about what translation of the problem makes the most sense. This is especially important because the same key

words may mean different things. For example, the key words "more than" may mean you should use a >symbol or they may mean you should write an addition expression.

Example

Antonio is buying milk for a breakfast event. Each container of milk costs $3. At most, he can spend $12 on milk for the event.

a. Write an inequality to represent, c, the number of containers of milk he can buy.

b. Could Antonio buy 4 containers of milk for the event? Explain.

Consider part a first.

Use a number, an operation sign, a variable, or an inequality symbol to represent each part of the problem. Since each container of milk costs $3, you can find the total cost, in dollars, of the milk he buys by multiplying 3 by the number of containers. The key words "at most" indicate that you should use a $\leq$ symbol.

$$\underline{\text{Each container}\ldots\text{costs \$3}}.\ \underline{\text{At most}},\ \text{he can spend}\ \underline{\$12}\ldots$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$
$$c \times 3 \qquad\qquad \leq \qquad\qquad 12$$

You may also want to consider that the value of c must be an integer greater than or equal to 0. Think about why that is for a moment.

The reason that the value of c must be an integer greater than or equal to zero is because Antonio cannot buy a negative number of containers nor can he buy a fraction of a container. Neither of those situations makes sense in real life. When using inequalities to represent real-life situations, you should always think about which values would make sense for the variable and which values would not make sense.

Next, consider part b.

Solve the inequality to help you.

$$c \times 3 \leq 12$$
$$\frac{c \times 3}{3} \leq \frac{12}{3}$$
$$c \times \frac{3}{3} \leq 4$$
$$c \times 1 \leq 4$$
$$c \leq 4$$

According to the inequality above, the number of containers, c, that he can buy must be less than or equal to 4.

Since 4 is a solution for this inequality, and since 4 is an integer that is greater than 0, he could buy 4 containers of milk.

Example

At the Stereo Store, Erika bought a $9 set of headphones and a DVD that was on sale for half its regular price. Erika spent more than $15 on these two purchases.

a. Write an inequality to represent d, the regular price of a DVD at the store.

b. List three possible values of d.

Consider part a first.

Use a number, an operation sign, a variable, or an inequality symbol to represent each part of the problem. The key words "more than", in this case, indicate that you should use a >symbol.

$9 set of headphones <u>and</u> $a\ DVD$...for <u>half the regular price</u>...spent <u>more than</u> $15...

$$9 \qquad\qquad\qquad + \qquad\qquad\qquad \frac{d}{2} \qquad\qquad\qquad > \qquad 15$$

Next, consider part b.

Solve the inequality to help you.

$$9 + \frac{d}{2} > 15$$
$$9 - 9 + \frac{d}{2} > 15 - 9$$
$$0 + \frac{d}{2} > 6$$
$$\frac{d}{2} > 6$$
$$\frac{d}{2} \times 2 > 6 \times 2$$
$$\frac{d}{\cancel{2}} \times \frac{\cancel{2}}{1} > 12$$
$$\frac{d}{1} > 12$$
$$d > 12$$

According to the inequality above, the regular price of a DVD, d, is more than 12 dollars.

So, three possible values of d are $12.50, $13, and $20. These are only 3 possible answers. **You could choose any amount that is greater than 12 dollars.**

Now let's go back to our introduction problem and help Tony with his dilemma on the train.

Real Life Example Completed

The Movie Tickets

Here is the original problem once again. Use what you have learned about inequalities and about solving inequalities to help you with the movie tickets. Reread the problem before you begin and underline any important information.

Marc and Kara have made several friends while swimming at the town pool. One rainy day, they decide to invite the group to go to a movie. Everyone is very excited. Grandma is so pleased that Kara and Marc have been making friends that she offers to treat.

"How many of you are going?" she asks Kara.

"Well, there are eight of us altogether," Kara replies.

"Alright, let me see," Grandma says, reaching into her wallet. "I have <u>forty-eight dollars</u> to contribute. Here it is."

"Thank you Grandma," the twins say, smiling.

Later at the movie theater, Kara takes out the money. They <u>have $48.00 to spend on tickets</u>. Last minute, one of the friends has brought her brother along. <u>Kara isn't sure that they have enough money for the brother too. If each ticket is $6.00, how many tickets can she buy without going over the $48.00?</u>

To figure this out, we need to write an inequality. The original group consisted of 8 kids without the extra brother. Each ticket is $6.00. We can use this in our inequality. If we have money left over, then we will know whether or not we can pay for the brother too.

We need the total to be less than or equal to $48.00.

Here is the inequality.

$6x \leq 48$

Now we can solve the inequality. The x represents the number of tickets that we can purchase if they are $6.00 each.

$$6x \leq 48$$
$$\frac{6x}{6} \leq 48$$
$$x \leq 8$$

This shows that the kids can purchase less than or equal to 8 tickets with the $48.00. They don't have enough to pay for the brother too.

Kara tells the group this information and they all chip in with their own money. All of the kids are able to attend the movie thanks to the generosity of the group.

Vocabulary

Here are the vocabulary words that are found in this lesson.

Equation

> a number sentence with an equal sign where the quantity on one side of the equals is the same as the quantity on the other side of the equals.

Inequality

> a number sentence where one side is not necessarily equal to the other side. There are several possible answers that will make an inequality a true statement.

Equivalent Inequalities

> Two inequalities that are written differently, but still express the same number relationships.

Technology Integration

EXAMPLE: PLOTTING
SIMPLE INEQUALITY
ON A NUMBER LINE

KHANACADEMY

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/5533

TWO-STEP
INEQUALITY

KHANACADEMY

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/121008

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/5535

MEDIA
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URL: https://www.ck12.org/flx/render/embeddedobject/5536

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URL: https://www.ck12.org/flx/render/embeddedobject/5537
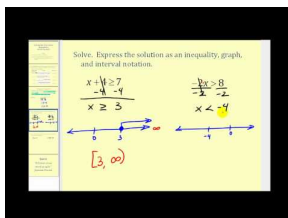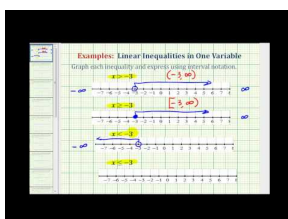
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/5538
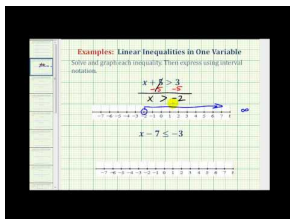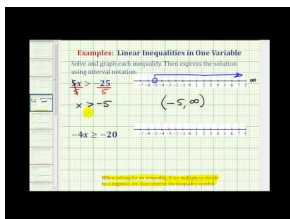
Click image to the left or use the URL below.
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PRACTICE

Directions: For problems 1-4, graph each inequality on the given number line.

1. $x < -3$
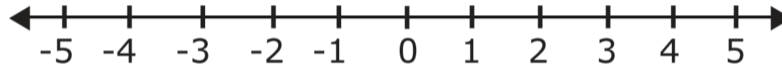
2. $x > -5$
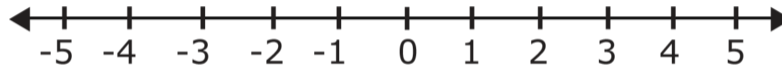
3. $n \leq 2$
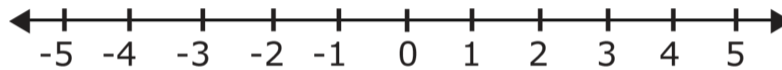
4. $1 \leq n$
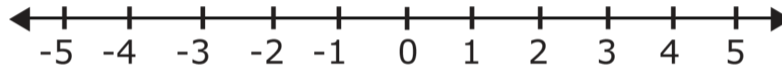
Directions: For problems 5-6, solve each inequality and then graph its solution on the given number line.

5. $x + 3 > 9$
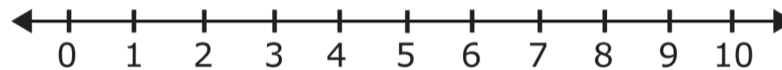
6. $n \div (-4) \geq 2$
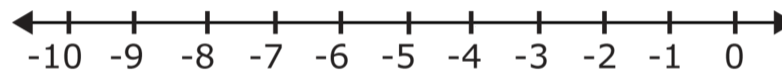
Directions: Solve each inequality.

7. $x + 4 < 10$

8. $x - 3 \geq 7$

9. $b + 5 \leq 15$

10. $a - 7 \geq 14$

11. $4y > 20$

12. $6x \leq 18$

13. $-4y < -12$

14. $-5x < -20$

15. $\frac{x}{2} = 10$

16. $\frac{x}{5} \leq 6$

17. $2x + 5 \geq 7$

18. $3y - 2 \leq 4$

19. $3a - 7 > 11$

20. $2b + 9 < 39$

Directions: Solve each problem.

21. Emma bought a fruit smoothie at a juice shop for t dollars, including tax. Emma paid with a $10 bill. She received less than $5 in change.

a. Write an inequality to represent t, the number of dollars, including tax, that Emma paid for the fruit smoothie.

b. List three possible values of t.

22. Kiet has 16 juice boxes for a family picnic and needs to buy more. Juice boxes are sold in packages of 8.

a. Write an inequality to represent p, the number of packages of juice boxes Kiet needs to buy in order to have at least 40 juice boxes total for the picnic.

b. If Kiet buys 4 packages of juice boxes, will that be enough?

3.11 Representing Solutions for Equations as Function Table (7.10b)

GUIDANCE

I. **Identify the Domain and Rangle of a Simple Linear Function**

We use the word "function" all the time in everyday speech. We say things like "It's a function of time" or "It's a function of price." This is a real life application of a mathematical concept called a function. You will learn how to apply functions to real-world examples, but first let's look at what a function is and how we can understand it better.

What is a function?

A *function* **is a set of ordered pairs in which the first element in any pair corresponds to exactly one second element.**

For example, look at this set of ordered pairs. Notice that braces, { }, are used to surround the set of ordered pairs.

$$\{(0,5), (\underline{1},\underline{6}), (2,7), (3,8)\}$$
$$\uparrow\uparrow$$

In (1, 6), 1 is the first element and

6 is the second element.

Each of the first elements–0, 1, 2, and 3–corresponds to exactly one second element. So, this set of ordered pairs represents a function.

Let's take a look at another set of ordered pairs.

$$\{(\underline{2},4), (5,3), (6,7), (\underline{2},8)\}$$

The first element, 2, corresponds

to two different second elements–4 and 8.

Since one of the first elements corresponds to two different second elements, the set of ordered pairs above does not represent a function.

We can use these criteria to determine whether or not a series of ordered pairs forms a function.

Now that you know how to identify a function, let's look at some of the key words associated with functions.

1. **Domain**
2. **Range**

The *domain* of a function is the set of all the first elements in a function. The *range* is the set of all the second elements in a function.

Let's look at an example and identify the domain and range of the series of ordered pairs.

Example

The ordered pairs below represent a function

{(0, -10), (2, -8), (4, -6), (6, -4)}

a. Identify the domain of the function.

b. Identify the range of the function.

Consider part a **first.**

The domain is the set of all the first elements in the function. These first elements are underlined below.

$\{(\underline{0}, -10), (\underline{2}, -8), (\underline{4}, -6), (\underline{6}, -4)\}$

The **domain (x - values)** of this function is {0, 2, 4, 6}.

Next, consider part b.

The range is the set of all the second elements in the function. These second elements are underlined below.

$\{(0, \underline{-10}), (2, \underline{-8}), (4, \underline{-6}), (6, \underline{-4})\}$

The **range (y - Values)** of this function is {-10, -8, -6, -4}.

II. **Evaluate a Given Function Rule Using an Input-Output Table**

An ***input-output table***, like the one shown below, can also be used to represent a function. Because of that, we can also call this kind of a table a *function table*.

TABLE 3.4:

Input number (x)	Output number (y)
0	0
1	3
2	6
3	9

Each pair of numbers in the table is related by the same ***function rule***. That rule is: multiply each input number $(x-$value) by 3 to find each output number $(y-$ value), $y = 3x$. You can use a rule like this to find other values for this function, too. You could say that an equation is another way of writing a function rule.

Working with function tables and function rules is a lot like being a detective! You have to use the clue of the

function rule to complete a table! Patterns are definitely involved in this work.

Now let's look at how we can use a function rule to complete a table.

Example

The rule for the input-output table below is: add 1.5 to each input number to find its corresponding output number, $y = x + 1.5$. Use this rule to find the corresponding output numbers for the given input numbers in the table.

TABLE 3.5:

Input number (x)	Output number (y)
0	
1	
2.5	
5	
10	

To find each missing output number, add 1.5 to each input number. Then write that output number in the table.

TABLE 3.6:

Input number (x)	Output number (y)	$y = x + 1.5$
0	1.5	$\leftarrow 0 + 1.5 = 1.5$
1	2.5	$\leftarrow 1.0 + 1.5 = 2.5$
2.5	4	$\leftarrow 2.5 + 1.5 = 4.0$
5	6.5	$\leftarrow 5.0 + 1.5 = 6.5$
10	11.5	$\leftarrow 10.0 + 1.5 = 11.5$

The table above shows five ordered pairs that match the given function rule. Let's write the answer in ordered pairs.

The answer is (0, 1.5) (1, 2.5) (2.5, 4) (5, 6.5) (10, 11.5).

Now let's look at how to create a function table given a rule.

Example

The rule for a function is: multiply each $x-$value by 4 and then subtract 2 to find each $y-$value, $y = 4x - 2$. Make a function table that shows three ordered pairs of values for a function that follows this rule.

First, choose three $x-$values for the table. You may choose any numbers, but let's select some small numbers that will be easy to work with, such as 1, 2, and 3. Then, to find the $y-$values, multiply each of those values by 4 and subtract 2 from that product, .

TABLE 3.7:

x	y	$y = 4x - 2$
1	2	$\leftarrow 1 \times 4 = 4$
		$4 - 2 = 2$
2	6	$\leftarrow 2 \times 4 = 8$
		$8 - 2 = 6$
3	10	$\leftarrow 3 \times 4 = 12$
		$12 - 2 = 10$

TABLE 3.7: (continued)

x	y	$y = 4x - 2$

The table above shows five ordered pairs that match the given function rule.

The answer is (1, 2) (2, 6) (3, 10).

We can also write a function rule in the form of an equation. Just like an equation shows the relationship between values, the function table does too. Let's look at an example.

Example

The equation $y = \frac{x}{3} + 1$ describes a function. Use this rule to find the missing values in the table below.

TABLE 3.8:

x	y
0	1
3	2
9	
	8

The table requires you to find the value of y when $x = 9$. To find the missing $y-$value, substitute the given $x-$value, 9, for x into the equation. Then solve for y.

$$y = \frac{x}{3} + 1$$
$$y = \frac{9}{3} + 1$$
$$y = 3 + 1$$
$$y = 4$$

So, when $x = 9, y = 4$. This means that (9, 4) is an ordered pair for this function.

The table also requires you to find the value of x when $y = 8$. To find the missing $x-$value, substitute the given $y-$value, 8, into the equation. Then solve for x as you would solve any two-step equation.

$$y = \frac{x}{3} + 1$$
$$8 = \frac{x}{3} + 1$$
$$8 - 1 = \frac{x}{3} + 1 - 1$$
$$7 = \frac{x}{3} + 0$$
$$7 = \frac{x}{3}$$

$$7 \times 3 = \frac{x}{3} \times 3$$
$$21 = \frac{x}{\cancel{3}} \times \frac{\cancel{3}}{1}$$
$$21 = \frac{x}{1} = x$$

So, when $y = 8, x = 21$. This means that (21, 8) is an ordered pair for this function.

The completed table will look like this.

TABLE 3.9:

x	y
0	1
3	2
9	4
21	8

III. Write a Function Rule From an Input-Output Table

Up until now, you have been given a function rule or an equation. Now it is time to use those detective skills and figure out the rule from a given table.

Remember, thinking like a detective looking for clues will help you write function rules.

One strategy that might be helpful in this task is to use guess and check to figure out the relationship between

the values in the domain and the values in the range. **Remember the** x **values are the domain and the** y **values are the range.**

Example

Identify the rule for this input-output table. Describe the rule in words.

TABLE 3.10:

Input number (x)	Output number (y)	(hint: equation)
10	1	
20	2	
30	3	
40	4	

Use guess and check to determine how to each pair of values is related.

First, notice that each output number $(y-$**value) is less than its corresponding input number** $(x-$**value). There are two operations that help numbers become smaller. So, the rule will involve either subtraction or division.**

Look for a subtraction rule first.

Consider the ordered pair (10, 1).

$10 - 1 = 9$, so the rule could be to subtract 9 from each input number to find the corresponding output number. Check to see if that rule works for the other pairs of values in the table.

Consider the ordered pair (20, 2).

$20 - 1 = 19$, not 2, so that rule does not work all the pairs of values in the table.

Look for a division rule next.

Consider the ordered pair (10, 1) again.

$10 \div 10 = 1$, so the rule could be to divide each input number by 10 to find the corresponding output number. Check to see if that rule works for the other pairs of values in the table.

Consider the ordered pair (20, 2).

$20 \div 10 = 2$, so the rule works for that ordered pair.

Consider the ordered pair (30, 3).

$30 \div 10 = 3$, so the rule works for that ordered pair.

Consider the ordered pair (40, 4).

$40 \div 10 = 4$, so the rule works for that ordered pair.

The rule for this function table is to divide each input number $(x-$**value) by 10 to find its corresponding output number** $(y-$**value).**

In this example, we wrote the function rule in words. We can also use an equation to express a function rule. Think about this past example.

The rule was "Divide each x value by 10 and that will give you the y value."

We could write $\frac{x}{10} = y$.

This would have been the same rule expressed as an equation.

Let's look at another example where we will write an equation to represent a function rule.

Example

Identify the rule for this function table. Describe the rule in words. Then write an equation to represent the relationship between the pairs of values in the table.

TABLE 3.11:

x	y
1	5
2	10
3	15
4	20

Use guess and check to determine how each pair of values is related.

For example, notice that each $y-$value is greater than its corresponding $x-$value. So, the rule will involve either addition or multiplication.

Look for an addition rule first.

Consider the ordered pair (1, 5).

$1+4=5$, so the rule could be to add 4 to each $x-$value to find the corresponding y-value. Check to see if that rule works for the other pairs of values in the table.

Consider the ordered pair (2, 10).

$2+5=7$, not 10, so that rule does not work all the pairs of values in the table.

Look for a multiplication rule next. Each $y-$value in the table is 5 more than the previous $y-$value, so the rule may involve multiplying by 5.

Consider the ordered pair (1, 5).

$1\times5=5$, so the rule could be to multiply each $x-$value by 5 to find the corresponding $y-$value. Check to see if that rule works for the other pairs of values in the table.

Consider the ordered pair (2, 10).

$2\times5=10$, so the rule works for that ordered pair.

Consider the ordered pair (3, 15).

$3\times5=15$, so the rule works for that ordered pair.

Consider the ordered pair (4, 20).

$4\times5=20$, so the rule works for that ordered pair.

The rule for this function table is "multiply each $x-$value by 5 to find its corresponding $y-$value".

Now that we have a rule in words, let's write an equation to show the same relationship. Remember, to find each $y-$value, you must multiply each $x-$value by 5. So, the equation would be:

$y=x\cdot5$**or** $y=5x$.

I see how this is like being a detective!

Yes it is. You will also become better at figuring out the rules over time.

One thing to consider is that sometimes, a function rule will involve more than one step. In other words, it will have two operations in it, not just one.

Example

Identify the rule for this function table. Describe the rule in words. Then write an equation to represent that rule.

TABLE 3.12:

x	y
1	6
2	11
3	16
4	21

Use guess and check to determine how to each pair of values is related.

For example, notice that each $y-$value is greater than its corresponding $x-$value. So, the rule will involve either addition or multiplication.

Since each $y-$value in the table is 5 more than the previous $y-$value, the rule may involve multiplying by 5.

Look for a two-step rule that involves multiplying by 5.

Consider the ordered pair $(1, 6)$.

$1 \times 5 = 5$ and $5 + 1 = 6$, so the rule could be to multiply each input number by 5 and then to add 1 to find each $y-$value. Check to see if that rule works for the other pairs of values in the table.

Consider the ordered pair $(2, 11)$.

$2 \times 5 = 10$ and $10 + 1 = 11$, so the rule works for that ordered pair.

Consider the ordered pair $(3, 16)$.

$3 \times 5 = 15$ and $15 + 1 = 16$, so the rule works for that ordered pair.

Consider the ordered pair $(4, 21)$.

$4 \times 5 = 20$ and $20 + 1 = 21$, so the rule works for that ordered pair.

The rule for this function table is to multiply each $x-$value by 5 and then add 1 to find its corresponding $y-$value.

Now that we have a rule in words, let's write an equation to show the same relationship. Remember, to find each $y-$value, we must multiply each $x-$value by 5 and then add 1. So, the equation would be

$y = 5x + 1$

–

GUIDED PRACTICE

Identify the domain and range of each function.

1. (1, 3) (2, 4) (5, 7) (9, 11)

2. (8, 12) (9, 22) (4, 7) (2, 5)

3. Complete the table given the rule "add 2". Write the answer in ordered pairs.

TABLE 3.13:

Input (x)	Output (y)	*(hint: equation)*
3		
5		
6		

4. Create a function table given the rule $x\,2 + 3$ (input times two, plus three). Use three values.

–

–

–

–

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–

–

Look at each function table and write each rule as an equation.

5.

TABLE 3.14:

x	y
5	10
7	14
9	18

6.

TABLE 3.15:

x	y
3	5
4	7
6	11

3.

TABLE 3.16:

x	y
2	1
4	3
6	5

IV. **Model and Solve Real-World Problems Involving Patterns of Change**

Functions can also help us describe real-world situations and solve real-world problems.

Example

The number of quarters needed to wash clothes in a washing machine at a laundromat is a function of the number of loads of laundry that need to be washed. This table shows the number of quarters needed to wash 1, 2, and 3 loads of laundry.

TABLE 3.17:

Number of Loads (x)	Number of Quarters (y)
1	4
2	8
3	12
4	?
5	?

a. Write an equation to describe the relationship between the pairs of values in the table above.

b. How many quarters are needed to wash 5 loads of laundry?

Consider part *a* first.

Use guess and check to determine how to each pair of values is related.

For example, notice that each $y-$value is greater than its corresponding $x-$value. So, the rule will involve either addition or multiplication.

Since each $y-$value in the table is 4 more than its corresponding $y-$value, the rule may involve multiplying by 4.

Look for a rule that involves multiplying by 4.

Consider the ordered pair (1, 4).

$1 \times 4 = 4$, so the rule could be to multiply each $x-$value (or number of loads) by 4 to find each $y-$value (or number of quarters). Check to see if that rule works for the other pairs of values in the table.

Consider the ordered pair (2, 8).

$2 \times 4 = 8$, so the rule works for that ordered pair.

Consider the ordered pair (3, 12).

$3 \times 4 = 12$, so the rule works for that ordered pair.

So, the rule for this function table could be to multiply each $x-$value by 4 to find its corresponding $y-$value. We could also say that the rule is to multiply the number of loads of laundry by 4 to find the number of quarters needed.

Now that we have our rule written out in words, let's write an equation that expresses the same thing.

Remember, to find each $y-$value, you must multiply each $x-$value by 4. So, the equation would be

$y = 4x$

Next, use the equation you wrote for part *a* to solve part *b*.

Part *b* asks how many quarters are needed to wash 5 loads of laundry. Substitute 5 for *x* in the equation.

$$y = 4x$$
$$y = 4 \cdot 5$$
$$y = 20$$

20 quarters would be needed to wash 5 loads of laundry.

Now we can go back to the problem in the introduction and help with the amusement park tickets.

Real Life Example Completed

A Day at the Amusement Park

The month has flown by and in two days Marc and Kara will head home. They have had a terrific time in Boston and while they are happy to be heading home, they are also sad to leave. For a final trip, Grandma and Grandpa decide to go to the amusement park. Everyone is excited.

At the amusement park, different rides take a different number of tickets. Kara and Marc figure out that there are 9 rides that they would like to ride. These are their favorites and any others are just a bonus. Most often the teens will need three tickets for each ride. Each ride is a function of the number of tickets needed. Marc and Kara go up to the ticket booth and come back with 21 tickets each. If they each have 21 tickets, how many rides can they go on for those tickets?

The first thing to note is that each ride is a function of the number of tickets needed. We can call the ride x and the ticket number y.

You know from this lesson that one of the best ways to work with a function is through a function table. Let's build one now.

TABLE 3.18:

x Rides	y Tickets

We can say that 1 ride is equal to 3 tickets. Let's choose some other ride numbers and work our way up to 21 tickets.

TABLE 3.19:

x Rides	y Tickets
1	3
2	6
3	9
	21

Look at the pattern. We can write an equation to show this pattern.

$y = 3x$

We want to figure out how many rides the teens can go on for 21 tickets. 21 is the y in the equation. We can use what we have learned about solving equations to solve for x. This will give us the number of rides they can go on for 21 tickets.

$$21 = 3x$$
$$7 = x$$

TABLE 3.20:

x Rides	y Tickets
1	3
2	6
3	9
7	21

The teens can go on 7 rides each for the 21 tickets. To ride all nine rides, they will need six more tickets each.

Vocabulary

Here are the vocabulary words that are found in this lesson.

Function

A set of ordered pairs in which one element corresponds to exactly one other element. Functions might be expressed as a set of ordered pairs or in a table.

Domain

the *x* value of an ordered pair or the *x* values in a set of ordered pairs.

Range

the *y* value of an ordered pair or the *y* values in a set of ordered pairs.

Input-Output Table

a way of showing a function using a table where the *x* value is shown to cause the *y* value through a function rule.

Function Table

another name for an input-output table.

Function Rule

a written **equation** that shows how the domain and range of a function are related through operations.

Technology Integration

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Khan Academy Functions
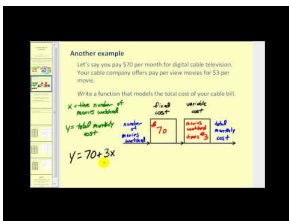
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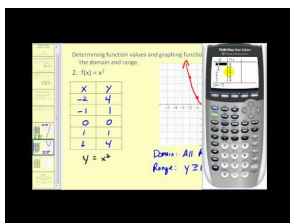
James Sousa, Introduction to Functions, Part1

James Sousa, Introduction to Functions, Part2
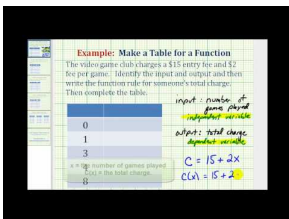
James Sousa, Write a Cost Function and Complete a Table of Values
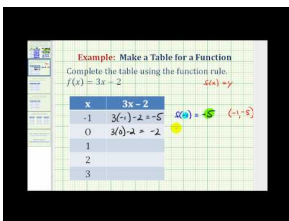
James Sousa, Complete a Table of Values Given a Function Rule

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James Sousa, Write a Function Rule Given a Table of Values

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James Sousa, Another Example of How to Write a Function Rule Given a Table of Values
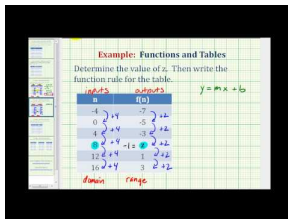
PRACTICE

Directions: Identify whether or not each series of ordered pairs forms a function.

1. (1, 3)(2, 6)(2, 5) (3, 7)

2. (2, 5) (3, 6) (4, 7) (5, 8)

3. (6, 1) (7, 2) (8, 3)

4. (5, 2) (5, 3) (5, 4) (5, 5)

5. (81, 19)(75, 18) (76, 18) (77, 19)

Directions: Identify the domain in numbers 1 - 5.

6.

7.

8.

9.

10.

<u>Directions:</u> Identify the range in numbers 1 - 5.

11.

12.

13.

14.

15.

<u>Directions:</u> Use the given rule or equation to complete the table.

16. The rule for the input-output below table is: multiply each input number by 7 and then add 2 to find each output number. Use this rule to find the corresponding output numbers for the given input numbers in the table. Fill in the table with those numbers.

TABLE 3.21:

Input number (x)	Output number (y)
0	
1	
2	
3	
4	

17. The rule for this function table is: subtract 6 from each $x-$value to find each $y-$value. Use this rule to find the missing numbers in the table. Fill in the table with those numbers.

TABLE 3.22:

x	y
0	
6	
	7
	16

18. The equation $y = \frac{x}{2} - 1$ describes a function. Use this rule to find the missing values in the table below.

TABLE 3.23:

x	y
2	0
4	1
8	
	6

<u>Directions:</u> Determine the rule for each table. Use that rule to answer the questions.

19. Identify the rule for this input-output table. Describe the rule in words.

TABLE 3.24:

Input number (x)	Output number (y)
1	3

TABLE 3.24: (continued)

Input number (x)	Output number (y)
2	4
3	5
4	6

20. Identify the rule for this function table.

TABLE 3.25:

x	y
1	2
2	4
3	6
4	8

a. Describe the rule in words.

b. Write an equation to represent the rule.

21. Identify the rule for this function table.

TABLE 3.26:

x	y
1	3
2	5
3	7
4	9

a. Describe the rule in words.

b. Write an equation to represent the rule.

Directions: Solve each problem.

22. The total number of cupcakes Shakir can bake, y, is a function of the number of batches of cupcake batter he makes, x. This table shows the total number of cupcakes Shakir will bake if he makes 1, 2 or 3 batches of cupcake batter.

TABLE 3.27:

Number of Batches of Batter (x)	Total Number of Cupcakes Baked (y)
1	12
2	24
3	36
4	?
5	?

a. Write a rule in words to describe the relationship between the pairs of values in the table above.

b. Write an equation to describe the relationship between the pairs of values in the table above.

c. What is the total number of cupcakes that Shakir will bake if he makes 5 batches of batter?

23. For a concert in the auditorium, Ms. Walsh set up 10 chairs on the stage for the performers and 20 chairs in each

row for the audience. This equation shows the relationship between r, the number of rows of chairs Ms. Walsh set up and t, the total number of chairs set up in the auditorium:

$t = 10 + 20r$.

a. Create a table to show the total number of chairs that would be set up in the auditorium if Ms. Walsh set up 0, 1, 2, 3, or 4 rows of chairs.

b. Identify the domain for the ordered pairs in the table.

c. Explain why the domain shown in the table is a reasonable domain for this function and why including a number less than 0 would not be reasonable.

3.12 Representing Solutions for Equations as Graphs (7.10b)

GUIDANCE

Marc and Kara have been having a terrific time at the amusement park. For them it has been a wonderful way to end their month with their Grandparents.

In the Write a Function Rule from an Input-Output Table Concept, we used tables. Well, we can use this data to create a visual display of the information. This information is not necessary for Marc and Kara to have a great time at the amusement park, but let's say that you were someone who worked at the amusement park and you were trying to figure out how to organize packs of tickets for people to purchase. If you wanted to create a group of tickets to be purchased then you could figure out how many rides someone could go on for the number of tickets in the booklet.

If the booklet had 6 tickets, then the person could go on two rides. If the booklet had 12 tickets in it, then the person could go on 4 rides. You can see how the number of rides is a function of the number of tickets. Using this chart can help the managers and the amusement park design new booklets of tickets.

TABLE 3.28:

x Rides	y Tickets
1	3
2	6
3	9
4	12
7	21

We can create a visual display of this data. How do we do this?

This is where graphing functions is important. A graph of a function can show the relationship between the x value and the y value. In this Concept, you will learn about graphs. We will come back to this problem at the end of the Concept.

Remember what a function is?

A *function* is a set of ordered pairs where one element in the domain is paired with exactly one element in the range. There is a relationship identified with a ***function rule*** between the values in the domain and the values in the range.

In this Concept, we will begin by focusing on a specific type of function called a ***linear function***. You may notice that the word "line" is part of the word "linear". That fact can help you remember that **when a linear function is graphed on a coordinate plane, its graph will be a straight line.**

You have already learned how to represent functions through a set of ordered pairs and through a table. We can also take the information in ordered pairs or in a table and represent a function in a graph.

How do we graph a linear function?

Let's look at a table of values and see how we can represent the function on a coordinate grid.

On a coordinate plane, graph the linear function that is represented by the ordered pairs in the table below.

TABLE 3.29:

x	y
-4	5
-2	3
0	1
2	-1
4	-3

The ordered pairs shown in the table are (-4, 5), (-2, 3), (0, 1), (2, -1) and (4, -3).

Plot those five points on the coordinate plane. Then connect them as shown below.
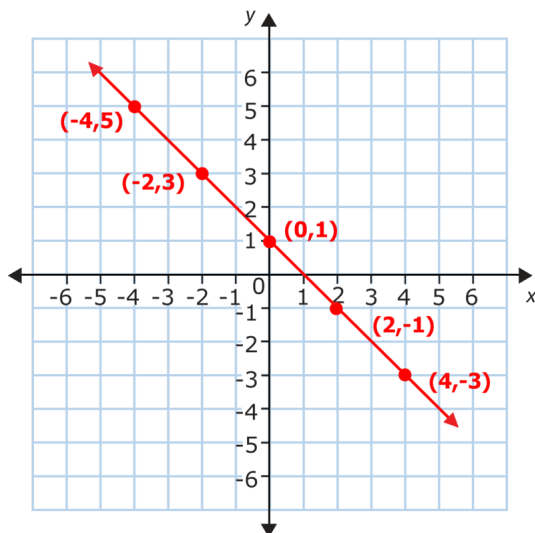
Notice that the graph of this function is a straight line. That is because this function is a linear function.

You can also graph a linear function if you are given an equation that represents the function. This will involve a few more steps. When you have an equation, you can use the equation to create a function table. Then plot several of the ordered pairs in the table and connect them with a line.

The equation $y = 2x - 1$ represents a linear function. Graph that function on a coordinate plane.

First, use the equation to create a function table and find several ordered pairs for the function. It is a good idea to use some negative $x-$values, some positive $x-$values and 0. For example, you can create a table to find the values of y when x is equal to -2, -1, 0, 1, and 2. You may need to use what you know about computing with integers to help you find those $y-$values.

TABLE 3.30:

x	y	$y = 2x - 1$
-2	-5	$\leftarrow 2x - 1 = 2(-2) - 1 = -4 - 1 = -4 + (-1) = -5$
-1	-3	$\leftarrow 2x - 1 = 2(-1) - 1 = -2 - 1 = -2 + (-1) = -3$
0	-1	$\leftarrow 2x - 1 = 2(0) - 1 = 0 - 1 = 0 + (-1) = -1$
1	1	$\leftarrow 2x - 1 = 2(1) - 1 = 2 - 1 = 1$
2	3	$\leftarrow 2x - 1 = 2(2) - 1 = 4 - 1 = 3$

The ordered pairs shown in the table are (-2, -5), (-1, -3), (0, -1), (1, 1) and (2, 3). This is the first way to show a function by using a table.

Plot those five points on the coordinate plane. Then connect them as shown below.
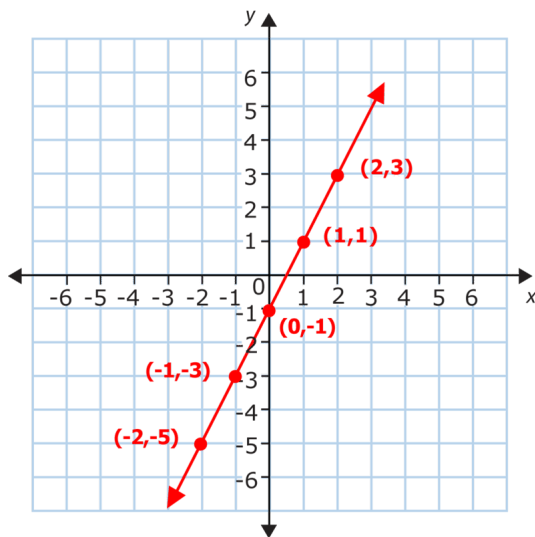
Answer the following questions about functions.

Example A

Is the graph above a positive graph or negative?

Solution: Positive

Example B

In (-3, 4) is the x value positive or negative?

Solution: Negative

Example C

In (-6, -7), which value is *y* value?

Solution: -7

Here is the original problem once again. Reread it and look at the graph at the end of it. Notice how the data from the table is represented in a visual way.

Marc and Kara have been having a terrific time at the amusement park. For them it has been a wonderful way to end their month with their Grandparents.

Think about the table from the last section. We can use this data to create a visual display of the information. This information is not necessary for Marc and Kara to have a great time at the amusement park, but let's say that you were someone who worked at the amusement park and you were trying to figure out how to organize packs of tickets for people to purchase. If you wanted to create a group of tickets to be purchased then you could figure out how many rides someone could go on for the number of tickets in the booklet.

If the booklet had 6 tickets, then the person could go on two rides. If the booklet had 12 tickets in it, then the person could go on 4 rides. You can see how the number of rides is a function of the number of tickets. Using this chart can help the managers and the amusement park design new booklets of tickets.

TABLE 3.31:

x Rides	*y Tickets*
1	3
2	6
3	9
4	12
7	21

We can create a visual display of this data. How do we do this?

Here is a graph representing the data from the table.
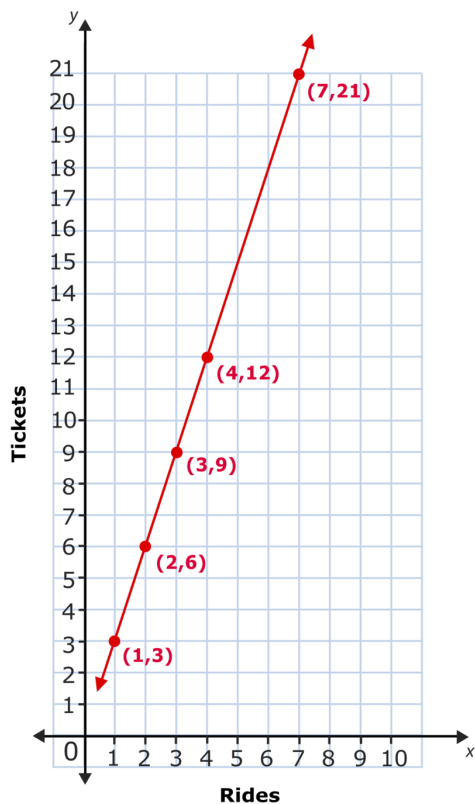
Notice that this is a linear graph showing the relationship between rides and tickets.

Vocabulary

Function

A pattern where one element of from the domain is paired with exactly one element from the range.

Function Rule

the pattern rule for a function.

Linear Function

a function that forms a straight line when graphed

GUIDED PRACTICE

Here is one for you to try on your own.

Identify the ordered pairs in this function.

TABLE 3.32:

x	y
1	**5**
2	**10**

TABLE 3.32: (continued)

x	y
3	15
4	20

Is it a linear function? Why or why not?

Answer

This is a linear function because the same rule applies to each input - output.

The rule of the function is $y = 5x$.

Video Review

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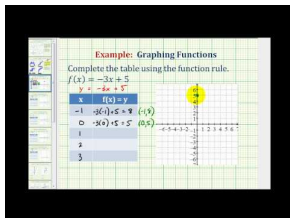
PRACTICE

1. Directions: On your own graph each function in the coordinate plane. Identify each rule(equation). There are two answers for each problem.

1.

TABLE 3.33:

Input	Output
1	4
2	5

TABLE 3.33: (continued)

Input	Output
3	6
4	7

2.

TABLE 3.34:

Input	Output
2	4
3	6
4	8
5	10

3.

TABLE 3.35:

Input	Output
1	3
2	6
4	12
5	15

4.

TABLE 3.36:

Input	Output
9	7
7	5
5	3
3	1

5.

TABLE 3.37:

Input	Output
8	12
9	13
11	15
20	24

6.

TABLE 3.38:

Input	Output
3	21
4	28

TABLE 3.38: (continued)

Input	Output
6	42
8	56

7.

TABLE 3.39:

Input	Output
2	5
3	7
4	9
5	11

8.

TABLE 3.40:

Input	Output
4	7
5	9
6	11
8	15

9.

TABLE 3.41:

Input	Output
5	14
6	17
7	20
8	23

10.

TABLE 3.42:

Input	Output
4	16
5	20
6	24
8	32

3.13 Representing Solutions for Inequalities (7.10b)

GUIDANCE

Marcia and Joe need to collect dragonflies and gather data about them for biology class. They need to capture at least 32 dragonflies for their project. They have already collected 3 dragonflies, and they can gather 2 dragonflies on average each hour. What is the minimum number of hours that Marcia and Joe need to search to get at least 32 dragonflies? Can you write an expression that models this information?

In this concept, you will learn to solve inequalities and graph solutions.

Inequalities are solved in almost the same way as regular equations. There are two small differences.

1. When solving for the variable in an inequality, if you **multiply or divide** each side of the inequality **by a negative number** the **direction of the inequality symbol changes**.
2. Unlike a regular equation, it is common to graph the solution set for an inequality on a number line use the following table to assist you.

Symbol	Word Description	What it Means	Type	Graph
<	is less than	"fewer than" "up to" "no more than"	exclusive, open circle, ○	
≤	is less than **or** equal to	"maximum" "no more than" "at most"	Inclusive, closed circle, ●	
>	is greater than	"more than" "exceeds" "in excess of"	exclusive, open circle, ○	
≥	is greater than **or** equal to	"minimum" "no less than" "at least"	Inclusive, closed circle, ●	

FIGURE 3.32

Let's look at an example.

Solve this inequality and graph its solution on a number line $n - 4 \le 3$.

Solve the inequality as you would solve an equation, by using inverse operations and applying the properties of equality (e.g. the addition property of equality). Since the 4 is subtracted from n, add 4 to both sides of the inequality to solve it.

$$
\begin{aligned}
n - 4 &\le 3 \\
n - 4 + 4 &\le 3 + 4 \\
n + 0 &\le 7 \\
n &\le 7
\end{aligned}
$$

The answer is $n \le 7$.

Next, graph the solution. The inequality $n \le 7$ is read as "n is less than or equal to 7." So, the solution of this inequality includes 7 and all numbers that are less than 7.

Draw a number line from 0 to 10. Add a closed circle at 7 to show that 7 is a solution for this inequality. Then draw an arrow pointing to the left showing all numbers less than 7.

The solution set is graphed above.

Let's look at another example.

Solve this inequality and graph its solution on a number line $-2n < 14$.

Since n is multiplied by -2, divide both sides of the inequality by -2 to isolate the variable.

When working with inequalities if you divide or multiply both sides of the equation by a negative number the inequality must change directions. This means changing the inequality symbol from a "less than" symbol (<) to a "greater than" symbol (>).

$$
\begin{aligned}
-2n &< 14 \\
\frac{-2n}{-2} &> \frac{14}{-2} \\
1n &> -7 \\
n &> -7
\end{aligned}
$$

The answer is $n > -7$.

Next, graph the solution. The inequality $n > -7$ is read as "n is greater than -7." So, the solution of this inequality includes all numbers that are greater than -7, but not -7.

Draw a number line from -10 to 0. Add an open circle at -7 to show that -7 is not a solution for this inequality. Then draw an arrow pointing to the right which shows all numbers greater than -7 are included in the solution.
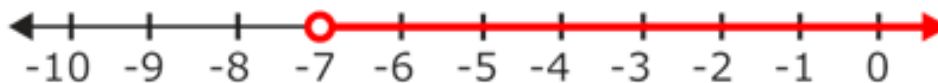
The answer is $n > -7$, and the graph of its solution is shown above.

Sometimes, it will take more than one step to solve an inequality. Solve these problems as if there were an equals sign, but remember that if you multiply or divide by a negative number the inequality will change direction.

Here is an example of a two-step inequality.

Solve the inequality $\frac{n}{3} + 9 \geq -9$.

This problem is done just like a two-step equality.

First, isolate the $\frac{n}{3}$ term. Subtract 9 from both sides of the inequality.

$$\begin{aligned} \frac{n}{3} + 9 &\geq -9 \\ \frac{n}{3} + 9 - 9 &\geq -9 - 9 \\ \frac{n}{3} &\geq -18 \end{aligned}$$

Now, you have a one-step inequality, which is easier to solve. Since you will not be multiplying or dividing by a negative number the inequality will not change direction. Multiply both sides of the inequality by 3, to isolate the variable n.

$$\begin{aligned} \frac{n}{3} \times 3 &\geq -18 \times 3 \\ \frac{3}{3} \times n &\geq -54 \\ 1n &\geq -54 \\ n &\geq -54 \end{aligned}$$

The answer is $n \geq -54$.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about the dragonfly project.

Marcia and Joe need to gather at least 32 dragonflies for their project. They have 3 already and can collect 2 each hour. Can you write an inequality that describes this situation?

First, let h be the number of hours spent collecting. They collect two per hour, or $2h$ dragonflies. They start with 3. So, if the students work h hours they will have $3 + 2h$ dragonflies. They need at least 32 dragonflies, so $3 + 2h$ needs be equal or greater than 32.

The following inequality describes this situation.

$$3 + 2h \geq 32$$

Now, solve the inequality.

First, isolate $2h$ by subtracting 3 from both sides of the equation.

$$
\begin{aligned}
3 - 3 + 2h &\geq 32 - 3 \\
0 + 2h &\geq 29 \\
2h &\geq 29
\end{aligned}
$$

Next, isolate the variable h, and divide both sides by 2.

$$
\begin{aligned}
\frac{2h}{2} &\geq \frac{29}{2} \\
\frac{2}{2}h &\geq 14.5 \\
1h &\geq 14.5 \\
h &\geq 14.5
\end{aligned}
$$

Finally, interpret the results. The students have to work at least 14.5 hours to collect 32 dragonflies. If you assume they need the full hour to collect the two dragonflies for that last hour, they should work 15 hours.

The answer is the students need to collect dragonflies for 15 hours.

Example 2

Antonio is buying milk for a breakfast event. Each container of milk costs $3. At most, he can spend $12 on milk for the event.

1. Write an inequality to represent, c, the number of containers of milk he can buy.
2. Could Antonio buy 4 containers of milk for the event? Explain.

Consider part a first.

Since each container of milk costs $3, you can find the total cost, in dollars, of the milk he buys by multiplying the number of containers by 3. The phrase "at most" indicates that you should use the $\leq$ symbol.

The answer is $3c \leq 12$.

Note that the value of c must also be an integer greater than or equal to 0. Think about why that is for a moment.

The reason that the value of c must be an integer greater than or equal to zero is because Antonio cannot buy a negative number of containers nor can he buy a fraction of a container. Neither of those situations makes sense in real life. When using inequalities to represent real-life situations, you should always think about which values would make sense for the variable and which values would not make sense.

Next, consider part b.

Solve $3c \leq 12$ to help you answer if he can buy 4 containers of milk.

First, divide both sides of the equation by 3 to isolate the variable.

$$
\begin{array}{rcl}
3c & \leq & 12 \\
\frac{3c}{3} & \leq & \frac{12}{3} \\
\frac{3}{3}c & \leq & 4 \\
1c & \leq & 4 \\
c & \leq & 4
\end{array}
$$

According to the inequality above, the number of containers, c, that he can buy must be less than or equal to 4.

So, the answer is yes, Antonio can buy 4 containers of milk for the event.

Solve each inequality.

Example 3

$x - 4 < 10$

Add 4 to both sides of the equation to isolate the variable x.

$$
\begin{array}{rcl}
x - 4 & < & 10 \\
x - 4 + 4 & < & 10 + 4 \\
x + 0 & < & 14 \\
x & < & 14
\end{array}
$$

The answer is $x < 14$.

Example 4

$2y + 4 \geq 12$

First, subtract both sides of the equation by -4 to isolate the term $2y$.

$$
\begin{array}{rcl}
2y + 4 & \geq & 12 \\
2y + 4 - 4 & \geq & 12 - 4 \\
2y & \geq & 8
\end{array}
$$

Now you have a single-step inequality, which is easier to solve. Divide both sides of the equation by 2 to isolate the variable y.

$$
\begin{array}{rcl}
\frac{2y}{2} & \geq & \frac{8}{2} \\
\frac{2}{2}y & \geq & 4 \\
1y & \geq & 4 \\
y & \geq & 4
\end{array}
$$

The answer is $y \geq 4$.

Example 5

$-4x \leq 16$

First, divide both sides of the equation by -4. You will have to change the direction of the inequality when you do this because you are dividing by a negative number.

$$
\begin{array}{rcl}
-4x & \leq & 16 \\
\frac{-4x}{-4} & \geq & \frac{16}{-4} \\
\frac{-4}{-4}x & \geq & -4 \\
1x & \geq & -4 \\
x & \geq & -4
\end{array}
$$

The answer is $x \geq -4$.

PRACTICE

Solve each inequality.

1. $x + 4 < 10$
2. $x - 3 \geq 7$
3. $b + 5 \leq 15$
4. $a - 7 \geq 14$
5. $4y > 20$
6. $6x \leq 18$
7. $-4y < -12$
8. $-5x < -20$
9. $\frac{x}{2} \leq 10$
10. $\frac{x}{5} \leq 6$
11. $2x + 5 \geq 7$
12. $3y - 2 \leq 4$
13. $3a - 7 > 11$
14. $2b + 9 < 39$
15. $2x - 8 < 42$

3.14 Representing Real Life Situations Using Equations (7.10c)

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GUIDANCE

Marisha's target heart rate is 0.70 times the quantity 220, minus her age. Her target heart rate is 140. What is her age?

After completing this Concept, you'll be able to solve real-world applications, like this one, that involve two-step equations.

Watch This

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Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/182908
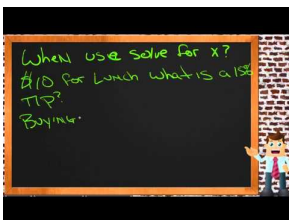
The hardest part of solving word problems is translating from words to an equation. First, you need to identify the unknown quantity in the word problem. What is the value for which you have to solve? That will be your **variable**. What operations are described in the problem, and how do they affect the unknown value? Your equation should use mathematics to describe two equal quantities, with the unknown value (in the form of a variable letter) on at least one side of the equation. Once you have restated the information in the word problem as a mathematical statement, the actual computation is often the easy part.

Example A
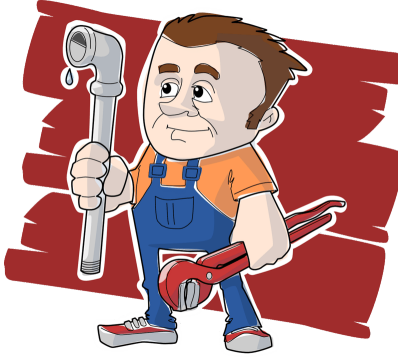
An emergency plumber charges $65 as a call-out fee plus an additional $75 per hour. He arrives at a house at 9:30 and works to repair a water tank. If the total repair bill is $196.25, at what time was the repair completed?

In order to solve this problem, we collect the information from the text and convert it to an equation.

Unknown: time taken in hours - this will be x

The bill is made up of two parts: a call out fee and a per-hour fee. The call out is a flat fee, and independent of x, it's the same no matter how many hours the plumber works. The per-hour part depends on the number of hours, x. So the total fee is $65 (regardless of the number of hours) plus $75x$ (where x is the number of hours), or $65 + 75x$.

Looking at the problem again, we also can see that the total bill is $196.25. So our statement of equal quantities (our equation) is: $196.25 = 65 + 75x$.

Solving for x:

$$196.25 = 65 + 75x \qquad \text{Subtract 65 from both sides.}$$
$$131.25 = 75x \qquad \text{Divide both sides by 75.}$$
$$1.75 = x \qquad \text{The job took 1.75 hours.}$$

Solution

The repair job was completed 1.75 hours after 9:30, so it was completed at 11:15AM.

Example B

When Asia was young, her Daddy marked her height on the door frame every month. Asia's Daddy noticed that between the ages of one and three, he could predict her height (in inches) by taking her age in months, adding 75 inches, and multiplying the result by one-third. Use this information to answer the following:

a) *Write an equation linking her predicted height, h, with her age in months, m.*

b) *Determine her predicted height on her second birthday.*

c) *Determine at what age she is predicted to reach three feet tall.*

Solution

a) To convert the text to an equation, first determine the type of equation we have. We are going to have an equation that links **two variables**. Our unknown will change, depending on the information we are given. For example, we could solve for height given age, or solve for age given height. However, the text gives us a way to determine **height,** so our equation will start with "$h =$".

The text tells us that we can predict her height by taking her age in months, adding 75, and multiplying by $\frac{1}{3}$. So our equation is $h = (m+75) \cdot \frac{1}{3}$, or $h = \frac{1}{3}(m+75)$.

b) To predict Asia's height on her second birthday, we substitute $m = 24$ into our equation (because 2 years is 24 months) and solve for h.

$$h = \frac{1}{3}(24+75)$$
$$h = \frac{1}{3}(99)$$
$$h = 33$$

Asia's height on her second birthday was predicted to be 33 inches.

c) To determine the predicted age when she reached three feet, substitute $h = 36$ into the equation and solve for m.

$$36 = \frac{1}{3}(m+75)$$
$$108 = m+75$$
$$33 = m$$

Asia was predicted to be 33 months old when her height was three feet.

Example C

To convert temperatures in Fahrenheit to temperatures in Celsius, take the temperature in degrees Fahrenheit, subtract 32, then divide the result by 1.8. This gives the temperature in degrees Celsius.

a) *Write an equation that shows the conversion process.*

b) *Convert 50 degrees Fahrenheit to degrees Celsius.*

a) The text gives the process to convert Fahrenheit to Celsius. We can write an equation using two variables. We will use f for temperature in Fahrenheit, and c for temperature in Celsius.

First we take the temperature in Fahrenheit and subtract 32. $f - 32$

Then divide by 1.8. $\dfrac{f-32}{1.8}$

This equals the temperature in Celsius. $c = \dfrac{f-32}{1.8}$

In order to convert from one temperature scale to another, simply substitute in for whichever temperature you know, and solve for the one you don't know.

b) To convert 50 degrees Fahrenheit to degrees Celsius, substitute $f = 50$ into the equation.

$$c = \frac{50-32}{1.8}$$
$$c = \frac{18}{1.8}$$
$$c = 10$$

50 degrees Fahrenheit is equal to 10 degrees Celsius.

Watch this video for help with the Examples above.
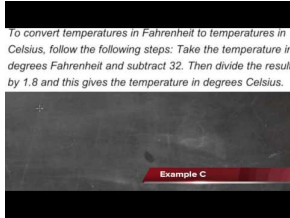
GUIDED PRACTICE

To convert temperatures in Fahrenheit to temperatures in Celsius, take the temperature in degrees Fahrenheit, subtract 32, then divide the result by 1.8.

a) *Convert 25 degrees Celsius to degrees Fahrenheit.*

b) *Convert -40 degrees Celsius to degrees Fahrenheit.*

Solution:

a) To convert 25 degrees Celsius to degrees Fahrenheit, substitute $c = 25$ into the equation:

$$25 = \frac{f - 32}{1.8}$$
$$45 = f - 32$$
$$77 = f$$

25 degrees Celsius is equal to 77 degrees Fahrenheit.

b) To convert -40 degrees Celsius to degrees Fahrenheit, substitute $c = -40$ into the equation.

$$-40 = \frac{f - 32}{1.8}$$
$$-72 = f - 32$$
$$-40 = f$$

-40 degrees Celsius is equal to -40 degrees Fahrenheit. (No, that's not a mistake! This is the one temperature where they are equal.)

PRACTICE

1. Jade is stranded downtown with only $10 to get home. Taxis cost $0.75 per mile, but there is an additional $2.35 hire charge. Write a formula and use it to calculate how many miles she can travel with her money.
2. Jasmin's dad, Andrew, is planning a surprise birthday party for her. He will hire a bouncy castle, and will provide party food for all the guests. The bouncy castle costs $150 for the afternoon, and the food will cost $3 per person. Andrew has a budget of $300. Write an equation and use it to determine the maximum number of guests he can invite.

The local amusement park sells summer memberships for $50 each. Normal admission to the park costs $25; admission for members costs $15.

3. If Darren wants to spend no more than $100 on trips to the amusement park this summer, how many visits can he make if he buys a membership with part of that money?
4. How many visits can he make if he does not?
5. If he increases his budget to $160, how many visits can he make as a member?
6. And how many as a non-member?

For an upcoming school field trip, there must be one adult supervisor for every five children.

7. Write an expression for the number of children and adults who will go on the field trip.
8. If the bus seats 40 people, how many children can go on the trip?
9. How many children can go if a second 40-person bus is added?
10. Four of the adult chaperones decide to arrive separately by car. Now how many children can go in the two buses?

3.15 Representing Real Life Situations using Inequalities (7.10c)

GUIDANCE

Inequalities are useful for solving real-world problems. In this Concept, you will see some examples of how to set up and solve real-world problems.

Example A

In order to get a bonus this month, Leon must sell at least 120 newspaper subscriptions. He sold 85 subscriptions in the first three weeks of the month. How many subscriptions must Leon sell in the last week of the month?

Solution: The number of subscriptions Leon needs is "at least" 120. Choose a variable, such as n, to represent the varying quantity-the number of subscriptions. The inequality that represents the situation is $n + 85 \geq 120$.

Solve by isolating the variable n: $n \geq 35$.

Leon must sell 35 or more subscriptions to receive his bonus.

Example B

The width of a rectangle is 12 inches. What must the length be if the perimeter is at least 180 inches? (Note: Diagram not drawn to scale.)
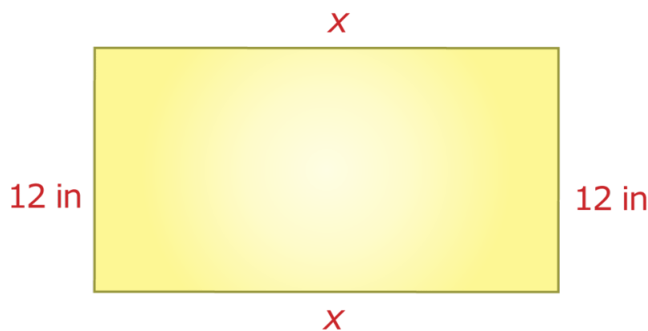
Solution: The perimeter is the sum of all the sides.

$$12 + 12 + x + x \geq 180$$

Simplify and solve for the variable x:

$$12 + 12 + x + x \geq 180 \rightarrow 24 + 2x \geq 180$$
$$2x \geq 156$$
$$x \geq 78$$

The length of the rectangle must be 78 inches or greater.

Example C

The speed of a golf ball in the air is given by the formula $v = -32t + 80$, where t is the time since the ball was hit. When is the ball traveling between 20 ft/sec and 30 ft/sec inclusive?
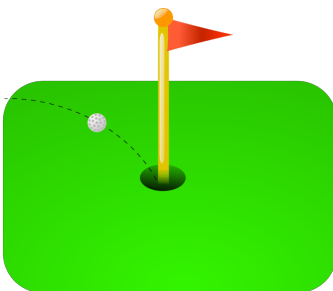
Solution: We want to find the times when the ball is traveling between 20 ft/sec and 30 ft/sec inclusive. Begin by writing the inequality to represent the unknown values, $20 \leq v \leq 30$.

Replace the velocity formula, $v = -32t + 80$, with the minimum and maximum values.

$$20 \leq -32t + 80 \leq 30$$

Separate the compound inequality and solve each separate inequality.

$$
\begin{array}{ccc}
20 \leq -32t + 80 & & -32t + 80 \leq 30 \\
32t \leq 60 & \text{and} & 50 \leq 32t \\
t \leq 1.875 & & 1.56 \leq t
\end{array}
$$

$1.56 \leq t \leq 1.875$. Between 1.56 and 1.875 seconds, the ball is traveling between 20 ft/sec and 30 ft/sec.

Inequalities can also be combined with dimensional analysis.

Video Review

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/79576

GUIDED PRACTICE

William's pick-up truck gets between 18 and 22 miles per gallon of gasoline. His gas tank can hold 15 gallons of gasoline. If he drives at an average speed of 40 miles per hour, how much driving time does he get on a full tank of gas?
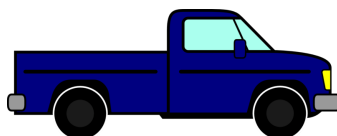
Solution: Use dimensional analysis to get from time per tank to miles per gallon.

$$\frac{t \ \cancel{hour}}{1 \ \cancel{tank}} \times \frac{1 \ \cancel{tank}}{15 \ gallons} \times \frac{40 \ miles}{1 \ \cancel{hour}} = \frac{40}{45} \frac{miles}{gallon}$$

Since the truck gets between 18 to 22 miles/gallon, you can write a compound inequality.

$$18 \leq \frac{40t}{15} \leq 22$$

Separate the compound inequality and solve each inequality separately.

$$18 \le \frac{40t}{15}$$
$$270 \le 40t \qquad \text{and}$$
$$6.75 \le t$$

$$\frac{40t}{15} \le 22$$
$$40t \le 330$$
$$t \le 8.25$$

Andrew can drive between 6.75 and 8.25 hours on a full tank of gas.

PRACTICE

For problems 1-5, write the inequality given by the statement. Choose an appropriate letter to describe the unknown quantity.

1. You must be at least 48 inches tall to ride the "Thunderbolt" Rollercoaster.
2. You must be younger than 3 years old to get free admission at the San Diego Zoo.
3. Charlie needs more than $1,800 to purchase a car.
4. Cheryl can have no more than six pets at her house.
5. The shelter can house no more than 16 rabbits.

6. The width of a rectangle is 16 inches. Its area is greater than 180 square inches.

    a. Write an inequality to represent this situation.
    b. Graph the possible lengths of the rectangle.

7. Ninety percent of some number is at most 45.

    a. Write an inequality to represent the situation.
    b. Write the solutions as an algebraic sentence.

8. Doubling Martha's jam recipe yields at least 22 pints.

    a. Write an inequality to represent the situation.
    b. Write the solutions using interval notation.

For problems 9-15, write the inequality and use it solve the problem.

9. At the San Diego Zoo, you can either pay $22.75 for the entrance fee or $71 for the yearly pass, which entitles you to unlimited admission. At most, how many times can you enter the zoo for the $22.75 entrance fee before spending more than the cost of a yearly membership?
10. Proteek's scores for four tests were 82, 95, 86, and 88. What will he have to score on his last test to average at least 90 for the term?
11. Raul is buying ties and he wants to spend $200 or less on his purchase. The ties he likes the best cost $50. How many ties could he purchase?
12. Virena's Scout Troop is trying to raise at least $650 this spring. How many boxes of cookies must they sell at $4.50 per box in order to reach their goal?
13. Using the golf ball example, find the times in which the velocity of the ball is between 50 *ft/sec and* 60 *ft/sec*.
14. Using the pick-up truck example, suppose William's truck has a dirty air filter, causing the fuel economy to be between 16 and 18 miles per gallon. How many hours can William drive on a full tank of gas using this information?
15. To get a grade of B in her Algebra class, Stacey must have an average grade greater than or equal to 80 and less than 90. She received the grades of 92, 78, and 85 on her first three tests. Between which scores must her grade fall on her last test if she is to receive a grade of B for the class?

Vocabulary:

- Equation
- Inequality
- One-variable
- Per (unit rate)

CHAPTER **4**

Constant Rates of Change and Constant of Proportionality

Chapter Outline

7.4a: represent constant rates of change in mathematical and real- world problems given pictorial, tabular, verbal, numeric,

7.4b: calculate unit rates from rates in mathematical and real-world problems

7.4c: determine the constant of proportionality ($k = y/x$) within mathematical and real-world problems

4.1 Ratios and Rates (7.4b)

GUIDANCE

The Top of the World - Mount Everest

Josh sat at the breakfast table with his nose in a book. His Mom poured some cereal into a bowl and put it down in front of him. Karen, Josh's sister came into the kitchen and sat down next to him.

"What are you reading?" she asked.

"I am reading a book about Mount Everest," Josh said without looking up.

"Alright Josh, now put the book down and eat something," their Mom instructed.

"It is terrific. Do you know that during the best year on Everest, and best meaning the year that the least number of people died, that 129 people summitted and only 8 died," Josh said smiling.

"That hardly seems like something to smile about," Mom said sipping her coffee.

"Yeah, how morbid," Karen chimed in.

"Listen it may be morbid, but it is a fact. During the worst year, only 98 people summitted and 15 died. That is a lot of people. I mean we can compare the number of people who summitted and the number of people who didn't make it," Josh said.

"Well, it proves that Everest is a dangerous place and not a trip to be taken lightly," Mom said.

"Yes, but think about how amazing it would be to stand on the top of the world!" Josh said.

"How do they figure out that the best year is the best and the worst year is the worst based on those numbers?" Karen asked.

"They compare the ratios," Josh explained. "Let me tell you how."

Do you know how? Given this information, Josh is going to show Karen how to simplify and compare ratios. In this lesson, you will learn how to write ratios and how to understand them better. Pay close attention and you can work with the ratios of the best and worst years on Everest at the end of the lesson.

What You Will Learn

In this lesson, you will learn how to execute the following skills.

- Write, compare and order ratios.
- Find and apply unit rates.
- Find and apply equivalent rates.
- Solve real-world problems involving ratios and rates.

Teaching Time

I. **Write, Compare and Order Ratios**

A *ratio* **is a comparison of two things or two quantities.** The key thing to notice about ratios is that we are comparing. There are many different ways to compare things in mathematics. For example, imagine that there are 25 students in a class; 12 boys and 13 girls. You could say that the ratio of boys to girls is 12 to 13. You could also write this ratio as $12 : 13$ or $\frac{12}{13}$. The ratio of boys to total students can be written as 12 to 25, $12 : 25$, or $\frac{12}{25}$. The ratio of girls to total students can be written as 13 to 25, $13 : 25$, or $\frac{13}{25}$. You will also see that the order that you compare makes a difference. If you are comparing boys to total students, then the number of boys comes first or if you are using fraction form, it is the numerator.

Are there several ways to write a ratio?

That is a great question. There are three different ways to write ratios. You can write them with a colon between the two values that you are comparing, you can write them using the word "to", and you can write them by putting the values in fraction form. You can choose which way you want to write a ratio and these ways are interchangeable too.

Take a minute to write the definition of a ratio and the three ways to write a ratio in your notebook.

We just looked at the three ways to write a ratio when you are making a simple comparison. Remember to read the information carefully so that you are clear what is being compared. Now let's look at an example where you might need to figure something out to write the ratio.

Example

There are 32 red and yellow candies in a bag. There are 10 yellow candies. What is the ratio of red candies to total candies in the bag?

We need the ratio of red candies to total candies. We know that there are 32 total candies. We need to find the number of red candies in the bag before writing the ratio.

$32 - 10 = 22$

Now write the ratio of red candies to total candies. **Because a ratio is a comparison, it can be simplified. Make sure to reduce the ratio to lowest terms. This makes it easier to understand the quantities that are being compared.**

$$\frac{\text{red candies}}{\text{total candies}} = \frac{22}{32} = \frac{11}{16}$$

We also could have written this ratio in two other ways.

22 to 32 then simplified to 11 to 16

OR

22 : 32 then simplified to 11 : 16

All of these answers would have been correct. Remember that you can interchange the form that you choose to write a ratio.

We can also compare ratios. This is when we have two or more ratios and we want to figure out which ones are larger and which ones are smaller. Let's look at an example.

GUIDED PRACTICE

Mr. Collison's class has 30 total students. Of these, 12 are boys. Mrs. Peterson's class has 25 students. Of these, 11 are boys. Which class has a higher ratio of boys to total students?

First, find the ratio of boys to total students for both classes. You need to do this first because these are the quantities that you are comparing.

Mr. Collison's class: $\frac{12}{30} = \frac{2}{5}$

Mrs. Peterson's class: $\frac{11}{25}$

Now compare the ratios the same way you compare fractions. Find a common denominator and compare the numerators.

The least common denominator is 25.

$\frac{2}{5}\left(\frac{5}{5}\right) = \frac{10}{25}$

$\frac{10}{25} < \frac{11}{25}$, since 10 is less than 11.

Mrs. Peterson's class has a higher ratio of boys to total students.

Note: In Example 2, even though Mr. Collison's class had a larger number of boys in the class, Mrs. Peterson's class had a larger ratio of boys to total students.

We can also order ratios. When you have more than two ratios, you can write them in order from least to greatest or from greatest to least. Let's look at an example.

Example

Order the following ratios from least to greatest: 10 to 15, $\frac{16}{36}$, 12 : 48

The first thing to notice is that these ratios are all in different forms. Let's write them in the same form first of all. Let's work with fraction form so that we can apply what we know about comparing and ordering fractions.

10 to 15 $= \frac{10}{15}$

$$\frac{16}{36}$$

12 : 48 $= \frac{12}{48}$

Now notice that none of these fractions are in simplest form. We can simplify them and that will make it much easier to order them.

$$\frac{10}{15} = \frac{2}{3}$$
$$\frac{16}{36} = \frac{4}{9}$$
$$\frac{12}{48} = \frac{1}{4}$$

If you understand fractions, we can simply order them right now. We know that one-fourth is the smallest part. That four is almost half of nine, so that would be the middle value, and that two-third would be the greatest part.

The answer is $\frac{1}{4}, \frac{4}{9}, \frac{2}{3}$.

Well in that case, you could rewrite them all using a common denominator. Then you will be able to order the numerators. Let's take a look at that.

$$\frac{1}{4} = \frac{9}{36}$$
$$\frac{4}{9} = \frac{16}{36}$$
$$\frac{2}{3} = \frac{24}{36}$$

You can see that our original work was correct.

II. **Find and Apply Unit Rates**

A *unit rate* is a special kind of ratio, where the second number, or the denominator, is equal to one. With a unit rate, you are comparing a quantity to one. Some common examples of unit rates are miles per gallon, price per pound, and pay rate per hour.

To find a unit rate, simplify the ratio so that you have a 1 in the denominator. You can simply divide the first number in the ratio by the second. Make sure you keep track of the units.

Write this information on unit rates down in your notebook. Then continue with the example.

Example

Kayla bought 5.5 pounds of apples. She paid a total of $7.15. What was the unit rate of the apples per pound?

A key word is here "per pound". When you see the word "per", you should know that you are working with unit rates.

You need to find the price per pound. We can write the ratio of price to pounds using the information in the problem. That is what we are comparing, so that is how we write the ratio. Then we can fill in the given information.

$\frac{price}{pounds} = \frac{\$7.15}{5.5\ pounds}$

Now divide to find the price per pound. We divide the price by the number of pounds that she bought.

$5.5\overline{)7.15}$

$\frac{\$7.15}{5.5\ pounds} = \frac{\$1.30}{1\ pound}$

The unit rate is $1.30 per pound.

Example

Brian worked for 8 hours yesterday and made a total of $86. What is his pay rate?

Write a ratio comparing pay rate to hours worked. The rate that we are looking for is what Brian made "per" hour. Even though the problem doesn't use the word "per" a pay rate is per hour.

$\frac{pay\ rate}{hours\ worked} = \frac{\$86}{8\ hours}$

Now divide to find the unit rate.

$\frac{\$86}{8\ hours} = \frac{\$10.75}{1\ hour}$

Brian's pay rate is $10.75 per hour.

You may also see rates that are equal. Let's look at equivalent rates.

III. **Find and Apply Equivalent Rates**

An *equivalent rate* is a rate that is equal but is written in a different form.'

To think about equivalent rates, think about equal fractions.

To find an equivalent rate, you need to write a fraction that is equivalent to the given fraction, or rate.

Example

Find an equivalent rate for this comparison.

$$\frac{\$2.00}{1} = \frac{?}{8}$$

Notice that we have a unit rate here. We know that the unit rate is two dollars for every one thing. We want eight. We can figure out how to work on this problem by thinking mathematically.

$1 \times 8 = 8$

Just like when we worked with fractions, whichever operation we perform with the denominator must be performed with the numerator too. We multiplied by eight, so we need to do that with the numerator too.

$$\frac{\$2.00}{1} = \frac{\$16.00}{8}$$

These two rates are equivalent.

As long as you multiply the numerator and the denominator by the same value, you will always create an equivalent rate!

IV. **Solve Real - World Problems Involving Ratios and Rates**

Now that you understand ratios and rates, we can look at applying this information when working with real - world problems. Let's look at some examples.

Example

Myra's team scored 10 goals in the last 3 games. At this rate, how many goals will Myra's team score in 6 games?

First, write the ratio to show the team's scoring rate.

$\frac{goals}{games} = \frac{10\ goals}{3\ games}$

You need to know the equivalent rate for 6 games. Notice that the second ratio has the games in the same spot of the denominator. Be sure that you write the ratios so that the same quantities are being compared. If you mix them up, you get a different result.

$\frac{10\ goals}{3\ games} = \frac{?\ goals}{6\ games}$

Look at the two fractions. The denominator is doubled in the second fraction. So, multiply the first fraction by an equivalent of 1 in order to get the second fraction.

$$\frac{10 \; goals}{3 \; games} \left(\frac{2}{2}\right) = \frac{20 \; goals}{6 \; games}$$

An equivalent rate is 20 goals in 6 games. So at this rate, Myra's team will score 20 goals.

Example

A store sells salmon for $6.99 per pound. What is the rate for 6 pounds of salmon?

You know the rate per pound. You need to find the rate for 6 pounds. So you can multiply the unit rate by 6 to find the equivalent ratio.

$$\frac{\$6.99}{1 \; pound} \left(\frac{6}{6}\right) = \frac{\$41.94}{6 \; pounds}$$

The rate for 6 pounds of salmon is $41.94.

Now let's go back to the problem from the introduction and work on solving that problem.

Real-Life Example Completed

The Top of the World - Mount Everest

Here is the original problem once again. First, reread it. Then you will need to write and simplify two ratios. One ratio will represent the numbers from the best year on Everest and one ratio will represent the numbers from the worst year on Everest. There are two parts to your answer.

Josh sat at the breakfast table with his nose in a book. His Mom poured some cereal into a bowl and put it down in front of him. Karen, Josh's sister came into the kitchen and sat down next to him.

"What are you reading?" she asked.

"I am reading a book about Mount Everest," Josh said without looking up.

"Alright Josh, now put the book down and eat something," their Mom instructed.

"It is terrific. Do you know that during the best year on Everest, and best meaning the year that the least number of people died, that 129 people summitted and only 8 died," Josh said smiling.

"That hardly seems like something to smile about," Mom said sipping her coffee.

"Yeah, how morbid," Karen chimed in.

"Listen it may be morbid, but it is a fact. During the worst year, only 98 people summitted and 15 died. That is a lot of people. I mean we can compare the number of people who summitted and the number of people who didn't make it," Josh said.

"Well, it proves that Everest is a dangerous place and not a trip to be taken lightly," Mom said.

"Yes, but think about how amazing it would be to stand on the top of the world!" Josh said.

"How do they figure out that the best year is the best and the worst year is the worst based on those numbers?" Karen asked.

"They compare the ratios," Josh explained. "Let me tell you how."

Now write your ratios. Remember to simplify them. There are two parts to your answer.

Solution to Real - Life Example

We are going to write two ratios that we can compare. One ratio will represent the best year on Everest and one will represent the worst year on Everest.

First, let's look at the numbers from the best year on Everest.

129 people summitted

8 died

The ratio is 129 : 8.

However, we want to simplify this ratio to get a better idea of the size of the ratio. Let's rewrite it in fraction form and simplify it.

$\frac{129}{8} = \frac{16}{1}$

This means that for every 16 people who summitted one did not make it.

Now let's look at the numbers for the worst year on Everest.

98 people summitted

16 people died

$$\frac{98}{16} = 6.5 \; to \; 1$$

This means that for every $6\frac{1}{2}$ people who summitted, one did not make it.

From these ratios, you can see how the best year compares to the worst year on Everest.

Vocabulary

Here are the vocabulary words that are found in this lesson.

Ratio

> a way of comparing two numbers or quantities. Ratios can be written in fraction form, with a colon or by using the word "to".

Unit Rate

a ratio that is comparing a quantity to one. The word "per" is a key word with unit rates.

Equivalent Rate

two rates that are equal although different values are being used to represent the same quantities.

Resources

www.mnteverest.net

PRACTICE

Directions: Look at each ratio. Then write it in the other two ways that it can be written.

1. 16 to 3
2. 4 to 5
3. 1 : 4
4. $\frac{12}{1}$
5. 6 : 11
6. 33 to 100
7. $\frac{4}{9}$
8. 3 to 4
9. 45 to 12
10. 12 : 12

Directions: Simplify each ratio and write your answer in fraction form.

11. 4 to 12
12. 5 : 20
13. 36 to 6
14. 18 : 36
15. 20 to 100

Directions: Use what you have learned about ratios to solve each problem.

In Kyle's drawer, there are 14 pairs of white socks and 8 pairs of black socks

16. Write the ratio of black socks to white socks.
17. Write the ratio of black socks to total socks.
18. Write the ratio of white socks to total socks.
19. There are 150 apartments in the Gray building. Of these, 60 are rented and the rest are owned. There are 65 apartments in the Black building. Of these, 45 are rented and the rest are owned. Which building has a larger ratio of owned apartments to total apartments, and by how much?
20. Susan drove a total of 182 miles in three and a quarter hours. If she drove the same speed the whole drive, what was her rate in miles per hour?
21. Holly works at a library reshelving books. She reshelved 960 books in 4 hours. What is Holly's rate of reshelving in books per hour?
22. Sam bought 9.5 pounds of peaches to make a pie. The peaches cost $15.39. What was the unit rate of the peaches?

23. A vacation house costs $1,100 to rent per month. If there are 30 days in the month, what is the daily rate?
24. Julie can type 55 words per minute. What is the equivalent rate of typing 15 minutes?
25. Don can wrap 8 presents in an hour. What is Don's rate for 12 presents?

4.2 Constant Rate of Change (7.4a)

GUIDANCE

What if when you bought a koi fish it measured 4 inches long? Three months later you measure it again and it is 6 inches long? At what rate is the fish growing? After completing this Concept, you'll be able to find the rate of change taking place in problems like this one.
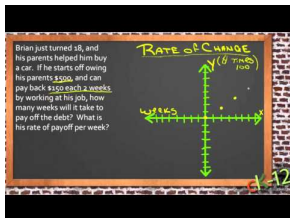
Watch This

MEDIA

Click image to the left or use the URL below.

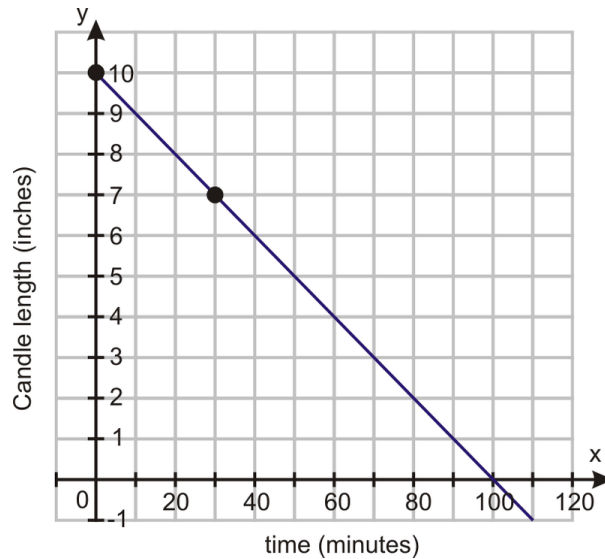URL: https://www.ck12.org/flx/render/embeddedobject/133274

The slope of a function that describes real, measurable quantities is often called a **rate of change**. In that case the slope refers to a change in one quantity (y)**per** unit change in another quantity (x). (This is where the equation $m = \frac{\Delta y}{\Delta x}$ comes in—remember that Δy and Δx represent the change in y and x respectively.)

Example A

A candle has a starting length of 10 inches. 30 minutes after lighting it, the length is 7 inches. Determine the rate of change in length of the candle as it burns. Determine how long the candle takes to completely burn to nothing.

Solution

First we'll graph the function to visualize what is happening. We have 2 points to start with: we know that at the moment the candle is lit ($time = 0$) the length of the candle is 10 inches, and after 30 minutes ($time = 30$) the length is 7 inches. Since the candle length depends on the time, we'll plot time on the horizontal axis, and candle length on the vertical axis.

The rate of change of the candle's length is simply the slope of the line. Since we have our 2 points $(x_1, y_1) = (0, 10)$ and $(x_2, y_2) = (30, 7)$, we can use the familiar version of the slope formula:

$$\text{Rate of change} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{(7 \text{ inches}) - (10 \text{ inches})}{(30 \text{ minutes}) - (0 \text{ minutes})}$$
$$= \frac{-3 \text{ inches}}{30 \text{ minutes}}$$
$$= -0.1 \text{ inches per minute}$$

Note that the slope is negative. A negative rate of change means that the quantity is decreasing with time—just as we would expect the length of a burning candle to do.

To find the point when the candle reaches zero length, we can simply read the $x-$intercept off the graph (100 minutes). We can use the rate equation to verify this algebraically:

$$\text{Length burned} = \text{rate} \times \text{time}$$
$$10 = 0.1 \times 100$$

Since the candle length was originally 10 inches, our equation confirms that 100 minutes is the time taken.

Example B

The population of fish in a certain lake increased from 370 to 420 over the months of March and April. At what rate is the population increasing?

Solution

Here we don't have two points from which we can get $x-$ and $y-$coordinates for the slope formula. Instead, we'll need to use the alternate formula, $m = \frac{\Delta y}{\Delta x}$.

The change in $y-$values, or Δy, is the change in the number of fish, which is $420 - 370 = 50$. The change in $x-$values, Δx, is the amount of time over which this change took place: two months. So $\frac{\Delta y}{\Delta x} = \frac{50 \text{ fish}}{2 \text{ months}}$, or **25 fish per month.**

Interpret a Graph to Compare Rates of Change

Example C

The graph below represents a trip made by a large delivery truck on a particular day. During the day the truck made two deliveries, one taking an hour and the other taking two hours. Identify what is happening in the first 3 stages of the trip (stages A through C).
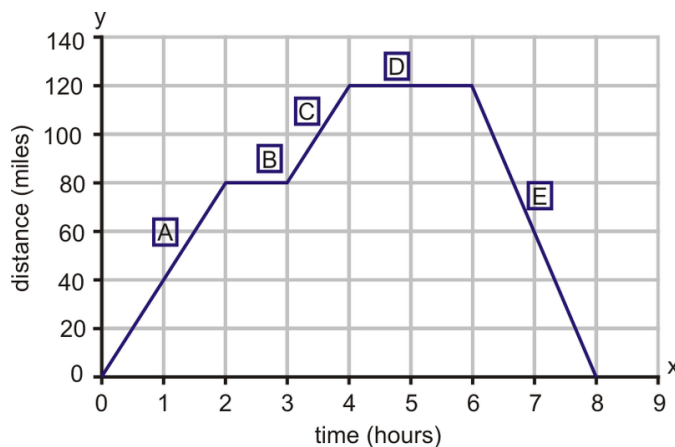
Solution:

The first 3 stages of the trip are:

A. The truck sets off and travels 80 miles in 2 hours.

B. The truck covers no distance for 2 hours.

C. The truck covers $(120 - 80) = 40$ miles in 1 hour.

A. Rate of change $= \frac{\Delta y}{\Delta x} = \frac{80 \text{ miles}}{2 \text{ hours}} = 40$ miles per hour

Notice that the rate of change is a **speed**—or rather, a **velocity.** (The difference between the two is that velocity has a direction, and speed does not. In other words, velocity can be either positive or negative, with negative velocity representing travel in the opposite direction. You'll see the difference more clearly in part E.)

Since velocity equals distance divided by time, the slope (or rate of change) of a distance-time graph is always a velocity.

So during the first part of the trip, the truck travels at a constant speed of 40 mph for 2 hours, covering a distance of 80 miles.

B. The slope here is 0, so the rate of change is 0 mph. The truck is stationary for one hour. This is the first delivery stop.

C. Rate of change $= \frac{\Delta y}{\Delta x} = \frac{(120-80) \text{ miles}}{(4-3) \text{ hours}} = 40$ miles per hour. The truck is traveling at 40 mph.

Watch this video for help with the Examples above.

MEDIA
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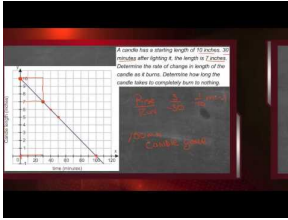
GUIDED PRACTICE

Example

Continue where we left of in Example C, by identifying what is happening in the last two stages of the trip (stages D and E).
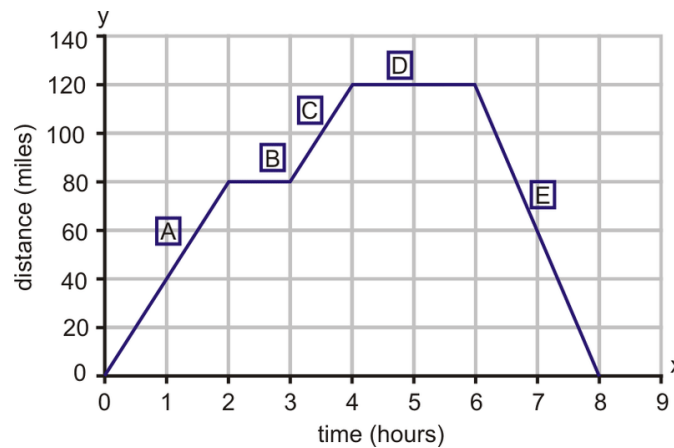
Solution:

The last two stages of the trip are:

D. The truck covers no distance for 1 hour.

E. The truck covers -120 miles in 2 hours.

Let's find the slopes:

D. Here the slope is 0, so the rate of change is 0 mph. The truck is stationary for two hours. This is the second delivery stop. At this point the truck is 120 miles from the start position.

E.

$$
\begin{aligned}
\text{Rate of change} &= \frac{\Delta y}{\Delta x} \\
&= \frac{(0-120)\text{ miles}}{(8-6)\text{ hours}} \\
&= \frac{-120\text{ miles}}{2\text{ hours}} \\
&= -60\text{ miles per hour.}
\end{aligned}
$$

The truck is traveling at *negative* 60 mph.

Wait - a negative speed? Does that mean that the truck is reversing? Well, probably not. It's actually the *velocity* and not the speed that is negative, and a negative velocity simply means that the distance *from the starting position* is decreasing with time. The truck is driving in the opposite direction - back to where it started from. Since it no longer has 2 heavy loads, it travels faster (60 mph instead of 40 mph), covering the 120 mile return trip in 2 hours. Its *speed* is 60 mph, and its *velocity* is -60 mph, because it is traveling in the opposite direction from when it started out.

Example

Jill's floral shop charges $24.00 for 2 dozen roses, 36.00 for 3 dozen roses, and $60.00 for 5 dozen roses. What is the rate of change?

Roses	Price
2 dz.	$24
3 dz.	$36
5 dz.	$60

FIGURE 4.1

$$\text{Rate of change} = \frac{\Delta y}{\Delta x} = \frac{(\$36-24)\text{ cost}}{(3-2)\text{ dzn.}} = 12 \text{ dollars per dozen.}$$

PRACTICE

For 1-6, the graph below is a distance-time graph for Mark's three and a half mile cycle ride to school. During this ride, he rode on cycle paths but the terrain was hilly. He rode slower up hills and faster down them. He stopped once at a traffic light and at one point he stopped to mend a punctured tire. The graph shows his distance from home at any given time. Identify each section of the graph accordingly.
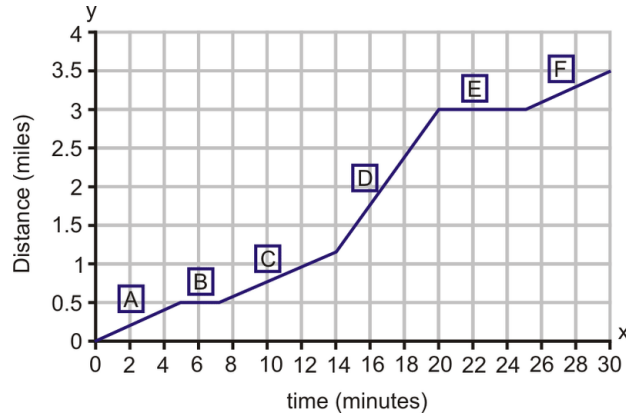
1. Section A.
2. Section B.
3. Section C.
4. Section D.
5. Section E.
6. Section F.

For 7-12, approximate the rate of change of each part of Mark's ride.

7. Section A.
8. Section B.
9. Section C.
10. Section D.
11. Section E.
12. Section F.

For 13 and 14, find the rate of change.

13. Jake ran five miles in fifteen minute, ten miles in thirty minutes, and twenty miles in sixty minutes.

14. Laura car can travel 81 miles with 3 gallons of gas, 108 miles with 4 gallons, and 135 miles with 5 gallons.

4.3 Calculating Unit Rates (7.4b)

GUIDANCE

There is a special kind of rate which is called a ***unit rate.***

A unit rate has a denominator of 1, meaning that it is the measure for one of whatever you are talking about: 1 mile, 1 pound, 1 foot, etc.

When we talk about unit rates, we are talking about singular measurement and single rates.

1 pound of apples is $1.79.

This is a unit rate. It tells us that for 1 pound of apples, we will pay $1.79. Based on this number, we can figure out the rate for 2 pounds, 3 pounds, 5 pounds, etc.

This problem told us what the unit rate was, but sometimes you will need to figure out the unit rate. How do we figure out the unit rate?

To figure out the unit rate, we have to write the ratio as a fraction with a denominator of one.

Rochelle ran 15 miles in 2 hours. Express her speed as a unit rate.

To express this as a unit rate, we need to figure out Rochelle's speed for one mile. Begin by setting up the rate as a fraction.

$$15 \; miles \; \text{in} \; 2 \; hours = \frac{15mi}{2h} = \frac{15}{2}$$

The second number in a unit rate is 1. Since $2 \div 2 = 1$, you can divide both terms by 2 to write this as a unit rate.

185

$$\frac{15mi}{2h} = \frac{15}{2} = \frac{15 \div 2}{2 \div 2}$$

Use long division to divide 15 by 2.

$$\begin{array}{r} 7.5 \\ 2 \overline{)\ 15.0} \\ -14 \\ \hline 10 \\ -10 \\ \hline 0 \end{array}$$

So, $\frac{15mi}{2h} = \frac{15 \div 2}{2 \div 2} = \frac{7.5}{1} = \frac{7.5mi}{1h}$ or 7.5 miles per hour.

The unit rate was 7.5 miles in 1 hour, or 7.5 miles per hour.

A quicker strategy for finding a unit rate is to simply divide the first term by the second term.

Notice that to find the unit rate in the first situation, you could have simply divided 15 miles by 2 hours to find the unit rate in miles per hour. Notice that that is exactly what we did. The only difference is that we showed *why* that strategy works by writing the original rate as a fraction and then showing how to simplify it to find an equivalent rate with a 1 in the denominator.

Try this out on this one.

Kyle ran 4 miles in 28 minutes. How fast did he run per mile?

"Per mile" let's us know that we are looking for his rate for 1 mile. That is the unit rate.

We write a fraction ratio and divide.

$$\frac{min}{miles} = \frac{28}{4}$$
$$28 \div 4 = 7$$
$$\frac{7}{1}$$

Kevin ran 1 mile in 7 minutes. That is the unit rate.

Examples

Try a few on your own. Find each unit rate.

Example A

5 pounds of apples for $2.00

Example B

45 miles for 3 gallons of gasoline

Example C

18 inches in 2 minutes

Solutions

Solution A: 1 pound for $.40

Solution B: 15 miles for 1 gallon

Solution C: 9 inches in 1 minute

Introduction

Here is the original problem once again.

Manuel loves to read. He found a series of mystery books that take place in the world of medieval knights, and he has been working his way through the series. There are twelve books in the series.

After the first 5 weeks of school, Manuel has finished 8 of the 12 books.

"How many have you finished?" his sister Sarina asked at breakfast.

"I have read 8 out of 12," Manuel said. "I have four more to go to finish the series."

"How long is that going to take you?" Sarina asked.

Manuel had to think for a few minutes about this. Then he took out his notebook to do some figuring.

Think about this with Manuel. At this rate, how many books is he reading per week? How many weeks will it take him to read all 12?

There are two questions here to figure out. Let's tackle the first one. We need to figure out how many books Manuel is reading per week. The word "per" lets us know that we want to figure out the number of books for one week. Let's write a ratio to compare books read to weeks.

$\frac{8 \ books}{5 \ weeks}$

Next, we need to figure that out for 1 week. To do this, we divide 8 by 5.

$5)\overline{8.0}$ with 1.6 on top

Manuel reads 1.6 books per week. We could also say that he reads a little more than $1\frac{1}{2}$ books per week.

Given this rate, how long will it take him to read 12 books? We can set up a pair of ratios to figure this out.

$$\frac{1.6 \; books}{1 \; week} = \frac{12 \; books}{x \; weeks}$$

If we divide 12 by 1.6, we will have the number of weeks.

$$1.6\overline{)12.00} \;\; 7.5$$

It will take Manuel $7\frac{1}{2}$ weeks to finish the series. In $2\frac{1}{2}$ more weeks he will finish the series.

Vocabulary

Rate

A special kind of ratio that compares two different quantities.

Per

a word that signals you that a rate is being used.

Unit Rate

a rate that is compared to 1. A rate can be for 1 pound, 1 mile, 1 second, 1 of any unit.

GUIDED PRACTICE

Example

Maria ran 5 miles in 60 minutes. Her brother Mario ran the same distance in half the time. What is Mario's rate per mile?

Answer

To figure this out, first notice that we are looking for Mario's unit rate. To find this, we need to start with figuring out Maria's. Then we know that Mario ran the same distance in half the time, so we can say that Mario's unit rate is one-half of Maria's. This means that the time that it takes Maria to run 1 mile, that Mario runs 1 mile in one-half the time.

$\frac{1}{2}$ **(Maria's unit rate) = Mario's unit rate**

First, we need to figure out Maria's unit rate.

$$\frac{60 \; min}{5 \; miles} = \frac{12 \; min}{1 \; mile}$$

Maria runs 1 mile in 12 minutes.

Mario runs the same distance in **half** the time. Mario runs one mile in 6 minutes.

Example

At a school book fair John bought 10 car racing decorative pencils for $2.00. How can John find the cost of each pencil?
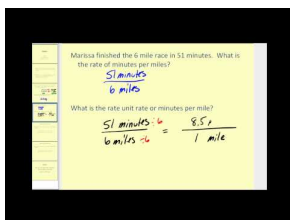
To figure this out, we need to find the unit cost of a pencil.

$\frac{2.00\ cost}{10\ pencils} = \frac{x\ cost}{1\ pencil}$

Notice that you can divide 10 by 10 to get one. Therefore we need to divide \$2.00 by 10.

So, $\frac{2.00\ cost}{10\ pencils} = \frac{2.00 \div 10}{10 \div 10} = \frac{.20\ cost}{1\ pencil}$ or .20 per pencil.

Video Review

MEDIA

Click image to the left or use the URL below.

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PRACTICE

<u>Directions:</u> Write a unit rate for each ratio.

1) 2 for \$10.00

2) 3 for \$15.00

3) 5 gallons for \$12.50

4) 16 pounds for \$40.00

5) 18 inches for \$2.00

6) 5 pounds of blueberries for \$20.00

7) 40 miles in 80 minutes

8) 20 miles in 4 hours

9) 10 feet in 2 minutes

10) 12 pounds in 6 weeks

11) 14 pounds for $7.00

12) 18 miles in 3 hours

13) 21 inches of cloth costs $7.00

14) 45 miles on 3 gallons of gasoline

15) 200 miles in 4 hours

4.4 Determining Constant of Proportionality (7.4c)

GUIDANCE

When two variables have a **proportional** relationship, the constant ratio between them is called the **Constant of proportionality, k** or unit rate. This **constant** of proportionality, **k** of the two variable quantities is expressed as

$k = \dfrac{y}{x}$ **or** $y = kx$**.** The constant of proportionality may also be called the constant of variation or direct variation.

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GUIDED PRACTICE

Example

Determine the constant of proportionality, k.

If the constant of proportionality between the two values, x and y, are equal then there is proportional relationship betwee

FIGURE 4.2

x	4	5	6	7
y	16	20	24	28

$k = \frac{y}{x} = \frac{16}{4}$
$k = 4;$ $k = \frac{y}{x} = \frac{20}{5}$
$k = 4;$ $k = \frac{y}{x} = \frac{24}{6}$
$k = 4;$ $k = \frac{y}{x} = \frac{28}{7}$
$k = 4$

As you can see the constant of proportionality for each are equal to each other, therefore there is a proportional relationship.

Example

The following data is from a proportional relationship.

FIGURE 4.3

x	6	9	15	30
y	15	22.5	37.5	75

Since this is a proportional relationship, we just have to select one set of values to determine the constant of proportionality.

$$k = \frac{y}{x} = \frac{15}{6}$$
$$k = 15 \div 6$$
$$k = 2.5$$

Example

The constant of proportionality, k, can also be identified from equations such as:

$y = 3x$ $\qquad y = .5x$ $\qquad y = \frac{3}{4}x$

To identify the constant of proportionality, k, look for the number that is in front of the variable when the equation is in the form $y = kx$.

$y = 3x$
$k = 3$ $\qquad y = .5x$
$k = .5$ $\qquad y = \frac{3}{4}x$
$k = \frac{3}{4}$

Example

In addition, you can identify the constant of proportionality, k, from a graph. For instance, look at the following graph.
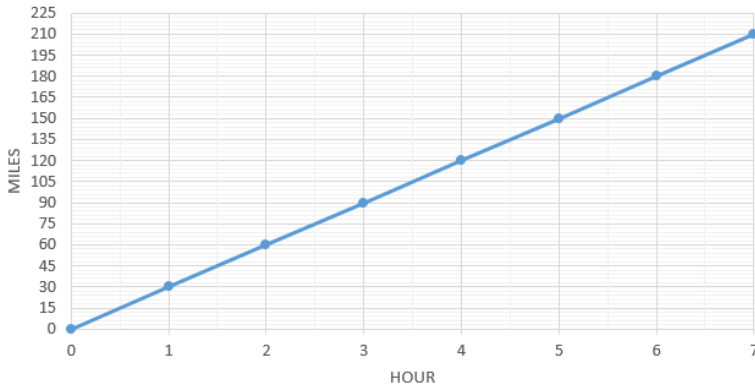
Can you determine the value of 'k'?

FIGURE 4.4

If you recall $k = \frac{y}{x}$, therefore we need to identify an ordered pair, (x,y). You may select any 'x' value and the corresponding 'y' value from the graph.
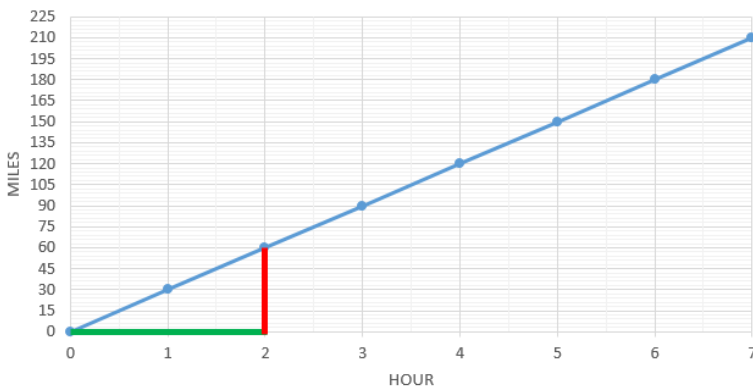

FIGURE 4.5

If we follow the green line to the right we see value of 'x' is 2 and if we follow the red line up we see the value of 'y' is 60.

Therefore, x=2 and y=60 the ordered pair is (2,60). Using $k = \frac{y}{x}$ we can substitute the values from the ordered pair, remember x=2 and y=60;

$Step1 : k = \frac{60}{2}$
$Step2 : k = \frac{30}{1}$
$Step3 : k = 30.$

So, the constant of proportionality is 30.

PRACTICE

Find the constant of proportionality from the table below. Show your work.

1.

x	1	2	3	5
y	5	10	15	25

Find the constant of proportionality from the table below. Show your work.

2.

x	2	4	6	8
y	11	22	33	44

FIGURE 4.6

Find the constant of proportionality from the table below. Show your work.

3.

x	3	5	7	9
y	6.75	11.25	15.75	20.25

Find the constant of proportionality from the table below. Show your work.

4.

x	$4\frac{1}{2}$	6	$7\frac{1}{2}$	9
y	$5\frac{2}{5}$	$7\frac{1}{5}$	9	$10\frac{4}{5}$

Find the constant of proportionality from the equations below:

5. $y = 2x$ 6. $y = 3x$

7. $y = .25x$ 8. $y = .4x$

FIGURE 4.7

9. $y = \frac{1}{2}x$ 10. $y = \frac{7}{8}x$

11. $y = \frac{11}{5}x$ 12. $y = \frac{2}{3}x$

13. Find the constant of proportionality from the graph below. Show your work.
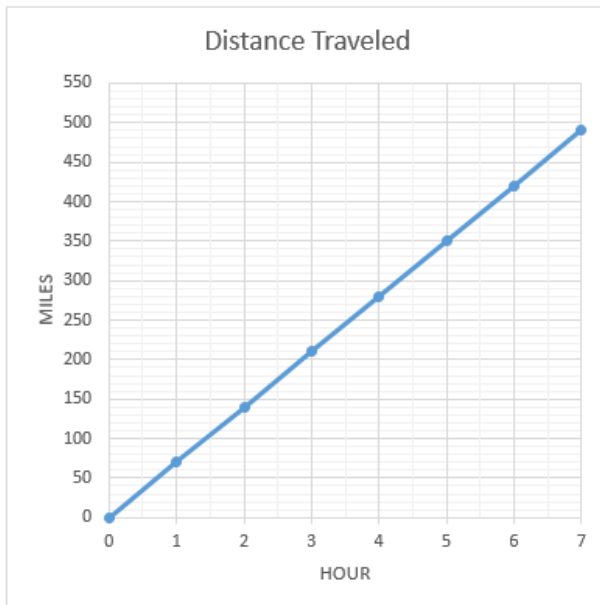
Distance Traveled

FIGURE 4.8

14. Find the constant of proportionality from the graph below. Show your work.

Pay Schedule

(Graph: Pay in Dollars vs Hour, line passing through origin up to about ($14, $105); points at ($2,$15), ($4,$30), ($6,$45), ($8,$60), ($10,$75), ($12,$90), ($14,$105))

15. Find the constant of proportionality from the graph below. Show your work.

Dessert

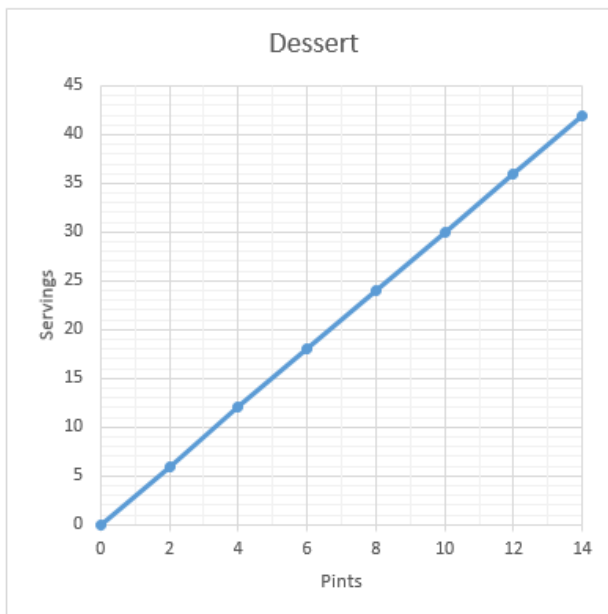(Graph: Servings vs Pints, line through origin; points at (2,6), (4,12), (6,18), (8,24), (10,30), (12,36), (14,42))

Vocabulary:

- Multiple Representation
- Constant Rate of Change
- Unit Rate
- Constant of Proportionality
- Proportional
- Linear
- Direct Variation
- Equation
- Value

CHAPTER **5**

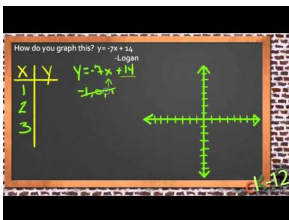# Proportion and Non-Proportional Relationships

Chapter Outline

7.7a: represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$

5.1 Graphs of Linear Relationships (7.7a)

GUIDANCE

If a bowling alley charged $3 for the first game and $2 for each additional game, could you graph the relationship and use it to find the cost of playing 5 games? After completing this Concept, you'll be able to graph and analyze linear functions like this one.

Watch This

| MEDIA |
| Click image to the left or use the URL below. |
| URL: https://www.ck12.org/flx/render/embeddedobject/133268 |

You're stranded downtown late at night with only $8 in your pocket, and your home is 6 miles away. Two cab companies serve this area; one charges $1.20 per mile with an additional $1 fee, and the other charges $0.90 per mile with an additional $2.50 fee. Which cab will be able to get you home?

Graph a Linear Equation

At the end of Lesson 4.1 we looked at ways to graph a function from a rule. A rule is a way of writing the relationship between the two quantities we are graphing. In mathematics, we tend to use the words **formula** and **equation** to describe the rules we get when we express relationships algebraically. Interpreting and graphing these equations is an important skill that you'll use frequently in math.

Example A

A taxi costs more the further you travel. Taxis usually charge a fee on top of the per-mile charge to cover hire of the vehicle. In this case, the taxi charges $3 as a set fee and $0.80 per mile traveled. Here is the equation linking the cost in dollars (y) *to hire a taxi and the distance traveled in miles* (x).

$$y = 0.8x + 3$$

Graph the equation and use your graph to estimate the cost of a seven-mile taxi ride.
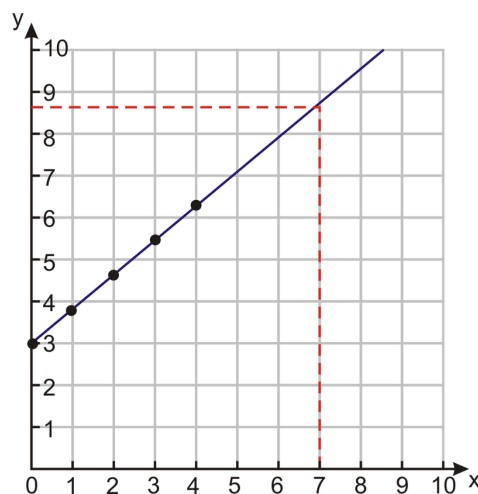
Solution

We'll start by making a table of values. We will take a few values for x (0, 1, 2, 3, and 4), find the corresponding $y-$values, and then plot them. Since the question asks us to find the cost for a seven-mile journey, we need to choose a scale that can accommodate this.

First, here's our table of values:

TABLE 5.1:

x	y
0	3
1	3.8
2	4.6
3	5.4
4	6.2

And here's our graph:

To find the cost of a seven-mile journey, first we find $x = 7$ on the horizontal axis and draw a line up to our graph. Next, we draw a horizontal line across to the $y-$axis and read where it hits. It appears to hit around half way between $y = 8$ and $y = 9$. Let's call it 8.5.

A seven mile taxi ride would cost approximately $8.50 ($8.60 exactly).

Here are some things you should notice about this graph and the formula that generated it:

- The graph is a straight line (this means that the equation is **linear**), although the function is **discrete** and really just consists of a series of points.

- The graph crosses the $y-$axis at $y = 3$ (notice that there's $a+3$ in the equation—that's not a coincidence!). This is the base cost of the taxi.
- Every time we move **over** by one square we move **up** by 0.8 squares (notice that that's also the coefficient of x in the equation). This is the rate of charge of the taxi (cost per mile).
- If we move over by three squares, we move up by 3×0.8 squares.

Example B

A small business has a debt of $500,000 incurred from start-up costs. It predicts that it can pay off the debt at a rate of $85,000 per year according to the following equation governing years in business (x) and debt measured in thousands of dollars (y).

$$y = -85x + 500$$

Graph the above equation and use your graph to predict when the debt will be fully paid.
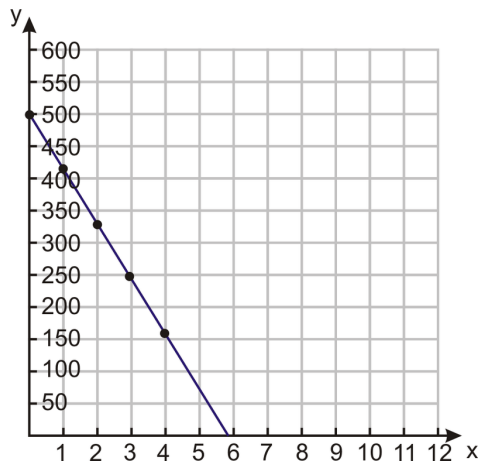
Solution

First, we start with our table of values:

TABLE 5.2:

x	y
0	500
1	415
2	330
3	245
4	160

Then we plot our points and draw the line that goes through them:

Notice the scale we've chosen here. There's no need to include any points above $y = 500$, but it's still wise to allow a little extra.

Next we need to determine how many years it takes the debt to reach zero, or in other words, what $x-$value will make the $y-$value equal 0. We know it's greater than four (since at $x = 4$ the $y-$value is still positive), so we need an $x-$scale that goes well past $x = 4$. Here we've chosen to show the $x-$values from 0 to 12, though there are many other places we could have chosen to stop.

To read the time that the debt is paid off, we simply read the point where the line hits $y = 0$ (the $x-$axis). It looks as if the line hits pretty close to $x = 6$. So **the debt will definitely be paid off in six years.**

**Analyze Graphs of Linear Functions**

We often use graphs to represent relationships between two linked quantities. It's useful to be able to interpret the information that graphs convey. For example, the chart below shows a fluctuating stock price over ten weeks. You can read that the index closed the first week at about $68, and at the end of the third week it was at about $62. You may also see that in the first five weeks it lost about 20% of its value, and that it made about 20% gain between weeks seven and ten. Notice that this relationship is discrete, although the dots are connected to make the graph easier to interpret.
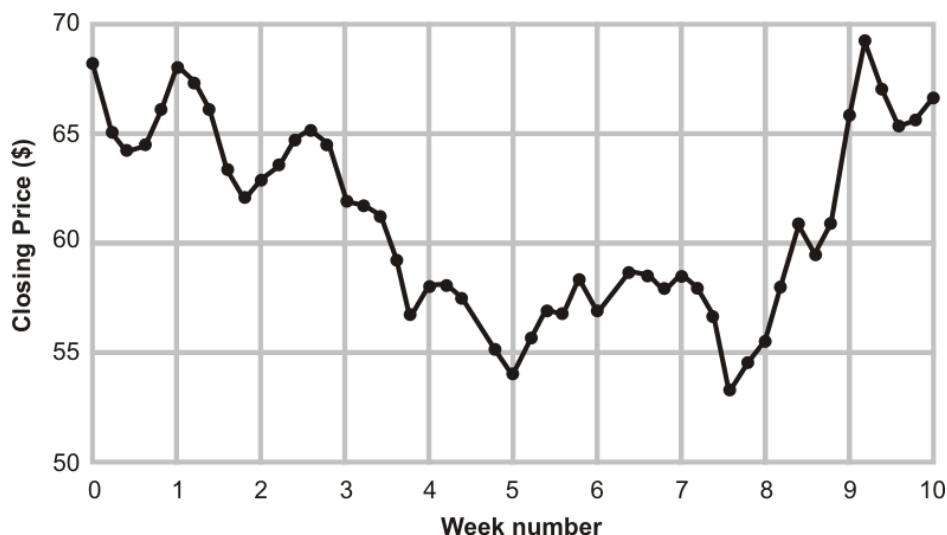
Analyzing graphs is a part of life - whether you are trying to decide to buy stock, figure out if your blog readership is increasing, or predict the temperature from a weather report. Many graphs are very complicated, so for now we'll start off with some simple linear conversion graphs. Algebra starts with basic relationships and builds to more complicated tasks, like reading the graph above.

Example C

Below is a graph for converting marked prices in a downtown store into prices that include sales tax. Use the graph to determine the cost including sales tax for a $6.00 pen in the store.
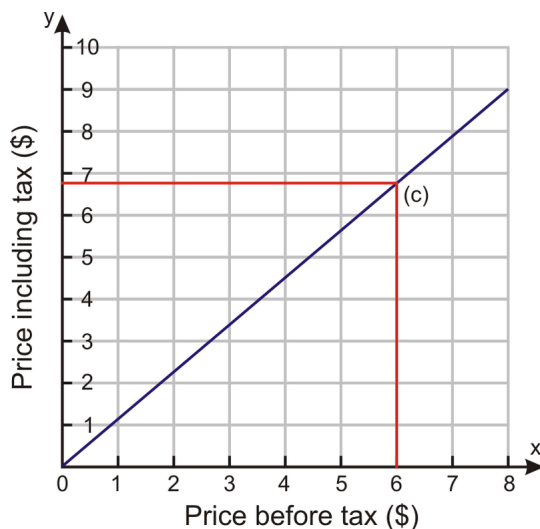
To find the relevant price with tax, first find the correct pre-tax price on the $x-$axis. This is the point $x = 6$.

Draw the line $x = 6$ up until it meets the function, then draw a horizontal line to the $y-$axis. This line hits at $y \approx 6.75$ (about three fourths of the way from $y = 6$ to $y = 7$).

The approximate cost including tax is \$6.75.

Example D

The graph for converting temperature from Fahrenheit to Celsius is shown below. Use the graph to convert the following:

a) $70°$ Fahrenheit to Celsius
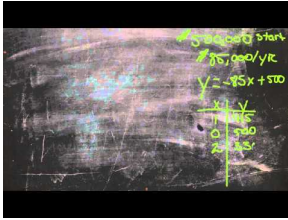
b) $0°$ Celsius to Fahrenheit
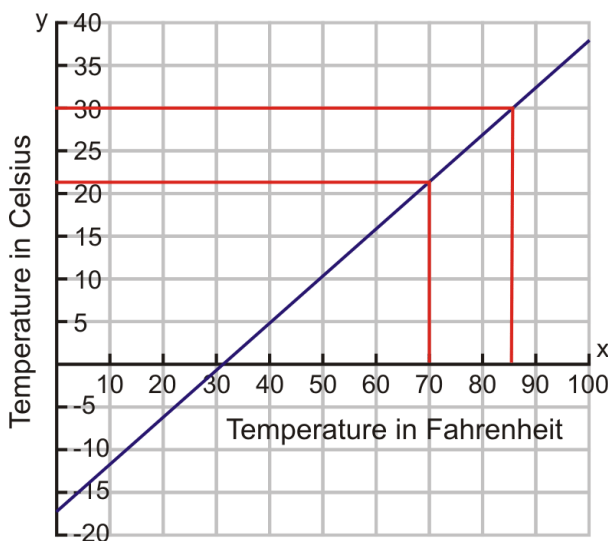
Solution:

a) To find $70°$ Fahrenheit, we look along the Fahrenheit-axis (in other words the $x-$axis) and draw the line $x = 70$ up to the function. Then we draw a horizontal line to the Celsius-axis ($y-$axis). The horizontal line hits the axis at a little over 20 (21 or 22).

$70°$ **Fahrenheit is approximately equivalent to** $21°$ **Celsius.**

b) To find $0°$ Celsius, we look at the Fahrenheit-axis (the line $y = 0$). The function hits the $x-$axis just right of 30.

$0°$ **Celsius is equivalent to** $32°$ **Fahrenheit.**

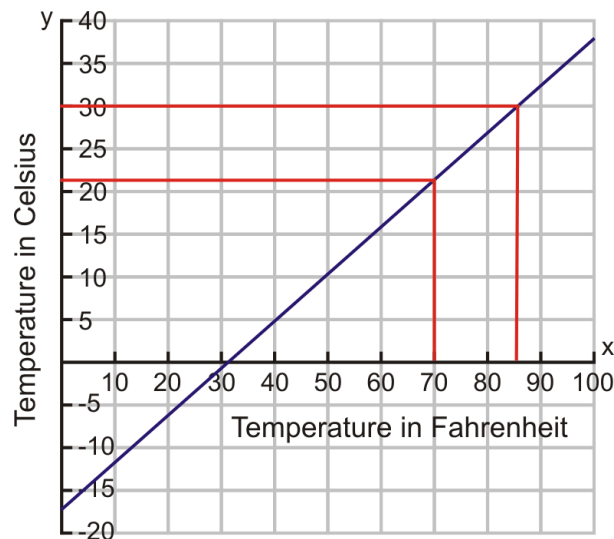Watch this video for help with the Examples above.

MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/133269

GUIDED PRACTICE

The graph for converting temperature from Fahrenheit to Celsius is shown below. Use the graph to convert the following:

a) $0°$ Fahrenheit to Celsius

b) $30°$ Celsius to Fahrenheit

Solution:

a) To find $0°$ Fahrenheit, we just look at the $y-$axis. (Don't forget that this axis is simply the line $x = 0$.) The line hits the $y-$axis just below the half way point between -15 and -20.

$0°$ **Fahrenheit is approximately equivalent to** $-18°$ **Celsius.**

b) To find $30°$ Celsius, we look up the Celsius-axis and draw the line $y = 30$ along to the function. When this horizontal line hits the function, we draw a line straight down to the Fahrenheit-axis. The line hits the axis at approximately 85.

$30°$ **Celsius is approximately equivalent to** $85°$ **Fahrenheit.**

PRACTICE

For 1-3, make a table of values for the following equations and then graph them.

1. $y = 2x + 7$
2. $y = 0.7x - 4$
3. $y = 6 - 1.25x$

4. *"Think of a number. Multiply it by 20, divide the answer by 9, and then subtract seven from the result."*

   a. Make a table of values and plot the function that represents this sentence.
   b. If you picked 0 as your starting number, what number would you end up with?
   c. To end up with 12, what number would you have to start out with?

5. At the airport, you can change your money from dollars into euros. The service costs \$5, and for every additional dollar you get 0.7 euros.

   a. Make a table for this and plot the function on a graph.
   b. Use your graph to determine how many euros you would get if you give the office \$50.
   c. To get 35 euros, how many dollars would you have to pay?
   d. The exchange rate drops so that you can only get 0.5 euros per additional dollar. Now how many dollars do you have to pay for 35 euros?

For 6-9, the graph below shows a conversion chart for converting between weight in kilograms and weight in pounds. Use it to convert the following measurements.
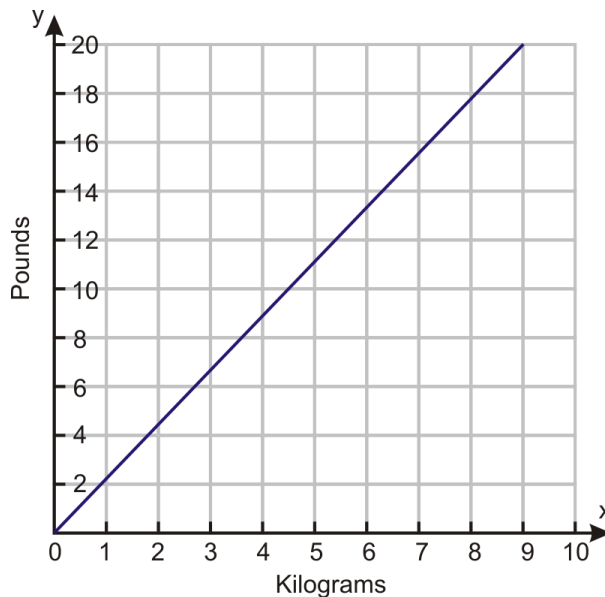
6. 4 kilograms into weight in pounds
7. 9 kilograms into weight in pounds
8. 12 pounds into weight in kilograms
9. 17 pounds into weight in kilograms

For 10-12, use the graph from problems 6-9 to answer the following questions.

10. An employee at a sporting goods store is packing 3-pound weights into a box that can hold 8 kilograms. How many weights can she place in the box?

11. After packing those weights, there is some extra space in the box that she wants to fill with one-pound weights. How many of those can she add?
12. After packing those, she realizes she misread the label and the box can actually hold 9 kilograms. How many more one-pound weights can she add?

5.2 Linear Equations (7.7a)

GUIDANCE

Alyssa, a grade eight student, was looking through the school newsletter when she saw an ad placed by a local farmer looking for students to pick strawberries. The farmer has offered to pay the students a daily rate of $15.00 plus $3.00 for each quart basket of strawberries they pick.

Alyssa is interested in applying for a job but she would like to model the information with some type of a linear equation so she can figure out how much money she could make picking strawberries in one day. Alyssa is confident she can pick 50 quarts of berries in one day.

How can Alyssa figure this out?

In this concept, you will learn to use slope-intercept form.
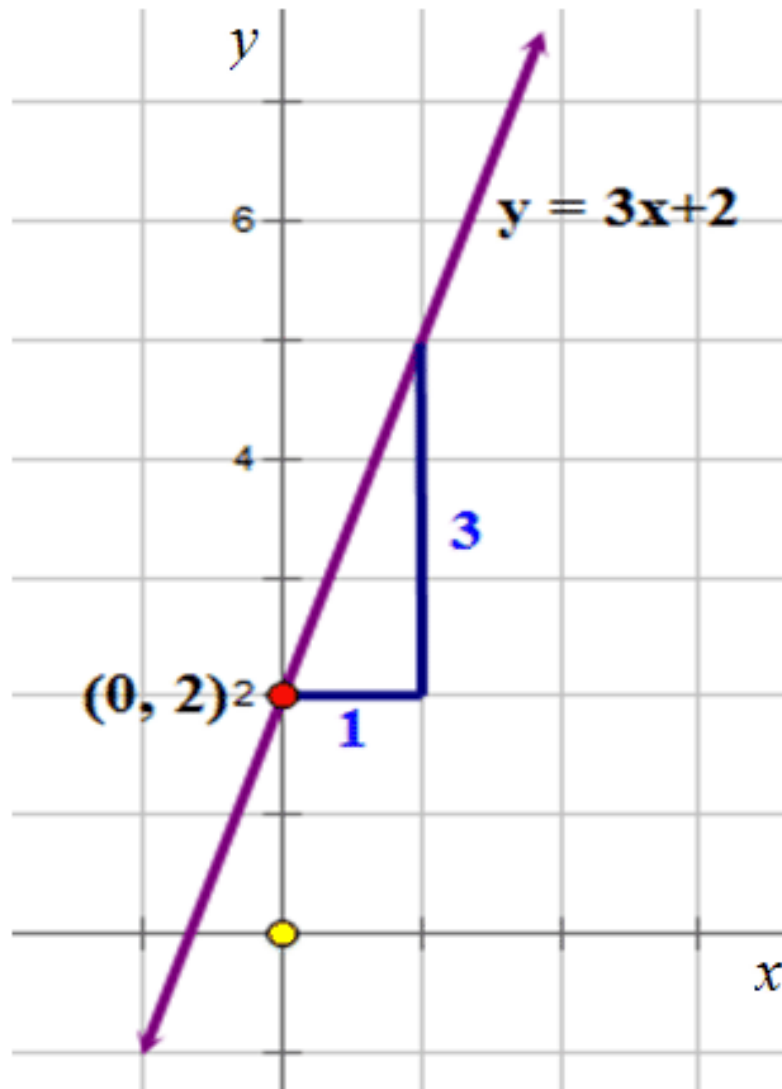
Slope Intercept Form

A linear equation can be expressed in **standard form** as $Ax + By = C$ such that $A \neq 0$ and $B \neq 0$. When the equation is written in this form the x- and y-intercepts are easily found using the formulas:

The $x-\text{intercept} = \frac{C}{A}$ and $y-\text{intercept} = \frac{C}{B}$. To find the slope of the line using the equation written in standard form requires using algebra to solve the equation for the variable 'y'.

Linear equations can also be expressed in slope-intercept form as $y = mx + b$ such that 'm' is the slope of the line and 'b' is the y-intercept of the line.

Let's look at an example:

Graph the linear equation $y = 3x + 2$.

The graph of the line crosses the *y*-axis at the point $(0,2)$. The slope of the line is one unit to the right (change in *x*) and up three units (change in *y*). The value of '*m*' is $\frac{3}{1}$.

A linear equation written in standard form can be changed to slope-intercept form using algebra.

Given the equation $3x+2y = 12$ written in standard form, rewrite it in slope-intercept form.

$$3x+2y = 12$$

First, subtract $3x$ from both sides of the equation to isolate the '*y*' term.

$$
\begin{aligned}
3x+2y &= 12 \\
3x-3x+2y &= 12-3x
\end{aligned}
$$

Next, simplify both sides of the equation.

$$
\begin{aligned}
3x-3x+2y &= 12-3x \\
2y &= 12-3x
\end{aligned}
$$

Next, rewrite the right side of the equation to correspond to $y = mx + b$.

$$\begin{aligned} 2y &= 12 - 3x \\ 2y &= -3x + 12 \end{aligned}$$

Then, divide both sides of the equation by '2' to solve for the variable 'y'.

$$\begin{aligned} 2y &= -3x + 12 \\ \frac{\overset{1}{\cancel{2}}y}{\cancel{2}} &= \frac{-3x}{2} + \frac{\overset{6}{\cancel{12}}}{\cancel{2}} \\ y &= -\tfrac{3}{2}x + 6 \end{aligned}$$

The answer is $y = -\frac{3}{2}x + 6$.

For the following equation, identify the slope and the y-intercept of the line:

$$y = -\frac{3}{2}x + 6$$

The equation is in the form

$$y = mx + b$$

The value of the slope (m) is $-\frac{3}{2}$ and the y-intercept (b) is $(0, 6)$.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Alyssa and the strawberries. She needs to figure out how much money she can make picking 50 quarts of strawberries in one day. How can she do this?

Alyssa can write an equation in slope intercept form to represent the information she knows.

First, write down the information given in the ad.

$3.00 for each quart of berries picked plus $15.00 per day.

Next, write down the slope-intercept form of an equation.

$$y = mx + b$$

Next, for the given information, what do the letters represent?

y = amount of money earned

m = amount paid for each quart of berries picked

x = number of quarts of berries picked

b = amount paid for each day worked

Next, fill in the known values into the equation.

$$
\begin{aligned}
y &= mx + b \\
y &= 3x + 15
\end{aligned}
$$

Then, using the equation fill in 50 for 'x' and simplify to calculate the total amount of money.

$$
\begin{aligned}
y &= 3x + 15 \\
y &= 3(50) + 15 \\
y &= 150 + 15 \\
y &= 165
\end{aligned}
$$

The answer is 165.

Alyssa can earn $165.00 in one day picking strawberries.

Example 2

Write the following equation in slope-intercept form and state the slope and the *y*-intercept of the line.

$$
-18x + 6y = 12
$$

First, add $18x$ to both sides of the equation and simplify.

$$
\begin{aligned}
-18x + 6y &= 12 \\
-18x + 18x + 6y &= 12 + 18x \\
6y &= 12 + 18x
\end{aligned}
$$

Next, divide both sides of the equation by '6' to solve for '*y*'.

$$
\begin{aligned}
6y &= 12 + 18x \\
\frac{\cancel{6}y}{\cancel{6}} &= \frac{\cancel{12}}{\cancel{6}} + \frac{\cancel{18}x}{\cancel{6}} \\
y &= 2 + 3x
\end{aligned}
$$

Then, express the equation in slope-intercept form.

$$
\begin{aligned}
y &= mx + b \\
y &= 3x + 2
\end{aligned}
$$

The slope of the line is '3' and the *y*-intercept is 2.

Example 3

For the following equation written in slope-intercept form, what is the slope and the y-intercept of the line?

$$y = -\frac{3}{4}x - 1$$

First, match the given equation with the general form of an equation written in slope-intercept form.

$$\begin{aligned} y &= mx + b \\ y &= -\frac{3}{4}x - 1 \end{aligned}$$

The slope of the line is represented by 'm' and has a value of $-\frac{3}{4}$. The y-intercept of the line is represented by 'b' and has a value of -1.

Example 4

Write the following equation in slope-intercept form and state the slope and the y-intercept of the line.

$$7x - 3y = 21$$

First, subtract $7x$ from both sides of the equation and simplify.

$$\begin{aligned} 7x - 3y &= 21 \\ 7x - 7x - 3y &= 21 - 7x \\ -3y &= 21 - 7x \end{aligned}$$

Next, divide both sides of the equation by '-3' to solve for 'y'.

$$\begin{aligned} -3y &= 21 - 7x \\ \frac{\overset{1}{\cancel{-3}}y}{\cancel{-3}} &= \frac{\overset{-7}{\cancel{21}}}{\cancel{-3}} - \frac{7}{-3}x \\ y &= -7 + \frac{7}{3}x \end{aligned}$$

Then, express the equation in slope-intercept form.

$$\begin{aligned} y &= mx + b \\ y &= \frac{7}{3}x - 7 \end{aligned}$$

The slope of the line is $\frac{7}{3}$ and the y-intercept is -7.

Example 5

Write the following equation in slope-intercept form and state the slope and the *y*-intercept of the line.

$$5x + y = -10$$

First, subtract $5x$ from both sides of the equation and simplify.

$$\begin{aligned} 5x + y &= -10 \\ 5x - 5x + y &= -10 - 5x \\ y &= -10 - 5x \end{aligned}$$

Then, express the equation in slope-intercept form.

$$\begin{aligned} y &= mx + b \\ y &= -5x - 10 \end{aligned}$$

The slope of the line is -5 and the *y*-intercept is -10.

PRACTICE

Look at each equation and identify the slope and the *y*-intercept by looking at each equation. There are two answers for each problem.

1. $y = 2x + 4$

2. $y = 3x - 2$

3. $y = 4x + 3$

4. $y = 5x - 1$

5. $y = \frac{1}{2}x + 2$

6. $y = -2x + 4$

7. $y = -3x - 1$

8. $y = \frac{-1}{3}x + 5$

Use what you have learned to write each in slope-intercept form and then answer each question.

9. $2x + 4y = 12$

10. Write this equation in slope-intercept form.

11. What is the slope?

12. What is the *y*-intercept?

13. $6x + 3y = 24$

14. Write this equation in slope-intercept form.

15. What is the slope?

16. What is the *y*-intercept?

17. $5x + 5y = 15$

18. Write this equation in slope-intercept form.
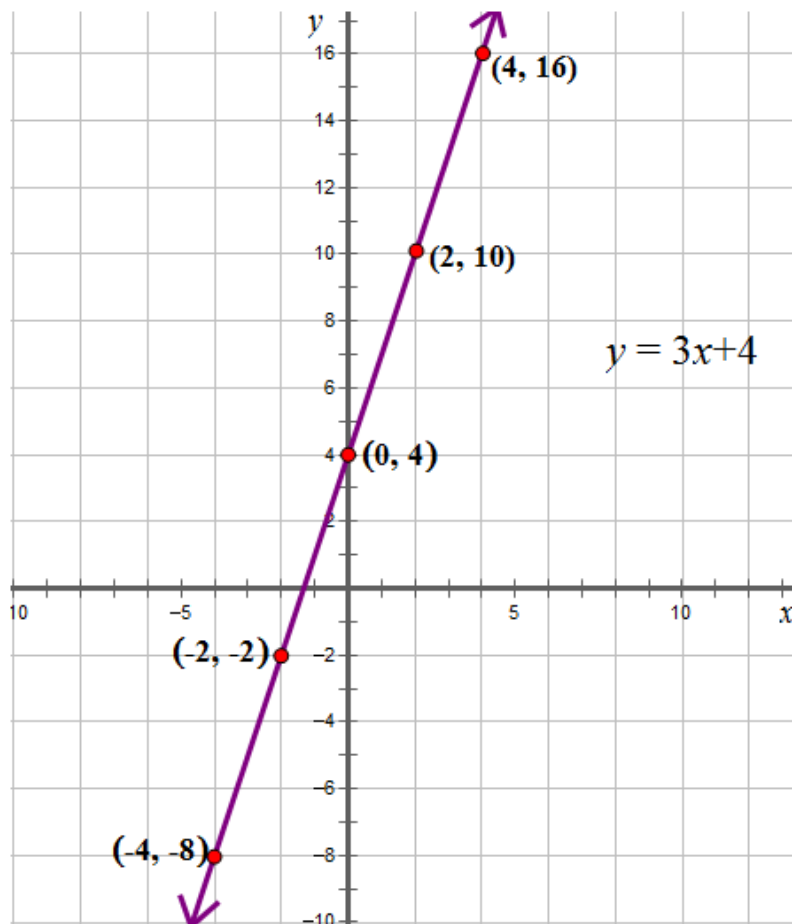
19. What is the slope?

20. What is the *y*-intercept?

5.3 Representation Table to Graph (7.7a)

GUIDANCE

Graphing Functions

Consider the following Cartesian graph that represents the equation $y = 3x + 4$.

The equation $y = 3x + 4$ is written in function form and can be used to create a table of values that will make the statement of equality true. Remember an equation written in function form can be used to determine values for the output 'y' based on the different input 'x' values substituted into the equation.

Using x-values of -4, -2, 0, 2 and 4, create a table of values to represent the equation $y = 3x + 4$.

TABLE 5.3:

x	y
-4	-8
-2	-2
0	4

<div align="center">**TABLE 5.3:** (continued)</div>

2	10
4	16

$$
\begin{aligned}
x &= -4 \\
y &= 3x+4 \\
y &= 3(-4)+4 \qquad \text{Substitute } x=-4 \text{ into the equation.} \\
y &= -12+4 \qquad \text{Perform the multiplication to clear the parenthesis.} \\
y &= -8 \qquad\quad\; \text{Simplify the right side of the equation.}
\end{aligned}
$$

Use this process to calculate the values of the output variable for each of the given input values.

$$
\begin{aligned}
\text{Given input value } x &= -2 \\
y &= 3x+4 \\
y &= 3(-2)+4 \\
y &= -6+4 \\
\text{The output value is } y &= -2
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 0 \\
y &= 3x+4 \\
y &= 3(0)+4 \\
y &= 0+4 \\
\text{The output value is } y &= 4
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 2 \\
y &= 3x+4 \\
y &= 3(2)+4 \\
y &= 6+4 \\
\text{The output value is } y &= 10
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 4 \\
y &= 3x+4 \\
y &= 3(4)+4 \\
y &= 12+4 \\
\text{The output value is } y &= 16
\end{aligned}
$$

The input value associated with the corresponding output value can be written as an ordered pair (x,y) such that $(-4,-8),(-2,-2),(0,8),(2,10)$ and $(4,16)$ are the ordered pairs that can be plotted to represent the equation $y = 3x+4$.

The ordered pairs are plotted on the Cartesian graph and are shown as red points. These points were then joined by a smooth straight line to draw the graph. The graph is a straight line such that the equation that produced this line was a **linear function**. The highest exponent of the variables of a linear function is one.

There are two special linear functions that produce a straight line graph. One of the straight lines is a vertical line that is parallel to the y-axis and the other is a horizontal line that is parallel to the x-axis.

Let's graph each of these special lines.

A line having $x = 5$ as its equation will pass through the point $(5,0)$ such that it will be parallel to the y-axis.
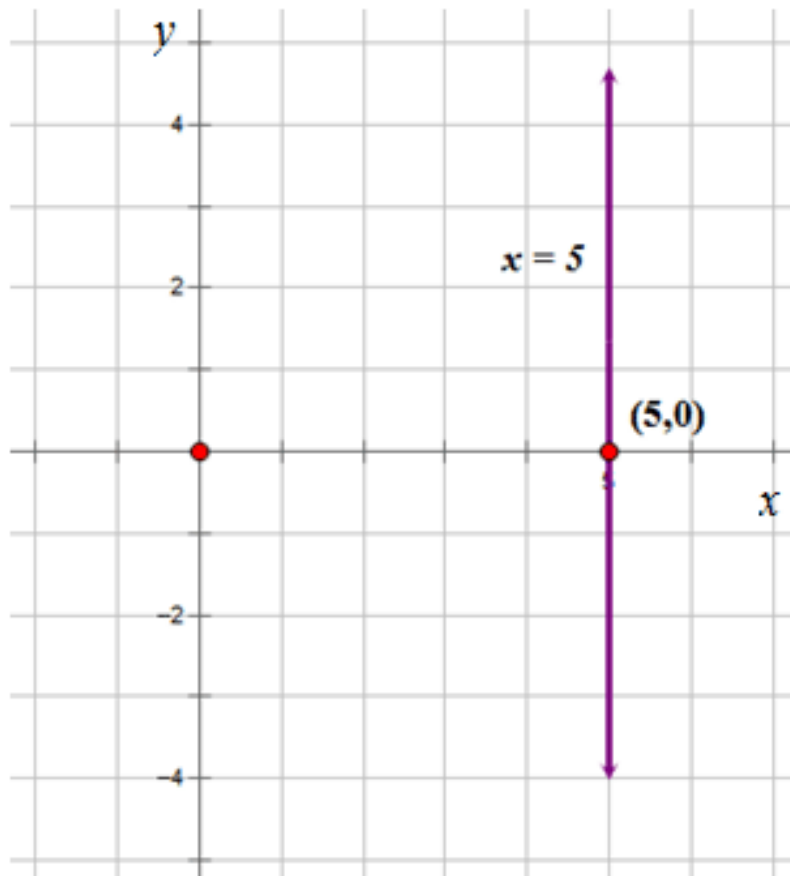
A line having $y = 4$ as its equation will pass through the point $(0, 4)$ such that it will be parallel to the x-axis.
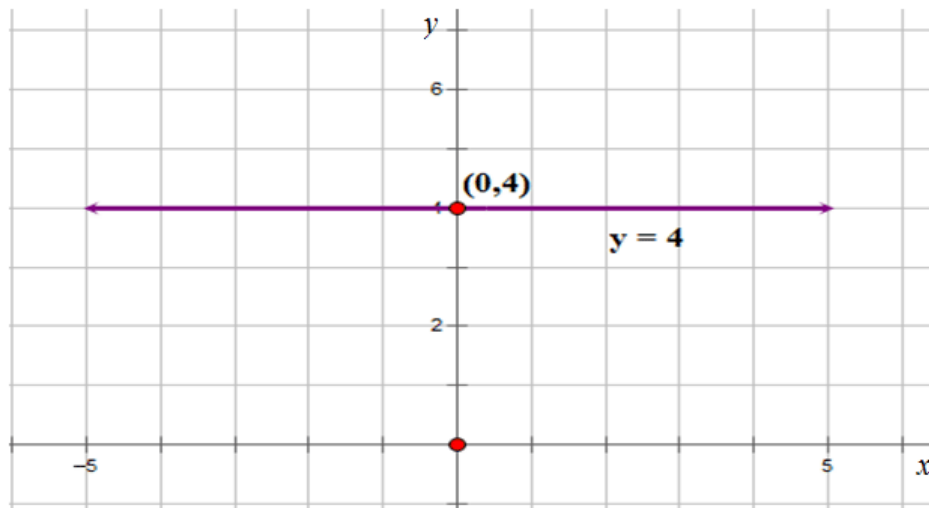
GUIDED PRACTICE

Example 1

Earlier, you were given a problem about the plotted plants and the soccer uniforms.

The students need to create a poster size graph to show how much money was raised. How can the students create this graph?

They can create a table of values and plot the ordered pairs from the table.

First, write an equation in function form to represent the sale of potted plants.

Let y represent the money raises and let x represent the number of potted plants sold.

$$y = 6.50x$$

Next, create a table of values and use the equation expressed in function form to calculate the output value for each input value.

TABLE 5.4:

x	y
50	
100	
150	
200	
250	

$$
\begin{aligned}
x &= 50 \\
y &= 6.50x \\
y &= 6.50(50) \quad &&\text{Substitute } x = 50 \text{ into the equation.} \\
y &= \$325.00 \quad &&\text{Perform the multiplication to clear the parenthesis.}
\end{aligned}
$$

Repeat the same process for the remaining input values.

$$
\begin{aligned}
\text{Given input value } x &= 100 \\
y &= 6.50x \\
y &= 6.50(100) \\
\text{The output value is } y &= \$650.00
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 150 \\
y &= 6.50x \\
y &= 6.50(150) \\
\text{The output value is } y &= \$975.00
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 200 \\
y &= 6.50x \\
y &= 6.50(200) \\
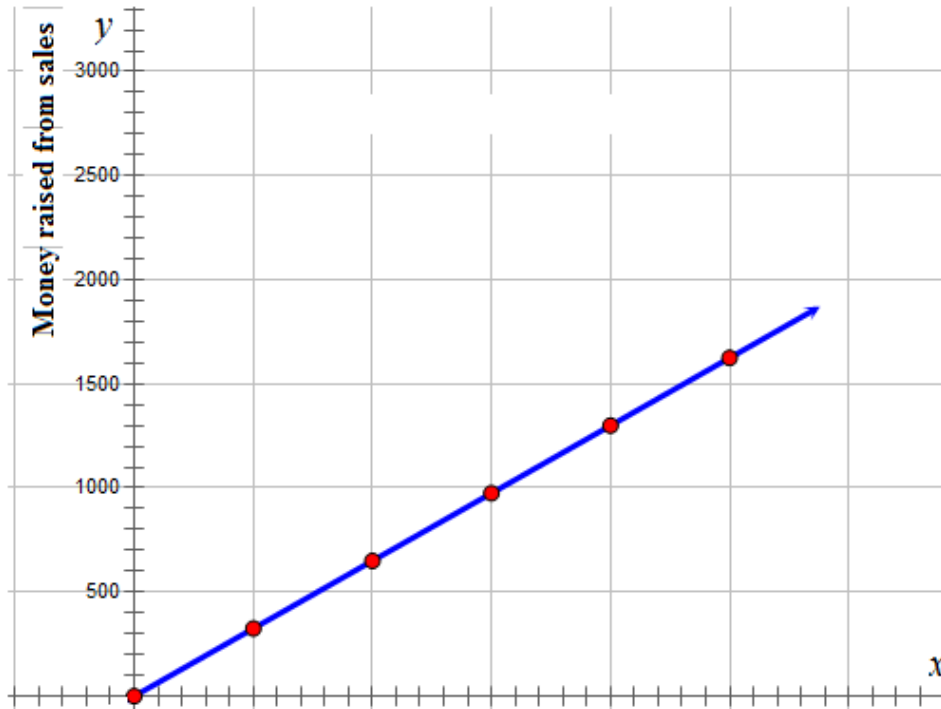\text{The output value is } y &= \$1300.00
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 250 \\
y &= 6.50x \\
y &= 6.50(250) \\
\text{The output value is } y &= \$1625.00
\end{aligned}
$$

Next, write the calculated 'y' values in the table.

TABLE 5.5:

x	y
50	325
100	650
150	975
200	1300
250	1625

Then, plot the ordered pairs shown in the table on a Cartesian grid.

The sports store will have to match the $1,625.00 raised by the students.

Example 2

For the following linear function written in function form, complete the table of values and plot the graph.

$$y = 3x - 4$$

TABLE 5.6:

x	y
-3	-13
-1	-7
1	-1
3	5
5	11

First, use the equation to calculate the output values 'y.'

$$
\begin{aligned}
x &= -3 \\
y &= 3x - 4 \\
y &= 3(-3) - 4 \qquad \text{Substitute } x = -3 \text{ into the equation.} \\
y &= -9 - 4 \qquad \text{Perform the multiplication to clear the parenthesis.} \\
y &= -13 \qquad \text{Simplify the right side of the equation.}
\end{aligned}
$$

Repeat the process to calculate the values for the variable 'y.'

$$
\begin{aligned}
\text{Given input value } x &= -1 \\
y &= 3x - 4 \\
y &= 3(-1) - 4 \\
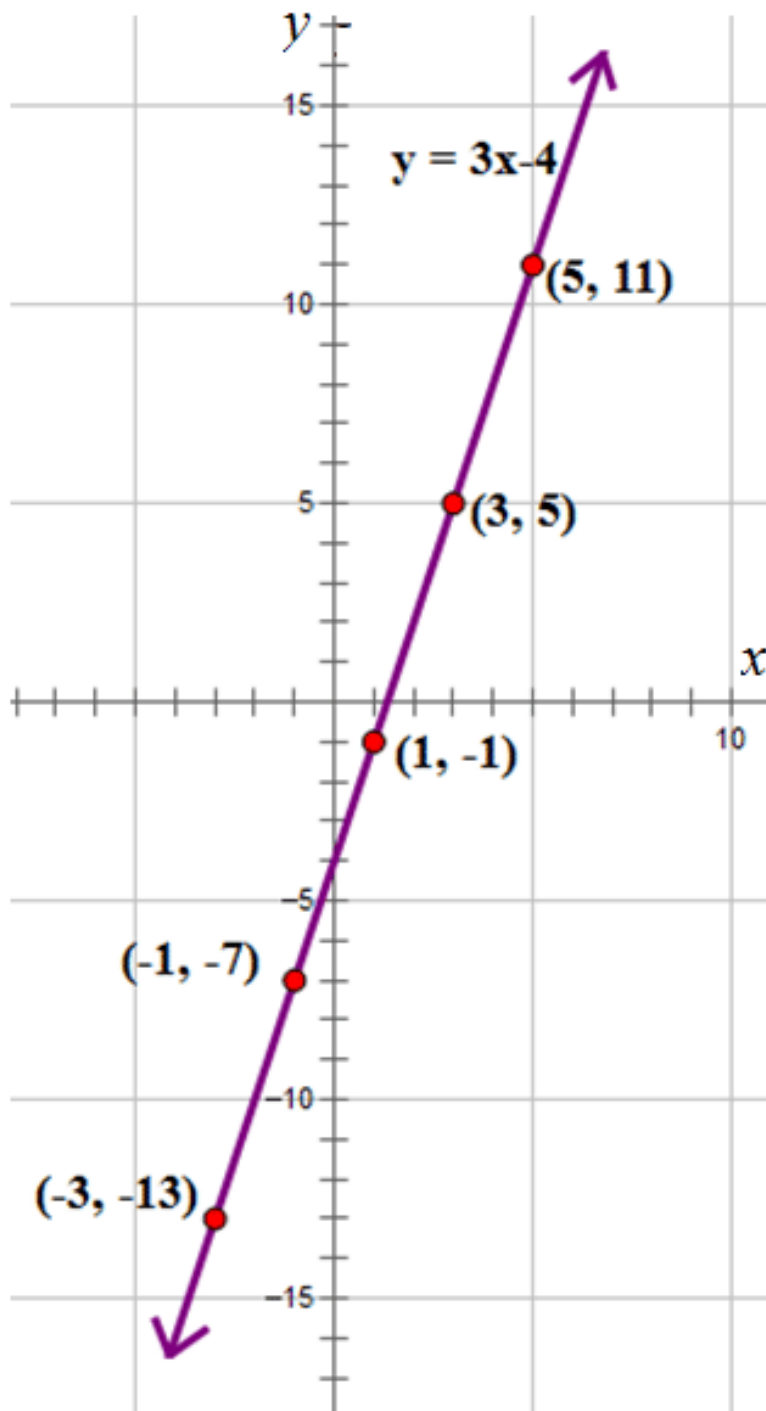y &= -3 - 4 \\
\text{The output value is } y &= -7
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 1 \\
y &= 3x - 4 \\
y &= 3(1) - 4 \\
y &= 3 - 4 \\
\text{The output value is } y &= -1
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 3 \\
y &= 3x - 4 \\
y &= 3(3) - 4 \\
y &= 9 - 4 \\
\text{The output value is } y &= 5
\end{aligned}
$$

$$
\begin{aligned}
\text{Given input value } x &= 5 \\
y &= 3x - 4 \\
y &= 3(5) - 4 \\
y &= 15 - 4 \\
\text{The output value is } y &= 11
\end{aligned}
$$

Write the calculated 'y' values in the table.

Plot the ordered pairs on the Cartesian grid and join the plotted points with a smooth, straight line.

$y = 3x-4$

$(5, 11)$

$(3, 5)$

$(1, -1)$

$(-1, -7)$

$(-3, -13)$

Example 3

For the following linear function create a table of values and plot the points to draw the graph:

$$y = 2x - 3$$

First, use the equation to calculate the output values 'y.'

$$\begin{aligned}
x &= -2 \\
y &= 2x - 3 \\
y &= 2(-2) - 3 \qquad && \text{Substitute } x = -2 \text{ into the equation.} \\
y &= -4 - 3 \qquad && \text{Perform the multiplication to clear the parenthesis.} \\
y &= -7 \qquad && \text{Simplify the right side of the equation.}
\end{aligned}$$

Repeat the process to calculate the values for the variable 'y.'

$$\begin{aligned}
\text{Given input value } x &= -1 \\
y &= 2x - 3 \\
y &= 2(-1) - 3 \\
y &= -2 - 3 \\
\text{The output value is } y &= -5
\end{aligned}$$

$$\begin{aligned}
\text{Given input value } x &= 0 \\
y &= 2x - 3 \\
y &= 2(0) - 3 \\
y &= 0 - 3 \\
\text{The output value is } y &= -3
\end{aligned}$$

$$\begin{aligned}
\text{Given input value } x &= 1 \\
y &= 2x - 3 \\
y &= 2(1) - 3 \\
y &= 2 - 3 \\
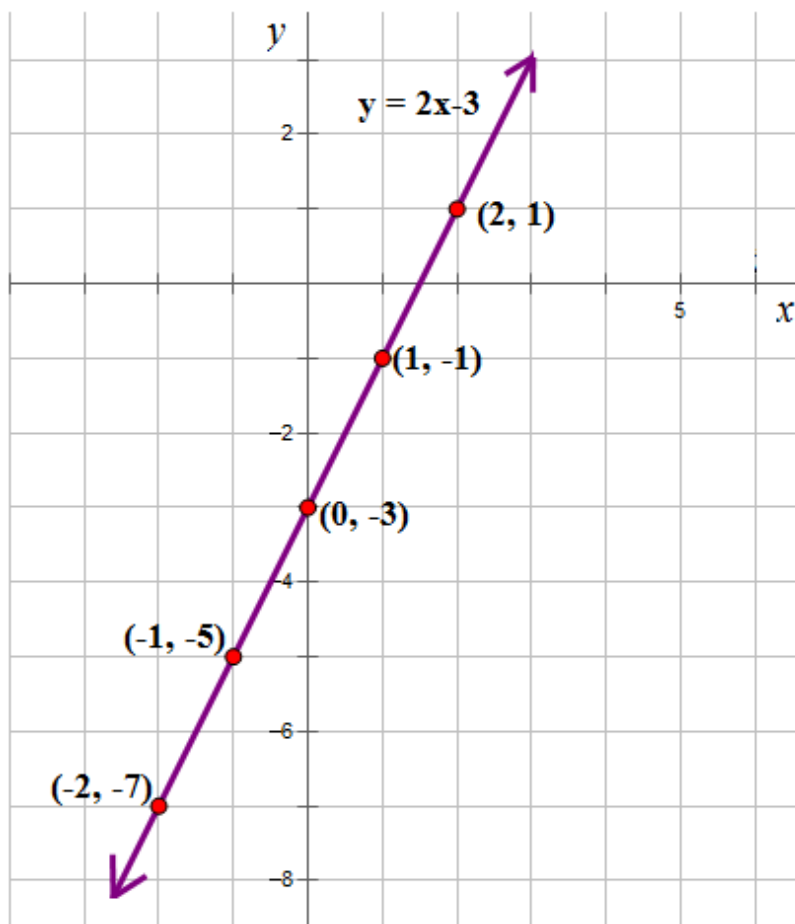\text{The output value is } y &= -1
\end{aligned}$$

$$\begin{aligned}
\text{Given input value } x &= 2 \\
y &= 2x - 3 \\
y &= 2(2) - 3 \\
y &= 4 - 3 \\
\text{The output value is } y &= 1
\end{aligned}$$

Write the calculated 'y' values in the table.

TABLE 5.7:

x	y
-2	-7
-1	-5
0	-3
1	-1
2	1

Plot the ordered pairs on the Cartesian grid and join the plotted points with a smooth, straight line.
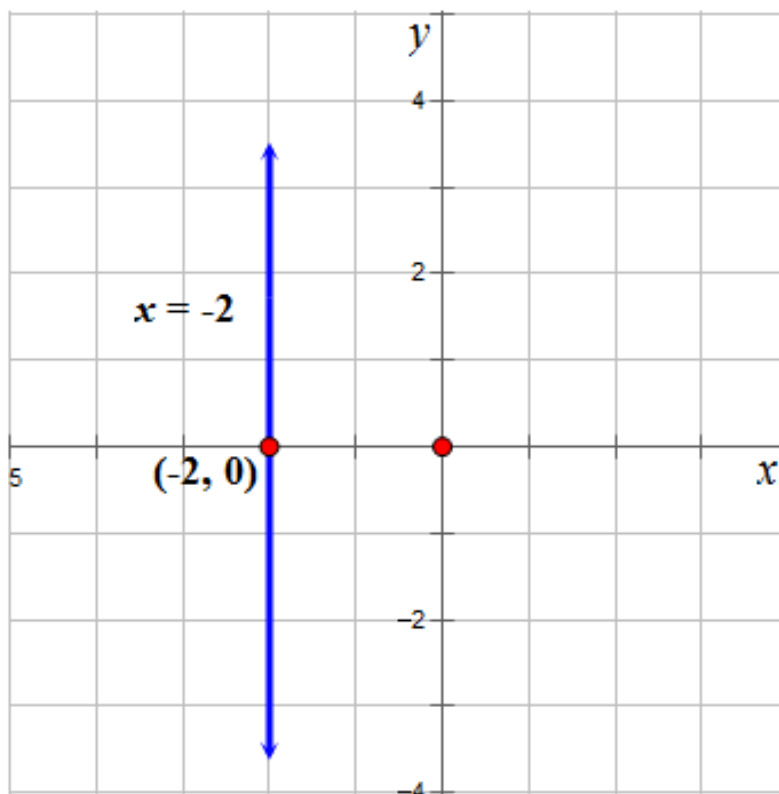
Example 4

Plot the graph of the line having $x = -2$ as its equation.

First, remember this is the graph of one of the special lines.

Next, describe what the graph will look like.

A vertical line passing through the point $(-2, 0)$ and parallel to the y-axis.

Then, graph the line on the Cartesian grid.

Example 5

Plot the graph of the line having $y = -3$ as its equation.

First, remember this is the graph of one of the special lines.

Next, describe what the graph will look like.

A horizontal line passing through the point $(0, -3)$ and parallel to the x-axis.

Then, graph the line on the Cartesian grid.
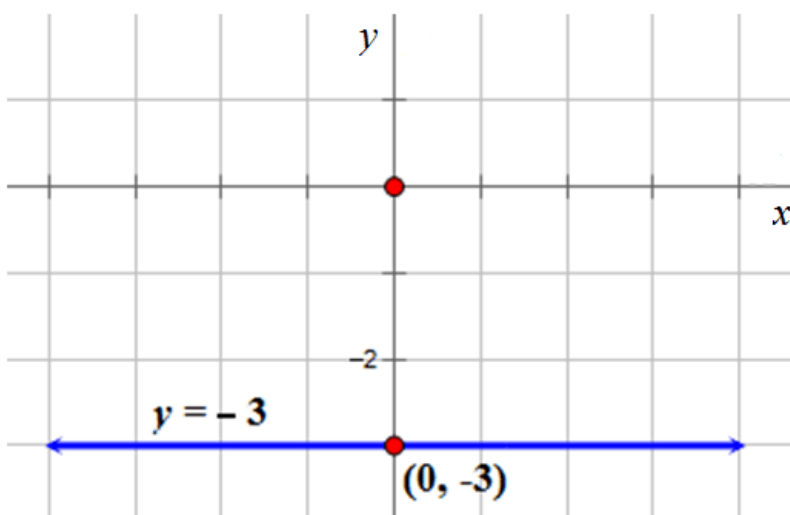
PRACTICE

Create a table of values for each equation and then graph it on the coordinate plane.

1. $y = 2x + 1$
2. $y = 3x + 2$
3. $y = -4x$
4. $y = -2x$
5. $y = -3x + 3$
6. $y = 2x + 3$
7. $y = 3x - 2$
8. $y = -8x$
9. $y = 3x + 1$
10. $y = 4x$
11. $y = -2x + 2$
12. $y = 2x - 2$
13. $y = x - 1$
14. $x = 4$
15. $y = -2$

5.4 Finding Slope (7.7a)

GUIDANCE

Learning Objectives

- Find positive and negative slopes.
- Recognize and find slopes for horizontal and vertical lines.
- Understand rates of change.
- Interpret graphs and compare rates of change.

Introduction

Wheelchair ramps at building entrances must have a slope between $\frac{1}{16}$ and $\frac{1}{20}$. If the entrance to a new office building is 28 inches off the ground, how long does the wheelchair ramp need to be?

We come across many examples of slope in everyday life. For example, a slope is in the pitch of a roof, the grade or incline of a road, or the slant of a ladder leaning on a wall. In math, we use the word **slope** to define steepness in a particular way.

$$\text{Slope} = \frac{\text{distance moved vertically}}{\text{distance moved horizontally}}$$

To make it easier to remember, we often word it like this:

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$
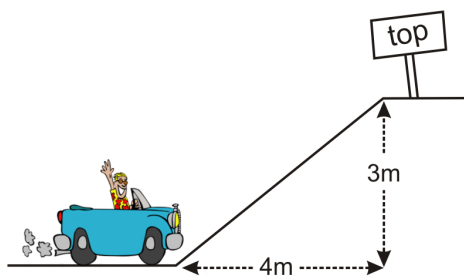
In the picture above, the slope would be the ratio of the **height** of the hill to the horizontal **length** of the hill. In other words, it would be $\frac{3}{4}$, or 0.75.

If the car were driving to the **right** it would **climb** the hill - we say this is a positive slope. Any time you see the graph of a line that goes up as you move to the right, the slope is **positive**.

If the car kept driving after it reached the top of the hill, it might go down the other side. If the car is driving to the **right** and **descending**, then we would say that the slope is **negative**.
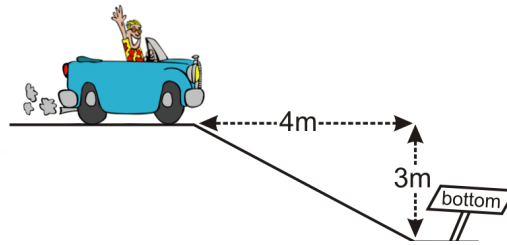
Here's where it gets tricky: If the car turned around instead and drove back down the left side of the hill, the slope of that side would still be positive. This is because the rise would be -3, but the run would be -4 (think of the $x-$axis - if you move from right to left you are moving in the **negative** $x-$direction). That means our slope ratio would be $\frac{-3}{-4}$, and the negatives cancel out to leave 0.75, the same slope as before. In other words, the slope of a line is the same no matter which direction you travel along it.
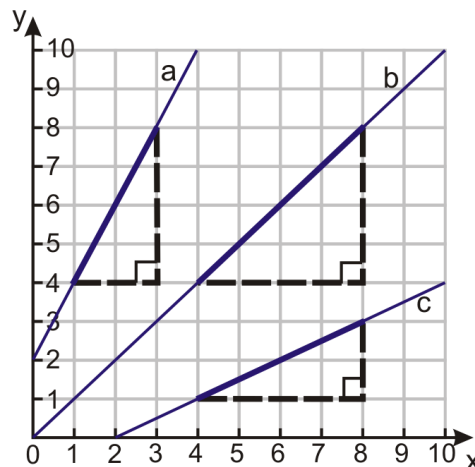
GUIDED PRACTICE

Find the Slope of a Line

A simple way to find a value for the slope of a line is to draw a right triangle whose hypotenuse runs along the line. Then we just need to measure the distances on the triangle that correspond to the rise (the vertical dimension) and the run (the horizontal dimension).

Example 1

Find the slopes for the three graphs shown.

Solution

There are already right triangles drawn for each of the lines - in future problems you'll do this part yourself. Note that it is easiest to make triangles whose vertices are **lattice points** (i.e. points whose coordinates are all integers).

a) The rise shown in this triangle is 4 units; the run is 2 units. The slope is $\frac{4}{2} = 2$.

b) The rise shown in this triangle is 4 units, and the run is also 4 units. The slope is $\frac{4}{4} = 1$.

c) The rise shown in this triangle is 2 units, and the run is 4 units. The slope is $\frac{2}{4} = \frac{1}{2}$.

Example 2

Find the slope of the line that passes through the points (1, 2) and (4, 7).

Solution

We already know how to graph a line if we're given two points: we simply plot the points and connect them with a line. Here's the graph:
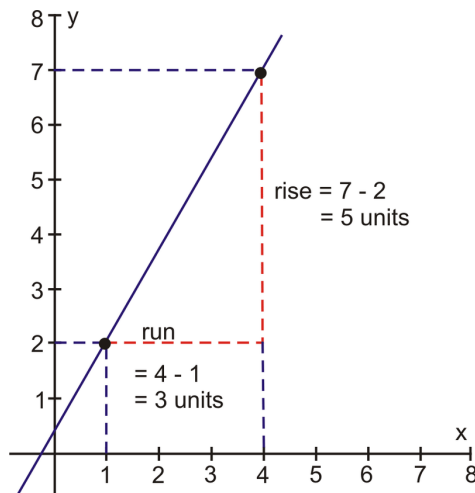
Since we already have coordinates for the vertices of our right triangle, we can quickly work out that the rise is $7 - 2 = 5$ and the run is $4 - 1 = 3$ (see diagram). So the slope is $\frac{7-2}{4-1} = \frac{5}{3}$.

If you look again at the calculations for the slope, you'll notice that the 7 and 2 are the $y-$coordinates of the two points and the 4 and 1 are the $x-$coordinates. This suggests a pattern we can follow to get a general formula for the slope between two points (x_1, y_1) and (x_2, y_2):

Slope between (x_1, y_1) and $(x_2, y_2) = \frac{y_2 - y_1}{x_2 - x_1}$

or $m = \frac{\Delta y}{\Delta x}$

In the second equation the letter m denotes the slope (this is a mathematical convention you'll see often) and the Greek letter delta (Δ) means **change**. So another way to express slope is *change in y* divided by *change in x*. In the next section, you'll see that it doesn't matter which point you choose as point 1 and which you choose as point 2.

Example 3

Find the slopes of the lines on the graph below.
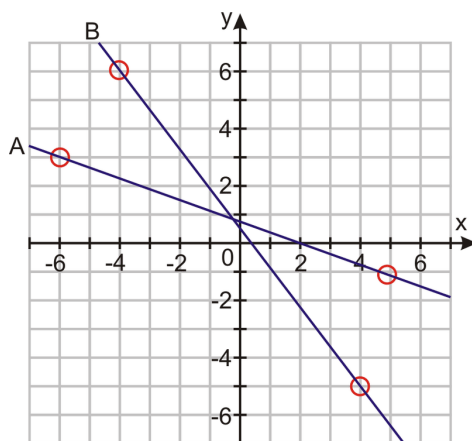
Solution

Look at the lines - they both slant down (or decrease) as we move from left to right. Both these lines have **negative slope.**

The lines don't pass through very many convenient lattice points, but by looking carefully you can see a few points that look to have integer coordinates. These points have been circled on the graph, and we'll use them to determine the slope. We'll also do our calculations twice, to show that we get the same slope whichever way we choose point 1 and point 2.

For Line *A*:

$$(x_1, y_1) = (-6, 3) \qquad (x_2, y_2) = (5, -1)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (3)}{(5) - (-6)} = \frac{-4}{11} \approx -0.364$$

$$(x_1, y_1) = (5, -1) \qquad (x_2, y_2) = (-6, 3)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3) - (-1)}{(-6) - (5)} = \frac{4}{-11} \approx -0.364$$

For Line *B*

$$(x_1, y_1) = (-4, 6) \qquad (x_2, y_2) = (4, -5)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-5) - (6)}{(4) - (-4)} = \frac{-11}{8} = -1.375$$

$$(x_1, y_1) = (4, -5) \qquad (x_2, y_2) = (-4, 6)$$
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(6) - (-5)}{(-4) - (4)} = \frac{11}{-8} = -1.375$$

You can see that whichever way round you pick the points, the answers are the same. Either way, **Line *A* has slope -0.364, and Line *B* has slope -1.375.**

Khan Academy has a series of videos on finding the slope of a line, starting at http://www.youtube.com/watch?v=hXP1Gv9IMBo.

Find the Slopes of Horizontal and Vertical lines

Example 4

Determine the slopes of the two lines on the graph below.
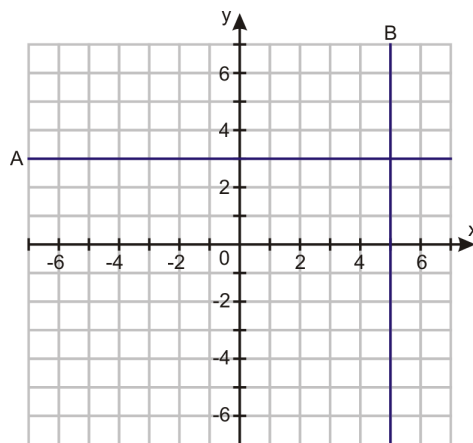
Solution

There are 2 lines on the graph: $A(y = 3)$ and $B(x = 5)$.

Let's pick 2 points on line *A*—say, $(x_1, y_1) = (-4, 3)$ and $(x_2, y_2) = (5, 3)$—and use our equation for slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3) - (3)}{(5) - (-4)} = \frac{0}{9} = 0.$$

If you think about it, this makes sense - if y doesn't change as x increases then there is no slope, or rather, the slope is zero. You can see that this must be true for all horizontal lines.

Horizontal lines ($y = constant$) all have a slope of 0.

Now let's consider line B. If we pick the points $(x_1, y_1) = (5, -3)$ and $(x_2, y_2) = (5, 4)$, our slope equation is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(4) - (-3)}{(5) - (5)} = \frac{7}{0}$. But dividing by zero isn't allowed!

In math we often say that a term which involves division by zero is **undefined.** (Technically, the answer can also be said to be infinitely large—or infinitely small, depending on the problem.)

Vertical lines ($x = constant$) all have an infinite (or undefined) slope.
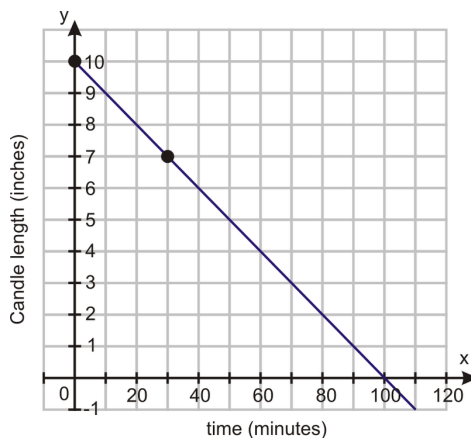
Find a Rate of Change

The slope of a function that describes real, measurable quantities is often called a **rate of change**. In that case the slope refers to a change in one quantity (y)**per** unit change in another quantity (x). (This is where the equation $m = \frac{\Delta y}{\Delta x}$ comes in—remember that Δy and Δx represent the change in y and x respectively.)

Example 5

A candle has a starting length of 10 inches. 30 minutes after lighting it, the length is 7 inches. Determine the rate of change in length of the candle as it burns. Determine how long the candle takes to completely burn to nothing.

Solution

First we'll graph the function to visualize what is happening. We have 2 points to start with: we know that at the moment the candle is lit ($time = 0$) the length of the candle is 10 inches, and after 30 minutes ($time = 30$) the length is 7 inches. Since the candle length depends on the time, we'll plot time on the horizontal axis, and candle length on the vertical axis.

The rate of change of the candle's length is simply the slope of the line. Since we have our 2 points $(x_1, y_1) = (0, 10)$ and $(x_2, y_2) = (30, 7)$, we can use the familiar version of the slope formula:

$$\text{Rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(7 \text{ inches}) - (10 \text{ inches})}{(30 \text{ minutes}) - (0 \text{ minutes})} = \frac{-3 \text{ inches}}{30 \text{ minutes}} = -0.1 \text{ inches per minute}$$

Note that the slope is negative. A negative rate of change means that the quantity is decreasing with time—just as we would expect the length of a burning candle to do.

To find the point when the candle reaches zero length, we can simply read the $x-$intercept off the graph (100 minutes). We can use the rate equation to verify this algebraically:

$$\text{Length burned} = \text{rate} \times \text{time}$$
$$10 = 0.1 \times 100$$

Since the candle length was originally 10 inches, our equation confirms that 100 minutes is the time taken.

Example 6

The population of fish in a certain lake increased from 370 to 420 over the months of March and April. At what rate is the population increasing?
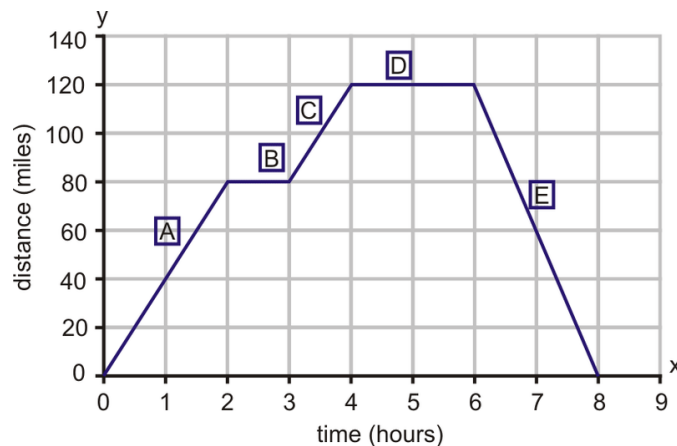
Solution

Here we don't have two points from which we can get $x-$ and $y-$coordinates for the slope formula. Instead, we'll need to use the alternate formula, $m = \frac{\Delta y}{\Delta x}$.

The change in $y-$values, or Δy, is the change in the number of fish, which is $420 - 370 = 50$. The change in $x-$values, Δx, is the amount of time over which this change took place: two months. So $\frac{\Delta y}{\Delta x} = \frac{50 \text{ fish}}{2 \text{ months}}$, or **25 fish per month.**

Interpret a Graph to Compare Rates of Change

Example 7

The graph below represents a trip made by a large delivery truck on a particular day. During the day the truck made two deliveries, one taking an hour and the other taking two hours. Identify what is happening at each stage of the trip (stages A through E).

Solution

Here are the stages of the trip:

a) The truck sets off and travels 80 miles in 2 hours.

b) The truck covers no distance for 2 hours.

c) The truck covers $(120 - 80) = 40$ miles in 1 hour.

d) The truck covers no distance for 1 hour.

e) The truck covers -120 miles in 2 hours.

Let's look at each section more closely.

A. Rate of change $= \frac{\Delta y}{\Delta x} = \frac{80 \text{ miles}}{2 \text{ hours}} = 40$ miles per hour

Notice that the rate of change is a **speed**—or rather, a **velocity.** (The difference between the two is that velocity has a direction, and speed does not. In other words, velocity can be either positive or negative, with negative velocity representing travel in the opposite direction. You'll see the difference more clearly in part E.)

Since velocity equals distance divided by time, the slope (or rate of change) of a distance-time graph is always a velocity.

So during the first part of the trip, the truck travels at a constant speed of 40 mph for 2 hours, covering a distance of 80 miles.

B. The slope here is 0, so the rate of change is 0 mph. The truck is stationary for one hour. This is the first delivery stop.

C. Rate of change $= \frac{\Delta y}{\Delta x} = \frac{(120-80) \text{ miles}}{(4-3) \text{ hours}} = 40$ miles per hour. The truck is traveling at 40 mph.

D. Once again the slope is 0, so the rate of change is 0 mph. The truck is stationary for two hours. This is the second delivery stop. At this point the truck is 120 miles from the start position.

E. Rate of change $= \frac{\Delta y}{\Delta x} = \frac{(0-120) \text{ miles}}{(8-6) \text{ hours}} = \frac{-120 \text{ miles}}{2 \text{ hours}} = -60$ miles per hour. The truck is traveling at *negative* 60 mph.

Wait - a negative speed? Does that mean that the truck is reversing? Well, probably not. It's actually the *velocity* and not the speed that is negative, and a negative velocity simply means that the distance *from the starting position* is decreasing with time. The truck is driving in the opposite direction - back to where it started from. Since it no longer has 2 heavy loads, it travels faster (60 mph instead of 40 mph), covering the 120 mile return trip in 2 hours. Its *speed* is 60 mph, and its *velocity* is -60 mph, because it is traveling in the opposite direction from when it started out.

Lesson Summary

- **Slope** is a measure of change in the vertical direction for each step in the horizontal direction. Slope is often represented as "*m*".
- Slope can be expressed as $\frac{\text{rise}}{\text{run}}$, or $\frac{\Delta y}{\Delta x}$.
- The slope between two points (x_1, y_1) and (x_2, y_2) is equal to $\frac{y_2 - y_1}{x_2 - x_1}$.
- **Horizontal lines** (where $y = a$ constant) all have a slope of 0.
- **Vertical lines** (where $x = a$ constant) all have an infinite (or undefined) slope.
- The slope (or **rate of change**) of a distance-time graph is a **velocity.**

PRACTICE

1. Use the slope formula to find the slope of the line that passes through each pair of points.

   a. (-5, 7) and (0, 0)
   b. (-3, -5) and (3, 11)
   c. (3, -5) and (-2, 9)
   d. (-5, 7) and (-5, 11)
   e. (9, 9) and (-9, -9)
   f. (3, 5) and (-2, 7)
   g. (2.5, 3) and (8, 3.5)

2. For each line in the graphs below, use the points indicated to determine the slope.
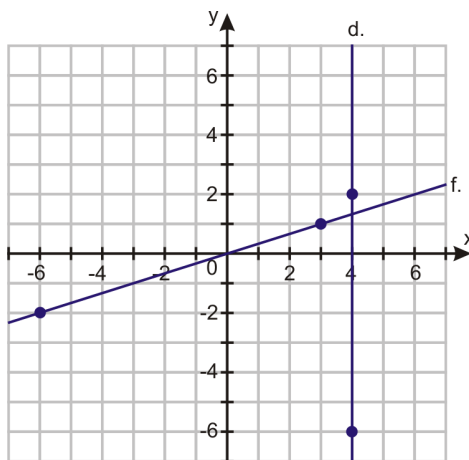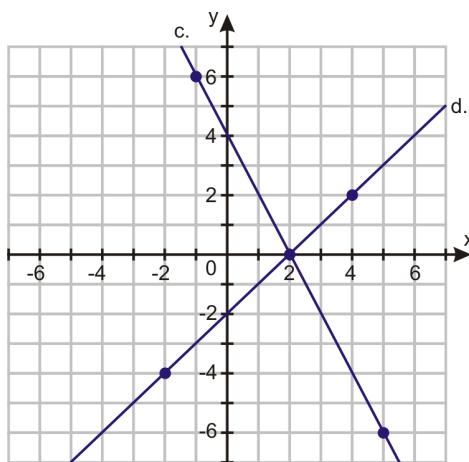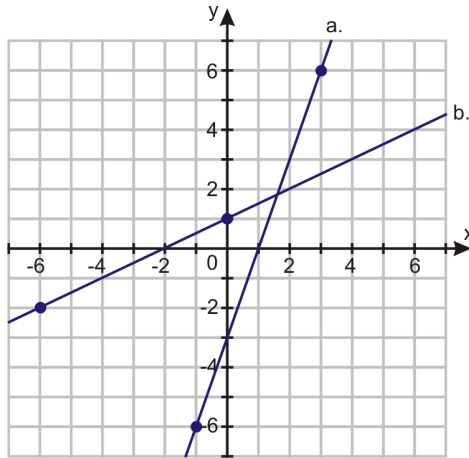
3. For each line in the graphs above, imagine another line with the same slope that passes through the point (1, 1), and name one more point on that line.

4. The graph below is a distance-time graph for Mark's three and a half mile cycle ride to school. During this ride, he rode on cycle paths but the terrain was hilly. He rode slower up hills and faster down them. He stopped once at a traffic light and at one point he stopped to mend a punctured tire. The graph shows his distance from home at any given time. Identify each section of the graph accordingly.
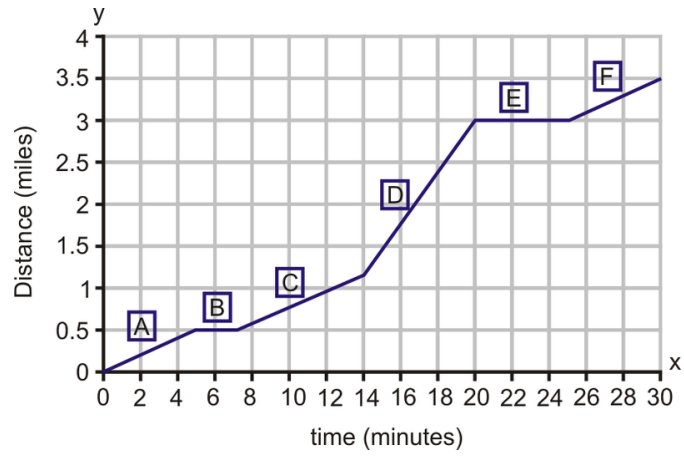
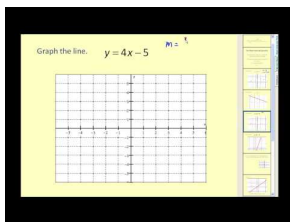## 5.5 Writing Equations from Tables and Graphs (7.7a)

GUIDANCE

You decide to buy a laptop for $800. In 3 years, the laptop will be worth $450. How much will the computer be worth after 6 years?

Writing a linear equation that relates the two prices will help you determine how much the computer will be worth after 6 years.

Finding the Equation of a Line

In the previous concept, we found the slope between two points. We will now find the entire equation of a line. Recall from Algebra I that the equation of a line in slope-intercept form is $y = mx + b$, where m is the slope and b is the $y-$intercept. You can find the slope either by using slope triangles or the Slope Formula. To find the $y-$intercept, or b, you can either locate where the line crosses the $y-$axis (if given the graph) or by using algebra.
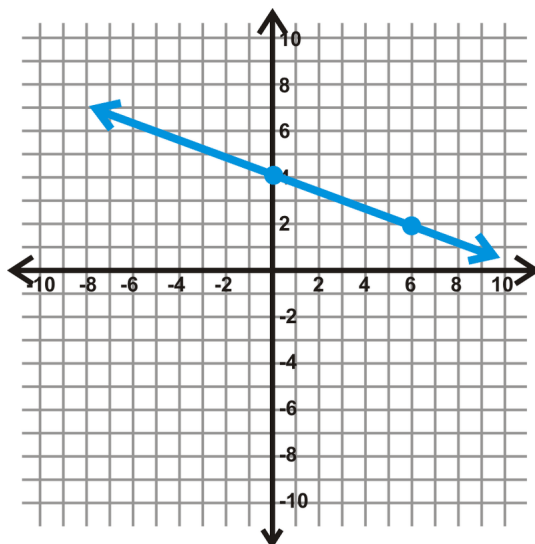

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URL: https://www.ck12.org/flx/render/embeddedobject/176996

Solve the following problems

Find the equation of the line below.

Analyze the line. We are given two points on the line, one of which is the $y-$intercept. From the graph, it looks like the line passes through the $y-$axis at (0, 4), making $b = 4$. Now, we need to find the slope. You can use slope triangles or the Slope Formula. Using slope triangles, we have:
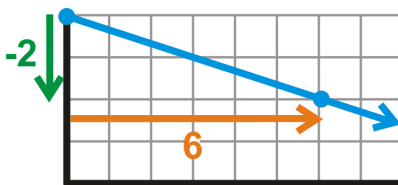
From this, we see that the slope is $-\frac{2}{6}$ or $-\frac{1}{3}$.

Plugging our found information into the slope-intercept equation, the equation of this line is $y = -\frac{1}{3}x + 4$.

<u>Alternate Method</u>: If we had used the Slope Formula, we would use (0, 4) and (6, 2), which are the values of the given points.

$$m = \frac{2-4}{6-0} = \frac{-2}{6} = -\frac{1}{3}$$

The slope of a line is -4 and the $y-$intercept is (0, 3). What is the equation of the line?

This problem explicitly tells us the slope and $y-$intercept. The slope is -4, meaning $m = -4$. The $y-$intercept is (0, 3), meaning $b = 3$. Therefore, the equation of the line is $y = -4x + 3$.

The slope of a line is $\frac{1}{2}$ and it passes through the point (4, -7). What is the equation of the line?

In this problem, we are given m and a point on the line. The point, (4, -7) can be substituted in for x and y in the equation. We need to solve for the $y-$intercept, or b. Plug in what you know to the slope-intercept equation.

$$y = mx + b$$
$$-7 = \frac{1}{2}(4) + b$$
$$-7 = 2 + b$$
$$-9 = b$$

From this, the equation of the line is $y = \frac{1}{2}x - 9$.

We can test if a point is on a line or not by plugging it into the equation. If the equation holds true, the point is on the line. If not, then the point is not on the line.

Find the equation of the line that passes through (12, 7) and (10, -1).

In this example, we are not given the slope or the $y-$intercept. First, we need to find the slope using the Slope Formula.

$$m = \frac{-1 - 7}{10 - 12} = \frac{-8}{-2} = 4$$

Now, plug in one of the points for x and y. It does not matter which point you choose because they are both on the line.

$$7 = 4(12) + b$$
$$7 = 48 + b$$
$$-41 = b$$

The equation of the line is $y = 4x - 41$.

GUIDED PRACTICE

Example 1

Earlier, you were asked how much will the computer be worth after 6 years.

To determine the equation of the line, rewrite the given information as points. The first could be (0, 800) and the second would be (3, 450). We already know that the y-intercept is 800 because the x-value is zero at that point. Find the slope.

$$\frac{800 - 450}{0 - 3} = -\frac{350}{3}$$

Therefore, the equation of the declining value of the laptop is $y = -\frac{350}{3}x + 800$. In 6 years, the laptop will be worth $y = -\frac{350}{3} \cdot 6 + 800 = -700 + 800 = 100$. The laptop will be worth $100.

Example 2

What is the equation of the line where the slope is 1 and passes through (5, 3)?

We are told that $m = 1, x = 5$, and $y = 3$. Plug this into the slope-intercept equation and solve for b.

$$3 = 1(5) + b$$
$$3 = 5 + b$$
$$-2 = b$$

The equation of the line is $y = x - 2$

Example 3

Find the equation of the line that passes through (9, -4) and (-1, -8).

First, find the slope.

$$m = \frac{-8 - (-4)}{-1 - 9} = \frac{-4}{-10} = \frac{2}{5}$$

Now, find the $y-$intercept. We will use the second point. Remember, it does not matter which point you use.

$$-8 = \frac{2}{5}(-1) + b$$
$$-8 = -\frac{2}{5} + b$$
$$-7\frac{3}{5} = b$$

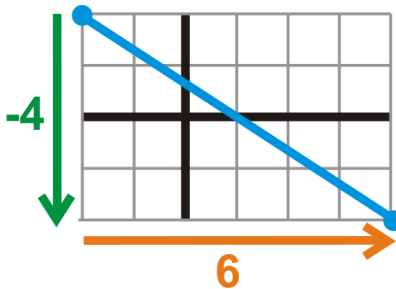The equation of the line is $y = \frac{2}{5}x - 7\frac{3}{5}$ or $y = \frac{2}{5}x - \frac{38}{5}$.

When your $y-$intercept is a fraction, make sure it is reduced. Double-check with your teacher on how s/he wants you to leave your answer.

Example 4

Find the equation of the line below.
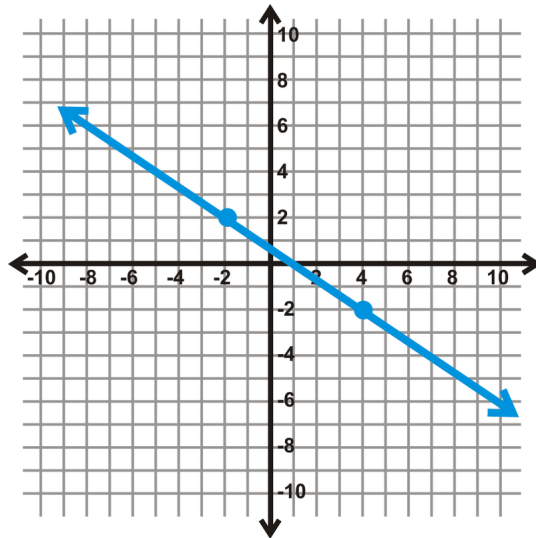
We can find the slope one of two ways: using slope triangles or by using the Slope Formula. We are given (by the drawn points in the picture) that (-2, 2) and (4, -2) are on the line. Drawing a slope triangle, we have:

We have that the slope is $-\frac{4}{6}$ or $-\frac{2}{3}$. To find the $y-$intercept, it looks like it is somewhere between 0 and 1. Take one of the points and plug in what you know to the slope-intercept equation.

$$2 = -\frac{2}{3}(-2) + b$$
$$2 = \frac{4}{3} + b$$
$$\frac{2}{3} = b$$

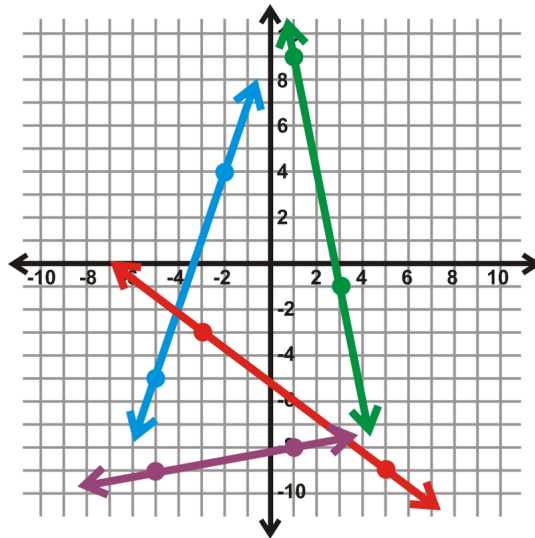The equation of the line is $y = -\frac{2}{3}x + \frac{2}{3}$.

PRACTICE

Find the equation of each line with the given information below.

1. slope = 2, $y-$intercept = (0, 3)
2. $m = -\frac{1}{4}$, $b = 2.6$
3. slope = -1, $y-$intercept = (0, 2)
4. $x-$intercept = (-2, 0), $y-$intercept = (0, -5)
5. slope = $\frac{2}{3}$ and passes through (6, -4)

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6. slope $= -\frac{3}{4}$ and passes through (-2, 5)
7. slope = -3 and passes through (-1, -7)
8. slope = 1 and passes through (2, 4)
9. passes through (-5, 4) and (1, 1)
10. passes through (5, -1) and (-10, -10)
11. passes through (-3, 8) and (6, 5)
12. passes through (-4, -21) and (2, 9)

For problems 13-16, find the equation of the lines in the graph below.

13. Green Line
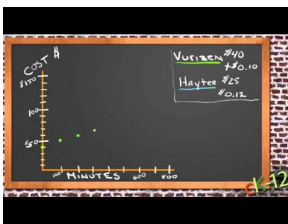14. Blue Line
15. Red Line
16. Purple Line
17. Find the equation of the line with zero slope and passes through (8, -3).
18. Find the equation of the line with zero slope and passes through the point (-4, 5).
19. Find the equation of the line with zero slope and passes through the point (a, b).
20. **Challenge** Find the equation of the line with an *undefined* slope that passes through (a, b).

5.6 Applications of Linear Relationships (7.7a)

What if you borrowed $2000 from your parents for a used car? At the end of the summer you pay them back $750 that you earned from your summer job. Throughout the school year, you will pay them another $125 per month from your part-time job. How many months will it take you to pay them back? After completing this Concept, you'll be able to solve problems like this one using graphs.

Watch This

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In this chapter, we've been solving problems where quantities are linearly related to each other. In this section, we'll look at a few examples of linear relationships that occur in real-world problems, and see how we can solve them using graphs. Remember back to our Problem Solving Plan:

1. **Understand the Problem**
2. **Devise a Plan—Translate**
3. **Carry Out the Plan—Solve**
4. **Look—Check and Interpret**

Example A

A cell phone company is offering its costumers the following deal: You can buy a new cell phone for $60 and pay a monthly flat rate of $40 per month for unlimited calls. How much money will this deal cost you after 9 months?

Solution

Let's follow the problem solving plan.

Step 1: The phone costs $60; the calling plan costs $40 per month.
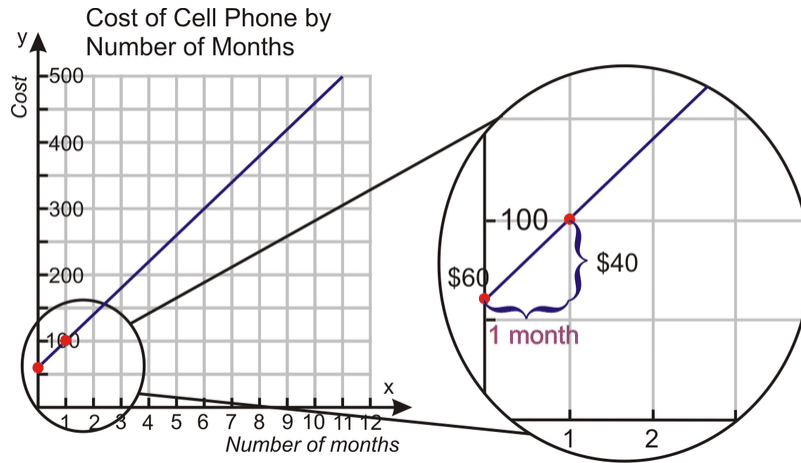
Let x = number of months.

Let y = total cost.

Step 2: We can solve this problem by making a graph that shows the number of months on the horizontal axis and the cost on the vertical axis.

Since you pay $60 when you get the phone, the $y-$intercept is $(0, 60)$.

You pay $40 for each month, so the cost rises by $40 for 1 month, so the slope is 40.

We can graph this line using the slope-intercept method.

Step 3: The question was "How much will this deal cost after 9 months?" We can now read the answer from the graph. We draw a vertical line from 9 months until it meets the graph, and then draw a horizontal line until it meets the vertical axis.
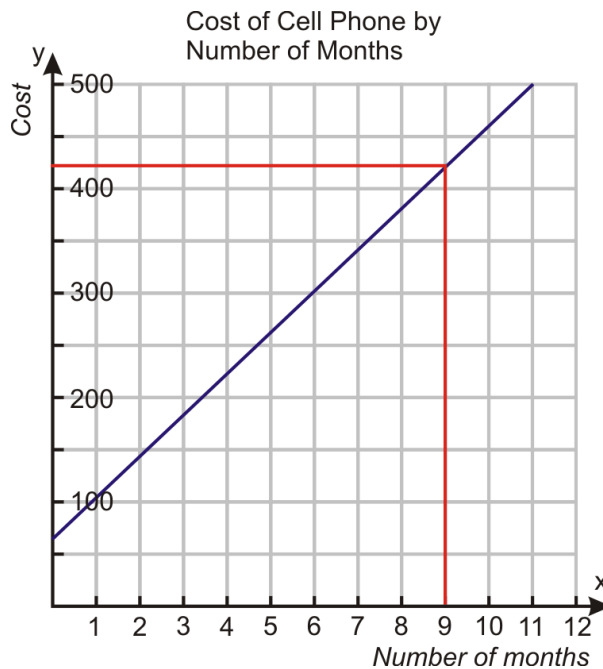
We see that after 9 months **you pay approximately $420.**

Step 4: To check if this is correct, let's think of the deal again.

Originally, you pay $60 and then $40 a month for 9 months.

$$\text{Phone} = \$60$$
$$\text{Calling plan} = \$40 \times 9 = \$360$$
$$\text{Total cost} = \$420.$$

The answer checks out.

Example B

A stretched spring has a length of 12 inches when a weight of 2 lbs is attached to the spring. The same spring has a length of 18 inches when a weight of 5 lbs is attached to the spring. What is the length of the spring when no weights are attached?

Solution

Step 1: We know: the length of the spring = 12 inches when weight = 2 lbs

the length of the spring = 18 inches when weight = 5 lbs

We want: the length of the spring when weight = 0 lbs

Let $x =$ the weight attached to the spring.

Let $y =$ the length of the spring.

Step 2: We can solve this problem by making a graph that shows the weight on the horizontal axis and the length of the spring on the vertical axis.
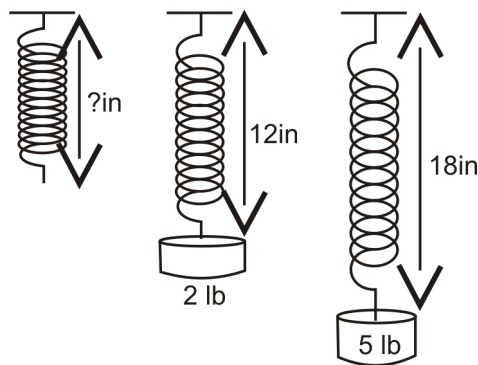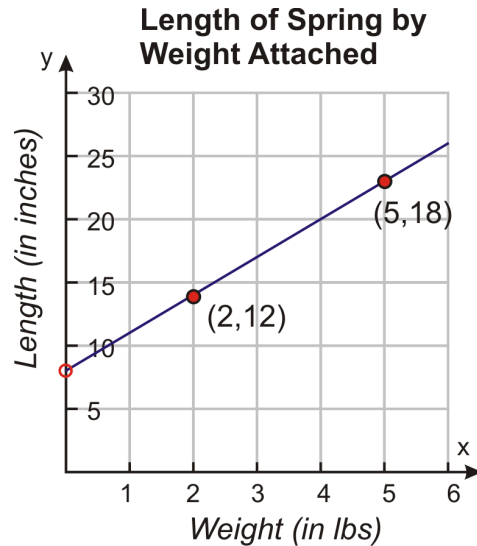
We have two points we can graph:

When the weight is 2 lbs, the length of the spring is 12 inches. This gives point (2, 12).

When the weight is 5 lbs, the length of the spring is 18 inches. This gives point (5, 18).

Graphing those two points and connecting them gives us our line.

Length of Spring by Weight Attached

Step 3: The question was: *"What is the length of the spring when no weights are attached?"*

We can answer this question by reading the graph we just made. When there is no weight on the spring, the $x-$value equals zero, so we are just looking for the $y-$intercept of the graph. On the graph, the $y-$intercept appears to be approximately 8 inches.

Step 4: To check if this correct, let's think of the problem again.

You can see that the length of the spring goes up by 6 inches when the weight is increased by 3 lbs, so the slope of the line is $\frac{6\ inches}{3\ lbs} = 2\ inches/lb$.

To find the length of the spring when there is no weight attached, we can look at the spring when there are 2 lbs attached. For each pound we take off, the spring will shorten by 2 inches. If we take off 2 lbs, the spring will be shorter by 4 inches. So, the length of the spring with no weights is 12 inches - 4 inches = 8 inches.

The answer checks out.

Example C

Christine took 1 hour to read 22 pages of Harry Potter. *She has 100 pages left to read in order to finish the book. How much time should she expect to spend reading in order to finish the book?*

Solution

Step 1: We know - Christine takes 1 hour to read 22 pages.

We want - How much time it takes to read 100 pages.

Let $x =$ the time expressed in hours.

Let $y =$ the number of pages.

Step 2: We can solve this problem by making a graph that shows the number of hours spent reading on the horizontal axis and the number of pages on the vertical axis.

We have two points we can graph:

Christine takes 1 hour to read 22 pages. This gives point (1, 22).

A second point is not given, but we know that Christine would take 0 hours to read 0 pages. This gives point (0, 0).

Graphing those two points and connecting them gives us our line.

Number of Pages Read by Time
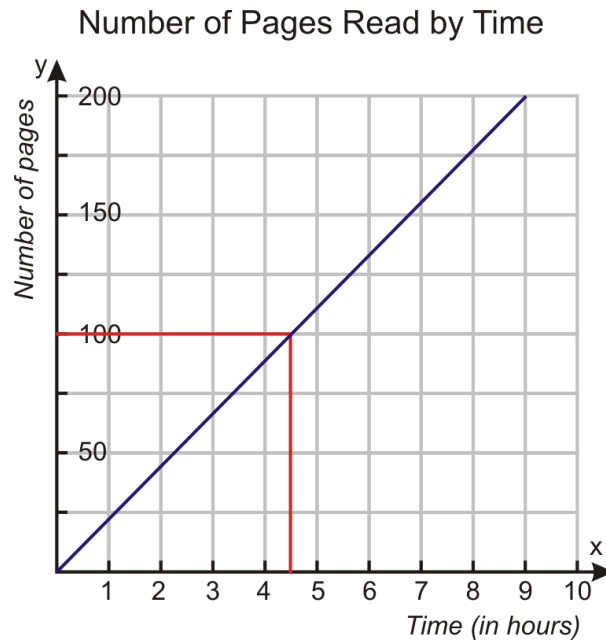
Step 3: The question was: "How much time should Christine expect to spend reading 100 pages?" We can find the answer from reading the graph - we draw a horizontal line from 100 pages until it meets the graph and then we draw the vertical until it meets the horizontal axis. We see that it takes **approximately 4.5 hours** to read the remaining 100 pages.

Step 4: To check if this correct, let's think of the problem again.

We know that Christine reads 22 pages per hour - this is the slope of the line or the rate at which she is reading. To find how many hours it takes her to read 100 pages, we divide the number of pages by the rate. In this case, $\frac{100 \ pages}{22 \ pages/hour} = 4.54 \ hours$. This is very close to the answer we got from reading the graph.

The answer checks out.

Watch this video for help with the Examples above.

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/133287

GUIDED PRACTICE

Aatif wants to buy a surfboard that costs $249. He was given a birthday present of $50 and he has a summer job that pays him $6.50 per hour. To be able to buy the surfboard, how many hours does he need to work?

Solution

Step 1: We know - The surfboard costs $249.

Aatif has $50.

His job pays $6.50 per hour.

We want - How many hours Aatif needs to work to buy the surfboard.

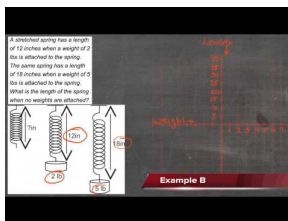Let $x =$ the time expressed in hours

Let $y =$ Aatif's earnings

Step 2: We can solve this problem by making a graph that shows the number of hours spent working on the horizontal axis and Aatif's earnings on the vertical axis.

Aatif has $50 at the beginning. This is the $y-$intercept: (0, 50).

He earns $6.50 per hour. This is the slope of the line.

We can graph this line using the slope-intercept method. We graph the $y-$intercept of (0, 50), and we know that for each unit in the horizontal direction, the line rises by 6.5 units in the vertical direction. Here is the line that describes this situation.

Aatif's Earnings by Hours Worked
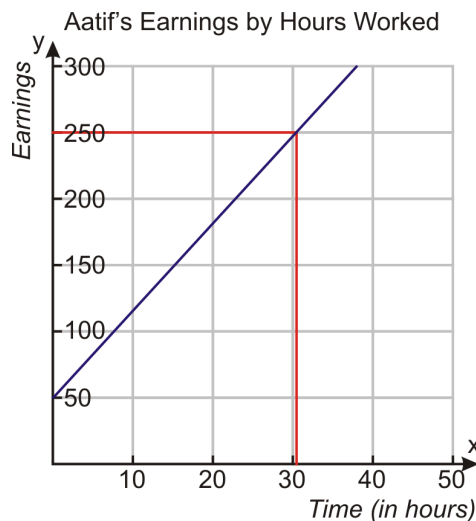
Step 3: The question was: *"How many hours does Aatif need to work to buy the surfboard?"*

We find the answer from reading the graph - since the surfboard costs $249, we draw a horizontal line from $249 on the vertical axis until it meets the graph and then we draw a vertical line downwards until it meets the horizontal axis. We see that it takes **approximately 31 hours** to earn the money.

Step 4: To check if this correct, let's think of the problem again.

We know that Aatif has $50 and needs $249 to buy the surfboard. So, he needs to earn $249 - $50 = $199 from his job.

His job pays $6.50 per hour. To find how many hours he need to work we divide: $\frac{\$199}{\$6.50/hour} = 30.6\ hours$. This is very close to the answer we got from reading the graph.

The answer checks out.

PRACTICE

Solve the following problems by making a graph and reading it.

1. A gym is offering a deal to new members. Customers can sign up by paying a registration fee of $200 and a

monthly fee of $39.

  a. How much will this membership cost a member by the end of the year?
  b. The old membership rate was $49 a month with a registration fee of $100. How much more would a year's membership cost at that rate?
  c. **Bonus:** For what number of months would the two membership rates be the same?

2. A candle is burning at a linear rate. The candle measures five inches two minutes after it was lit. It measures three inches eight minutes after it was lit.

  a. What was the original length of the candle?
  b. How long will it take to burn down to a half-inch stub?
  c. Six half-inch stubs of candle can be melted together to make a new candle measuring $2\frac{5}{6}$ inches (a little wax gets lost in the process). How many stubs would it take to make three candles the size of the original candle?

3. A dipped candle is made by taking a wick and dipping it repeatedly in melted wax. The candle gets a little bit thicker with each added layer of wax. After it has been dipped three times, the candle is 6.5 mm thick. After it has been dipped six times, it is 11 mm thick.

  a. How thick is the wick before the wax is added?
  b. How many times does the wick need to be dipped to create a candle 2 cm thick?

4. Tali is trying to find the thickness of a page of his telephone book. In order to do this, he takes a measurement and finds out that 55 pages measures $\frac{1}{8}$ *inch*. What is the thickness of one page of the phone book?

5. Bobby and Petra are running a lemonade stand and they charge 45 cents for each glass of lemonade. In order to break even they must make $25.

  a. How many glasses of lemonade must they sell to break even?
  b. When they've sold $18 worth of lemonade, they realize that they only have enough lemons left to make 10 more glasses. To break even now, they'll need to sell those last 10 glasses at a higher price. What does the new price need to be?

6. Dale is making cookies using a recipe that calls for 2.5 cups of flour for two dozen cookies. How many cups of flour does he need to make five dozen cookies?

7. To buy a car, Jason makes a down payment of $1500 and pays $350 per month in installments.

  a. How much money has Jason paid at the end of one year?
  b. If the total cost of the car is $8500, how long will it take Jason to finish paying it off?
  c. The resale value of the car decreases by $100 each month from the original purchase price. If Jason sells the car as soon as he finishes paying it off, how much will he get for it?

8. Anne transplants a rose seedling in her garden. She wants to track the growth of the rose so she measures its height every week. On the third week, she finds that the rose is 10 inches tall and on the eleventh week she finds that the rose is 14 inches tall. Assuming the rose grows linearly with time, what was the height of the rose when Anne planted it?

9. Ravi hangs from a giant spring whose length is 5 m. When his child Nimi hangs from the spring its length is 2 m. Ravi weighs 160 lbs and Nimi weighs 40 lbs. Write the equation for this problem in slope-intercept form. What should we expect the length of the spring to be when his wife Amardeep, who weighs 140 lbs, hangs from it?

10. Nadia is placing different weights on a spring and measuring the length of the stretched spring. She finds that for a 100 gram weight the length of the stretched spring is 20 cm and for a 300 gram weight the length of the stretched spring is 25 cm.

  a. What is the unstretched length of the spring?
  b. If the spring can only stretch to twice its unstretched length before it breaks, how much weight can it hold?

11. Andrew is a submarine commander. He decides to surface his submarine to periscope depth. It takes him 20 minutes to get from a depth of 400 feet to a depth of 50 feet.

     a. What was the submarine's depth five minutes after it started surfacing?

     b. How much longer would it take at that rate to get all the way to the surface?

12. Kiersta's phone has completely run out of battery power when she puts it on the charger. Ten minutes later, when the phone is 40% recharged, Kiersta's friend Danielle calls and Kiersta takes the phone off the charger to talk to her. When she hangs up 45 minutes later, her phone has 10% of its charge left. Then she gets another call from her friend Kwan.

     a. How long can she spend talking to Kwan before the battery runs out again?

     b. If she puts the phone back on the charger afterward, how long will it take to recharge completely?

13. Marji is painting a 75-foot fence. She starts applying the first coat of paint at 2 PM, and by 2:10 she has painted 30 feet of the fence. At 2:15, her husband, who paints about $\frac{2}{3}$ as fast as she does, comes to join her.

     a. How much of the fence has Marji painted when her husband joins in?

     b. When will they have painted the whole fence?

     c. How long will it take them to apply the second coat of paint if they work together the whole time?

Vocabulary:

- Linear Relationships
- Slope
- Direct Variation
- Proportional
- Non-proportional
- Slope-intercept form
- Function
- Equation

CHAPTER **6**

Rates and Percentages – Financial Literacy I

Chapter Outline

7.4d: solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems

7.13e: calculate and compare simple interest and compound interest earnings

8.12d: calculate and compare simple interest and compound interest earnings

7.13f: analyze and compare monetary incentives, including sales, rebates, and coupons

6.1 Percent – Estimation (7.4d)

GUIDANCE

Discount Day at the Supermarket

The supermarket is having a special discount day in honor of its one year anniversary. On Saturday, discount day will begin and every customer will receive 15% off of his/her total order.

Saturday, the store is full of customers. There are free balloons being given out to the children along with coffee and donuts for the parents. Everyone is in a terrific mood and business is booming.

Many people are making large purchases. Mr. Kemp is excited to see how much business the store is getting, but at the same time he can't help thinking about the profit he is losing by offering the 15% discount.

Mr. Kemp peers over the service desk to hear one of the girls at the cash register say, "Well, your total is $345.00 before the discount."

Then the girl showed the customer her new total. The customer paid and left, smiling to Mr. Kemp as she passed by.

If this is the total before the discount, how much did the customer end up paying? How much money did the store lose by offering such a discount?

What You Will Learn

In this lesson you will learn to do the following:

- Find a percent of a number.
- Find prices involving discounts.
- Find prices involving sales tax.
- Estimate tips.
- Find amounts involving simple interest.

GUIDED PRACTICE

Percents are found in real life all around us. We work with percents every day. In fact, they are so common that sometimes we don't even realize that we are using them. This lesson takes what we have learned about percents and applies it in some different real world situations. Let's begin by learning how to find the percent of a number.

I. **Find a Percent of a Number**

We can find the percent of another number. When we find the percent of a number, we want to figure out what part of the number is equal to the amount of the percent. Let's look at an example.

Example

What is 10% of 25?

This is an example where we are looking for the **percent of a number.** We want to figure out 10% of 25. **Said another way, we want to find a part of 25 that is the same as ten percent.**

How can we figure out this problem?

We can figure out the percent of a number in two different ways. One is to use a *proportion* and one is to use key words and multiplication. Let's look at using a proportion first.

How can we find the percent of a number using a proportion?

Remember that a proportion is created when two ratios are equal. We can compare the percent out of 100 with a part of another number.

We know that we need to find 10% of 25. **The percent is out of 100 so we can write our first ratio by changing 10% into a fraction.**

$$10\% = \frac{10}{100}$$

Next, we change the 25 into a proportion. Now we are looking for what part of 25 is equal to 10%, so that is going to be what we need to find out of 25. Here is what it looks like.

$$\frac{x}{25}$$

Our proportion is $\frac{10}{100} = \frac{x}{25}$.

Oh, I see - we used a variable for the unknown part of 25! Now we can solve the proportion.

That's correct! We solve the proportion to find our answer. Do you remember how to solve the proportion? We can do this by using cross products.

$$\frac{10}{100} \bowtie \frac{x}{25}$$

$$10(25) = 250$$

$$100 \text{ times } x = 100x$$

Next, we can write an equation.

$$100x = 250$$

Now what? I can't remember how to solve this!

To solve this equation, we have to think "What times 100 is equal to 250?" We could also use the *inverse operation* **of "times 100", and divide 250 by 100.**

$$x = 250 \div 100$$

Remember!

When we divide by 100, we move the decimal point two places to the left!

$$250 \div 100 = 2\underset{\leftarrow}{5}0. = 2.5$$

Our answer is 2.5

Let's review the steps of using a proportion!!

Finding the % of a Number using a Proportion:

1. Write the % as a fraction out of 100.
2. Use a variable and write a ratio for the number we are looking for.
3. Use cross products and solve for the missing number.

We can also use key words and multiplication to find the percent of a number.

Example

What is 10% of 25?

First, we look for any key words that mean an operation. The word "OF" means multiplication, so we are going to use multiplication to find an answer.

Next, we turn 10% into a decimal.

10% = .10

We are looking for 10% of 25, so we multiply the decimal .10 times 25 to find our answer.

$$
\begin{array}{r}
25 \\
\times \ .10 \\
\hline
00 \\
+ \ 25 \\
\hline
250
\end{array}
$$

Finally we put the decimal point into our product. We have two decimal places in .10 so we put it in two places counting from right to left.

Our answer is 2.5.

Notice that our answers are the same!! You can use either way to find the correct answer!!

It is time for you to practice a few of these on your own.

1. **What is 10% of 54?**
2. **What is 25% of 80?**
3. **What is 5% of 78?**

Take a few minutes to check your work with a peer.

II. **Find Prices Involving Discounts**

Figuring out a *discount* is a time when we use a percent in a real life situation. A discount is an amount of money that is taken off of the original price. Think about shopping! We use discounts all the time when we shop. In fact, we often use mental math to figure out a discount.

We can find a discount and then a final price of the item we are purchasing. This involves two steps.

1. **Figure out the amount of the discount**
2. **Subtract that amount from the original price**

How do we figure out the amount of a discount?

To understand this, let's look at an example.

Example

Tracy went shopping for a new pair of sneakers. She chose a pair of blue ones that were $58.00. The sign said that they were 15% off of the original price. What is the amount of the discount? How much did Tracy end up paying for the sneakers?

Our first step is to figure out the amount of the discount.

We need to find 15% of 58.

Good question! The original price of the sneakers is $58.00. The percent of the discount is 15%, so we can write that we need to find 15% of 58. That will give us the amount of money that Tracy did not have to pay.

Let's multiply to find the amount of the discount.

$$
\begin{array}{r}
15\% \text{ of } 58 = 58 \\
\times\ \ .15 \\
\hline
290 \\
+\ \ 58 \\
\hline
8.70
\end{array}
$$

The amount of the discount is $8.70.

Now we can subtract the amount of the discount from the original sales price and we will know how much Tracy paid for the sneakers.

$$
\begin{array}{r}
58.00 \\
-\ \ 8.70 \\
\hline
\$49.30
\end{array}
$$

Tracy paid $49.30 for the sneakers.

Figure these out on your own. First figure out the amount of the discount, then figure out the new price.

1. **If a $50.00 shirt is 25% off, how much would you pay for the shirt?**
2. **If a video game that usually costs $45.50 is 30% off, how much would you pay for the game?**

Now check your answers with a partner.

III. **Find Prices Involving Sales Tax**

When we figured out a price with a discount, we subtracted because a discount is an amount taken off of an original price. *Sales tax* is just the opposite!

What is sales tax?

Sales tax is an amount that is ***added*** to a price. Many states have sales tax. When you shop in those states, you have to add a sales tax to your total. Sales tax is a percentage.

How can we calculate a price that includes sales tax?

First, we figure out the amount of the sales tax. Then we add that to the original price to figure out the new price.

Let's look at an example.

Example

The state of Maine has a sales tax of 5%. If you purchased a book for $25.00, how much would you pay for the book if you bought it in Maine?

First, we figure out the amount of sales tax for a $25.00 book. We need to find 5% of 25.

We change 5% to a decimal, .05, and then multiply.

$$
\begin{array}{r}
25 \\
\times\ \ .05 \\
\hline
1.25
\end{array}
$$

The amount of sales tax is $1.25.

Now, because this is sales tax, we need to add this amount to our original price.

$$\begin{array}{r} \$25.00 \\ +\quad 1.25 \\ \hline \$26.25 \end{array}$$

We would pay $26.25 for the book in the state of Maine.

Try a few of these on your own. Figure out the sales tax and then add it to the original price for a new total.

1. **What would you pay for a $35.00 book if the sales tax is 5%?**
2. **What would you pay for a $99.00 a night hotel room if the sales tax is 7%?**

Take a few minutes to check your work. Did you remember to ADD the amount of the sales tax?

IV. **Estimate Tips**

When you eat in a restaurant, you usually pay a tip to the server. **A *tip* is a percent of the cost of the meal. It is also called a *gratuity*.**

When ordering out, we can use estimation to figure out the tip for the server. It is customary to give the server 15% of the total of the meal.

We can use what we have learned about percents to estimate tips.

Let's look at an example.

Example

John ate out for lunch. His total bill came to $12.00. How much should he tip the server?

How can we figure this out?

We know that we need to find approximately 15% of $12.00. 15% is between 10% and 20%. It is easy to find 10% because we can multiply by .10 or move the decimal point one place to the left in the total.

This would mean that 10% of $12.00 is $1.20. If 10% of $12.00 is 1.20, then 20% is 2.40.

We want to choose a number between these two amounts for 15%.

A reasonable estimate would be $1.75 or $2.00.

Sometimes people find it easier to calculate 20% using mental math so they often give the server a larger tip because it is easier to figure out!

Estimate the following tips.

1. **A tip on $25.00**
2. **A tip on $18.00**

Check your answers with a friend. Are your tip amounts reasonable?

V. Find Amounts Involving Simple Interest

Interest **is another amount that is ADDED to a total.** You hear the word "interest" when talking about borrowing money. When someone borrows money from the bank, the bank charges them a small percent of the amount that the person borrowed for each period of time that they have the money. The percent is often calculated *annually* or per year. **In this way, the bank says, "We will loan you this money, but until you pay it back, you must pay us for each month or year that you have it."**

How can we calculate interest?

There are three main things that you need to know to calculate interest. You need to know the amount that was borrowed or *the principal*, the *rate* or the percentage the bank is charging to loan the money, and the time that you are keeping the money.

Here is a formula we can use to calculate interest.

$$I = prt$$
$$Interest = principal \times rate \times time$$

We take the principal, multiply it with the rate, and multiply that with the length of time that the money has been borrowed, to find the interest.

Let's look at an example.

Example

Carrie borrowed $500.00 from the bank. The bank charges 5% interest annually. If it takes Carrie 1 year to pay back the money, how much interest will she pay?

To figure this out, let's use our formula.

$$I = prt$$

The principal is $500.00.

The rate is 5% $= .05$

The time is 1 year.

$$I = (500)(.05)(1)$$

$$\begin{array}{r} 500 \\ \times\ \ .05 \\ \hline 25.00 \end{array}$$

Carrie will pay $25.00 in interest.

What if it had been 3 years before she had paid back the money?

If this was the case, we would have used this formula.

$$I = (500)(.05)(3)$$
$$I = (25.00)(3)$$
$$I = \$75.00$$

Carrie would have paid $75.00 in interest.

Try a few of these on your own.

1. Mark borrowed $250.00 at 4% for 3 years. How much interest did he pay?
2. Kris borrowed $300.00 at 2% for 2 years. How much interest did he pay?

Check your answers with a partner.

Real Life Example Completed

Discount Day at the Supermarket

Mr. Kemp is having a difficult time thinking about all of the money that he is losing by offering discount day at the grocery store. Here is the original problem. Reread it and underline all of the important information.

The supermarket is having a special discount day in honor of its one year anniversary. On Saturday, discount day will begin and every customer will receive 15% off of his/her total order.

Saturday, the store is full of customers. There are free balloons being given out to the children along with coffee and donuts for the parents. Everyone is in a terrific mood and business is booming.

Many people are making large purchases. Mr. Kemp is excited to see how much business the store is getting, but at the same time he can't help thinking about the profit he is losing by offering the 15% discount.

Mr. Kemp peers over the service desk to hear one of the girls at the cash register say, "Well, your total is $345.00 before the discount."

Then the girl showed the customer her new total. The customer paid and left, smiling to Mr. Kemp as she passed by.

If this is the total before the discount, how much did the customer end up paying? How much money did the store lose by offering such a discount?

There are two questions to answer in solving this problem. First, we need to figure out the amount of the discount given the total and the 15% off.

$345 \times .15 = \$51.75$

The discount is \$51.75.

This is the amount that the store lost.

What did the customer end up paying?

To figure this out, we take the discount and subtract it from the original total.

345 - 51.75 = \$293.25

WOW!! That customer saved a lot of money by shopping on discount day!

Vocabulary

Here are the vocabulary words that are found in this lesson.

Percent

   a part of a whole 100, written using a % sign.

Proportion

   two equal ratios.

Sales Tax

> a percent added to a total

Discount

> an amount taken off of an original price

Tip

> 15 - 20% of a total bill paid to a server

Gratuity

> another word for tip

Interest

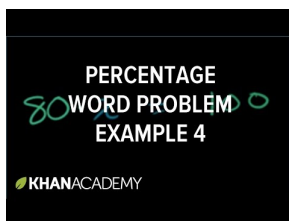> the sum of money a person pays a bank for borrowing money

Principal

> the amount of money borrowed
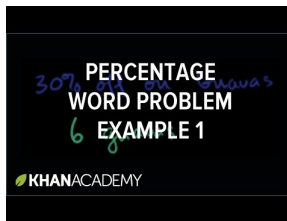
Rate

> a percent that the bank charges for borrowing money

Annually

> per year

Technology Integration
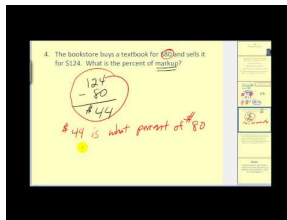

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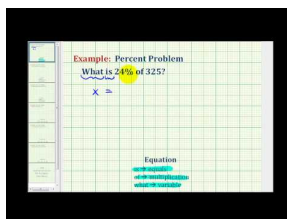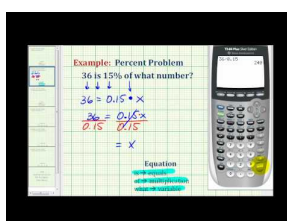
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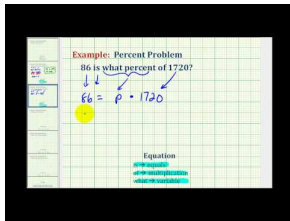
PRACTICE

Directions: Find the percent of each number.

1. What is 2% of 10?

2. What is 5% of 50?

3. What is 10% of 30?

4. What is 25% of 18?

5. What is 20% of 36?

6. What is 11% of 40?

7. What is 8% of 80?

8. What is 15% of 45?

9. What is 20% of 100?

10. What is 25% of 250?

11. What is 4% of 60?

12. What is 5% of 85?

Directions: Calculate each new price based on the discount and the original price.

13. Original price: $19.95, discount 15%

14. Original price: $29.95, discount 20%

15. Original price: $18.00, discount 10%

16. Original price: $47.50, discount 10%

17. Original price: $75.00, discount 30%

18. Original price: $125.00, discount 20%

19. Original price: $225.50, discount 10%

20. Original price: $456.00, discount 25%

Directions: Calculate the total amount paid including sales tax if the sales tax is 4%.

21. Total: $56.75

22. Total: $43.25

23. Total: $65.00

24. Total: $25.50

25. Total: $18.75

26. Total: $59.00

27. Total: $21.50

28. Total: $44.50

29. Total: $125.50

30. Total: $430.00

6.2 Percent – Proportions (7.4d)

GUIDANCE

A senator wants to start a program to encourage more people to vote in his state. In County A, of the 39150 potential voters, 32,100 people voted. In neighboring County B, of the 81400 potential voters, 57,800 people voted. Which county needs the program more?

In this concept, you will use proportions to figure out percent.

Proportions

A **percent** is a part of a whole that represents a quantity out of 100. Fractions and decimals are also parts of a whole. Sometimes, you will be given information, but not a percent. You will need to know how to figure out the percent. Percentages, fractions, decimals and proportions can all help to you solve problems and figure out percent.

You began using proportions to figure out a percent when writing fractions as percent. Remember that proportions involve comparing quantities. A **proportion** is a comparison between two equal **ratios**. Because both of these are comparing, you can use proportions to help us figure out a percent.

First, write the proportion using *a* over *b*.

$$\frac{a}{b}$$

This is equal to the percent which is out of 100.

$$\frac{p}{100}$$

Here is the proportion:

$$\frac{a}{b} = \frac{p}{100}$$

Now let's apply this proportion to a problem.

What percent is 12 out of 45?

First, write a ratio comparing your given values to the missing percent.

$$\frac{12}{45} = \frac{p}{100}$$

Next, cross multiply.

$$\begin{array}{rcl} \frac{12}{45} & = & \frac{p}{100} \\ 45 \times p & = & 12 \times 100 \\ 45 \times p & = & 1200 \end{array}$$

Then, divide by 45 to isolate p.

$$\begin{array}{rcl} 45 \times p & = & 1200 \\ \frac{45p}{45} & = & \frac{1200}{45} \\ p & = & 26.7 \end{array}$$

The answer is 26.7.

Therefore 12 out of 45 is 26.7%.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about the voters.

County A has 39150 people and 32100 voted. County B has 81400 people and 57800 voted. Which county had the highest proportion of people voting?

First, write each ratio comparing your given values to the missing percent.

County A

$$\frac{32100}{39150} = \frac{p}{100}$$

County B

$$\frac{57800}{81400} = \frac{p}{100}$$

Next, cross multiply.

County A

$$\begin{aligned}
\frac{32100}{39150} &= \frac{p}{100} \\
39150 \times p &= 32100 \times 100 \\
39150 \times p &= 3210000
\end{aligned}$$

County B

$$\begin{aligned}
\frac{57800}{81400} &= \frac{p}{100} \\
81400 \times p &= 57800 \times 100 \\
81400 \times p &= 5780000
\end{aligned}$$

Then, divide to isolate p.

County A

$$\begin{aligned}
39150 \times p &= 3210000 \\
\frac{39150p}{39150} &= \frac{3210000}{39150} \\
p &= 81.99
\end{aligned}$$

County B

$$\begin{aligned}
81400 \times p &= 5780000 \\
\frac{81400p}{81400} &= \frac{5780000}{81400} \\
p &= 71.0
\end{aligned}$$

The answer is that 82% of the people in County A voted and 71% of the people in County B voted. Therefore, County A had the highest voter turnout and County B needs the senator's program more.

Example 2

John ran 8 out of 9 miles. What percent of the total miles did he run?

First, write a ratio comparing your given values to the missing percent.

$$\frac{8}{9} = \frac{p}{100}$$

Next, cross multiply.

$$\begin{aligned}
\frac{8}{9} &= \frac{p}{100} \\
9 \times p &= 8 \times 100 \\
9 \times p &= 800
\end{aligned}$$

Then, divide by 9 to isolate p.

$$9 \times p = 800$$
$$\frac{9p}{9} = \frac{800}{9}$$
$$p = 88.9$$

The answer is 88.9.

Therefore, John ran 88.9% of the total miles.

Example 3

What percent is 18 out of 50?

First, write a ratio comparing your given values to the missing percent.

$$\frac{18}{50} = \frac{p}{100}$$

Next, cross multiply.

$$\frac{18}{50} = \frac{p}{100}$$
$$50 \times p = 18 \times 100$$
$$50 \times p = 1800$$

Then, divide by 50 to isolate p.

$$50 \times p = 1800$$
$$\frac{50p}{50} = \frac{1800}{50}$$
$$p = 36$$

The answer is 36. Therefore 18 out of 50 is 36%.

Example 4

What percent is 22 out of 40?

First, write a ratio comparing your given values to the missing percent.

$$\frac{22}{40} = \frac{p}{100}$$

Next, cross multiply.

$$\frac{22}{40} = \frac{p}{100}$$
$$40 \times p = 22 \times 100$$
$$40 \times p = 2200$$

Then, divide by 40 to isolate p.

$$
\begin{aligned}
40 \times p &= 2200 \\
\frac{40p}{40} &= \frac{2200}{40} \\
p &= 55
\end{aligned}
$$

The answer is 55.

Therefore 22 out of 40 is 55%.

Example 5

What percent is 78 out of 80?

First, write a ratio comparing your given values to the missing percent.

$$
\frac{78}{80} = \frac{p}{100}
$$

Next, cross multiply.

$$
\begin{aligned}
\frac{78}{80} &= \frac{p}{100} \\
80 \times p &= 78 \times 100 \\
80 \times p &= 7800
\end{aligned}
$$

Then, divide by 80 to isolate p.

$$
\begin{aligned}
80 \times p &= 7800 \\
\frac{80p}{80} &= \frac{7800}{80} \\
p &= 97.5
\end{aligned}
$$

The answer is 97.5.

Therefore 78 out of 80 is 97.5%.

PRACTICE

Find p in the given problems using cross product. Round to the nearest tenths place.

1. $\frac{7}{15} = \frac{p}{100}$
2. $\frac{52}{3810} = \frac{p}{100}$
3. $\frac{16}{17} = \frac{p}{100}$
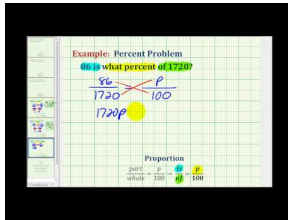4. $\frac{3}{4} = \frac{p}{100}$
5. $\frac{3}{5} = \frac{p}{100}$
6. $\frac{1}{5} = \frac{p}{100}$
7. A dentist filled cavities in 8 of his 30 patients on Tuesday. What percent had cavities filled?

8. A florist delivered 18 out of 25 bouquets. What percent was delivered?

9. The baker sold 3 out of 4 dozen rolls. What percent was sold?

10. What percent is 85 out of 5000?

11. What percent is 15 out of 30?

12. What percent is 88 out of 1200?

13. What percent is 99 out of 200?

14. What percent is 100 out of 330?

15. What percent is 224 out of 5400?

Resources

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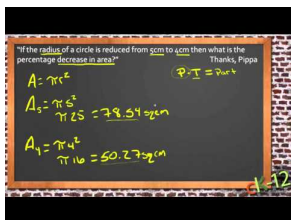Click image to the left or use the URL below.
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6.3 Percent – Equations (7.4d)

GUIDANCE

What if you knew that 25% of a number was equal to 24? How could you find that number? After completing this Concept, you'll be able to use the percent equation to solve problems like this one.

Watch This

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The percent equation is often used to solve problems. It goes like this:

$$\text{Rate} \times \text{Total} = \text{Part}$$
or
$$R\% \text{ of Total is Part}$$

Rate is the ratio that the percent represents (*R%* in the second version).

Total is often called the *base unit*.

Part is the amount we are comparing with the base unit.

Example A

Find 25% of $80.

Solution

We are looking for the **part**. The **total** is $80. 'of' means multiply. *R%* is 25%, so we can use the second form of the equation: 25% of $80 is Part, or $0.25 \times 80 = \text{Part}$.

$0.25 \times 80 = 20$, so the Part we are looking for is **$20**.

Example B

Express $90 as a percentage of $160.

Solution

This time we are looking for the **rate**. We are given the **part** ($90) and the **total** ($160). Using the rate equation, we get Rate $\times$ 160 = 90. Dividing both sides by 160 tells us that the rate is 0.5625, or 56.25%.
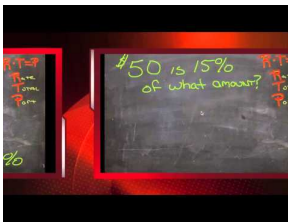
Example C

$50 is 15% of what total sum?

Solution

This time we are looking for the **total**. We are given the **part** ($50) and the **rate** (15%, or 0.15). Using the rate equation, we get $0.15 \times$ Total $= \$50$. Dividing both sides by 0.15, we get Total $= \frac{50}{0.15} \approx 333.33$. So **$50 is 15% of $333.33.**

Watch this video for help with the Examples above.

GUIDED PRACTICE

$96 is 12% of what total sum?

Solution:

This time we are looking for the **total**. We are given the **part** ($96) and the **rate** (12%, or 0.12). Using the rate equation, we get $0.12 \times$ Total $= \$96$. Dividing both sides by 0.15, we get Total $= \frac{96}{0.12} = 800$. So **$96 is 12% of $800.**

PRACTICE

Find the following.

1. 30% of 90
2. 27% of 19
3. 16.7% of 199

4. 11.5% of 10.01
5. 0.003% of 1,217.46
6. 250% of 67
7. 34.5% of y
8. 17.02% of y
9. x% of 280
10. a% of 0.332
11. y% of $3x$

6.4 Percent – Percent of Change (7.4d)

GUIDANCE

Bronze Gym's membership is currently at 2,100 members. The owner, Jezzie, would like to increase this number by 15%. How many new members does she have to recruit? What will the new membership total be?

In this concept, you will learn to use the percent of change to find new amounts.

Percent of Change

You can find the percent of change if you know an original amount and how much it either increased or decreased. At times, however, you are given the percent of increase or decrease and need to calculate a new amount.

Let's look at how you can calculate a new amount by using the percent of change.

A restaurant manager has noticed an increase of 4% in the cost of utilities. In order to pay for the increased costs, he decides to increase prices by 4% as well. Not all items are priced the same. The chicken platter currently costs $5.99 and the steak platter cost $7.99. If the prices are increased by 4%, what will the new prices be?

First, calculate the new price of the chicken platter. You know that the original price is $5.99 and the price is going up 4%. Use the formula:

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original amount}} \times 100$$

Next, put in the amounts that you know.

$$4\% = \frac{\text{amount of change}}{5.99} \times 100$$

Then, divide by 100.

$$
\begin{aligned}
4\% &= \frac{\text{amount of change}}{5.99} \times 100 \\
\frac{4}{100} &= \frac{\frac{\text{amount of change}}{5.99} \times 100}{100} \\
0.04 &= \frac{\text{amount of change}}{5.99}
\end{aligned}
$$

Then, multiply by 5.99.

$$
\begin{aligned}
0.04 &= \frac{\text{amount of change}}{5.99} \\
0.04 \times 5.99 &= \frac{\text{amount of change}}{5.99} \times 5.99 \\
\text{amount of change} &= 0.2396
\end{aligned}
$$

The answer is 0.2396.

The chicken platter will increase in price by 24¢. The new price will be $6.23.

Now, do the same calculations for the steak platter. The original price of the steak platter is $7.99 and it too will be increased by 4%.

First, put in the amounts that you know.

$$4\% = \frac{\text{amount of change}}{7.99} \times 100$$

Then, divide by 100.

$$
\begin{aligned}
4\% &= \frac{\text{amount of change}}{7.99} \times 100 \\
\frac{4}{100} &= \frac{\frac{\text{amount of change}}{7.99} \times 100}{100} \\
0.04 &= \frac{\text{amount of change}}{7.99}
\end{aligned}
$$

Then, multiply by 7.99.

$$
\begin{aligned}
0.04 &= \frac{\text{amount of change}}{7.99} \\
0.04 \times 7.99 &= \frac{\text{amount of change}}{7.99} \times 7.99 \\
\text{amount of change} &= 0.3196
\end{aligned}
$$

The answer is 0.3196.

The steak platter will increase in price by 32¢. The new price will be $8.31.

Let's look at another question.

Find the new amount if 60 is decreased by 27%.

273

First, use the formula to put in the amounts that you know.

$$27\% = \frac{\text{amount of change}}{60} \times 100$$

Then, divide by 100.

$$
\begin{aligned}
27\% &= \frac{\text{amount of change}}{60} \times 100 \\
\frac{27}{100} &= \frac{\frac{\text{amount of change}}{60} \times 100}{100} \\
0.27 &= \frac{\text{amount of change}}{60}
\end{aligned}
$$

Then, multiply by 60.

$$
\begin{aligned}
0.27 &= \frac{\text{amount of change}}{60} \\
0.27 \times 60 &= \frac{\text{amount of change}}{60} \times 60 \\
\text{amount of change} &= 16.2
\end{aligned}
$$

The answer is 16.2.

Since this is a decrease, the new amount would be:

$$60 - 16.2 = 43.8$$

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Bronze Gym's flagging membership.

The gym currently has 2,100 members and Jezzie, the owner, wants to increase this number by 15%. How many new members does she need to recruit?

First, use the formula to put in the amounts that you know.

$$15\% = \frac{\text{amount of change}}{2100} \times 100$$

Then, divide by 100.

$$
\begin{aligned}
15\% &= \frac{\text{amount of change}}{2100} \times 100 \\
\frac{15}{100} &= \frac{\frac{\text{amount of change}}{2100} \times 100}{100} \\
0.15 &= \frac{\text{amount of change}}{2100}
\end{aligned}
$$

Then, multiply by 2100.

$$0.15 = \frac{\text{amount of change}}{2100}$$
$$0.15 \times 2100 = \frac{\text{amount of change}}{2100} \times 2100$$
$$\text{amount of change} = 315$$

The answer is 315. Since this is an increase, the new amount would be $2100 + 315 = 2415$.

Therefore, Jezzie will need to recruit 315 more members to bring her total membership to 2415.

Example 2

The number of students participating in the chess club increased in one year. It started off with 35 students and had an increase of 55%. Figure out the new number of students in the chess club given this increase.

First, use the formula to put in the amounts that you know.

$$55\% = \frac{\text{amount of change}}{35} \times 100$$

Then, divide by 100.

$$55\% = \frac{\text{amount of change}}{35} \times 100$$
$$\frac{55}{100} = \frac{\frac{\text{amount of change}}{35} \times 100}{100}$$
$$0.55 = \frac{\text{amount of change}}{35}$$

Then, multiply by 35.

$$0.55 = \frac{\text{amount of change}}{35}$$
$$0.55 \times 35 = \frac{\text{amount of change}}{35} \times 35$$
$$\text{amount of change} = 19.25$$

The answer is 19.25.

Since this is an increase, the new amount would be:

$$35 + 19 = 54$$

The chess club would have 54 students.

Example 3

Find the new amount if 10 is increased by 18%.

First, use the formula to put in the amounts that you know.

$$18\% = \frac{\text{amount of change}}{10} \times 100$$

Then, divide by 100.

$$
\begin{aligned}
18\% &= \frac{\text{amount of change}}{10} \times 100 \\
\frac{18}{100} &= \frac{\frac{\text{amount of change}}{10} \times 100}{100} \\
0.18 &= \frac{\text{amount of change}}{10}
\end{aligned}
$$

Then, multiply by 10.

$$
\begin{aligned}
0.18 &= \frac{\text{amount of change}}{10} \\
0.18 \times 10 &= \frac{\text{amount of change}}{10} \times 10 \\
\text{amount of change} &= 1.8
\end{aligned}
$$

The answer is 1.8.

Since this is an increase, the new amount would be:

$$10 + 1.8 = 11.8$$

Example 4

Find the new amount if 16 is decreased by 20%.

First, use the formula to put in the amounts that you know.

$$20\% = \frac{\text{amount of change}}{16} \times 100$$

Then, divide by 100.

$$
\begin{aligned}
20\% &= \frac{\text{amount of change}}{16} \times 100 \\
\frac{20}{100} &= \frac{\frac{\text{amount of change}}{16} \times 100}{100} \\
0.20 &= \frac{\text{amount of change}}{16}
\end{aligned}
$$

Then, multiply by 16.

$$
\begin{aligned}
0.20 &= \frac{\text{amount of change}}{16} \\
0.20 \times 16 &= \frac{\text{amount of change}}{16} \times 16 \\
\text{amount of change} &= 3.2
\end{aligned}
$$

The answer is 3.2.
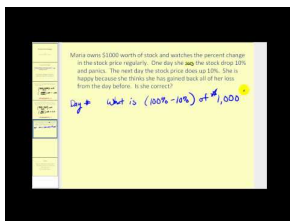
Since this is a decrease, the new amount would be

$$16 - 3.2 = 12.8$$

Example 5

Find the new amount if 250 is increase by 30%.

First, use the formula to put in the amounts that you know.

$$30\% = \frac{\text{amount of change}}{250} \times 100$$

Then, divide by 100.

$$30\% = \frac{\text{amount of change}}{250} \times 100$$
$$\frac{30}{100} = \frac{\frac{\text{amount of change}}{250} \times 100}{100}$$
$$0.30 = \frac{\text{amount of change}}{100}$$

Then, multiply by 250.

$$0.30 = \frac{\text{amount of change}}{250}$$
$$0.30 \times 250 = \frac{\text{amount of change}}{250} \times 250$$
$$\text{amount of change} = 75$$

The answer is 75.

Since this is an increase, the new amount would be:

$$250 + 75 = 325$$

PRACTICE

Use percent to find the new amount. You may round to the nearest whole number when necessary.

1. 82 increased by 90%
2. 64 decreased by 10%
3. 9 increased by 55%
4. 25,470 decreased by 77%
5. 75 increased by 10%
6. 33 decreased by 5%
7. 99 increased by 15%
8. 40 decreased by 8%
9. 56 increased by 25%
10. 900 decreased by 30%
11. 800 increased by 23%
12. 789 increased by 12%
13. 880 decreased by 10%
14. 450 increased by 45%
15. 855 decreased by 18%

Resources

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6.5 Percent – Sales Tax and Tip (7.4d)

GUIDANCE

Jerod and a few of his friends went to the local Chinese restaurant after seeing a matinee movie. The total of the food they ordered came to $47.50. But the restaurant also charges 6% sales tax on the total. The server has left the tip amount up to them as well. If using a credit card, how much will they pay at the restaurant's register, including sales tax and tip?

In this concept, you will learn to find total bills involving tips and taxes.

Finding Total Bills Involving Tips and Taxes

When calculating tips, you add the amount back to the total of the bill for the final cost of the meal. Sometimes, the tip is left in cash on the table. When this happens, you don't have to add it to the price of the meal. However, when using a credit card, you have to add the tip to the total to be sure that the server receives his/her tip. You also have to add any taxes.

Taxes are monies that are collected by the government to help pay for services such as schools, libraries, roads, and police and fire protection. A sales tax is a tax assessed on an item bought in a store. The rate of sales tax is given as a percent. You can find the amount of sales tax by using the following formula:

$$\text{Amount of sales tax} = \text{price} \times \text{rate of sales tax}$$

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After you calculate the amount of a tax using the percentage and the formula, you have to add it to an original amount. With sales tax, this is especially important.

Let's look at an example.

Emily bought a jacket for $85. If the sales tax is 7.5%, find the total cost of the jacket.

First, find the amount of sales tax.

$$7.5\% \text{ of } \$85 = 0.075 \times \$85 = \$6.375$$

You need to round $6.375 to the nearest penny, which is $6.38.

Then, add the original price of the jacket and the sales tax.

$$\$85 + \$6.38 = \$91.38$$

The total cost of the jacket, with sales tax, is $91.38.

Let's look at another example.

Mr. and Mrs. Green and their four children went out to dinner. The cost of the meal was $72. The restaurant added a tip of 18% to the bill. What was the total cost of the dinner?

First, find the amount of the tip.

$$18\% \text{ of } \$72 = 0.18 \times \$72 = \$12.96$$

Then, add the cost of the meal and the tip.

$$\$72 + \$12.96 = \$84.96$$

The total cost of the dinner, with tip, was $84.96.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Jerod's restaurant bill.

The total amount of the food came to $47.50. But the restaurant also charges 6% sales tax on the total. The excellent server has left the tip amount up to them as well. If using a credit card, how much will they pay at the restaurant's register, including sales tax and tip?

First, figure out the sales tax. To do this, convert the percent to a decimal and then multiply by the total cost of the meal.

$$0.06 \times \$47.50 = \$2.85$$

Next, figure out the tip. Because the server provided very good service, they decide to leave an 18% tip. Convert the percent to a decimal and multiply by the cost of the meal.

$$0.18 \times \$47.50 = \$8.55$$

Then, add it all up.

$$\$47.50 + \$2.85 + \$8.55 = \$58.90$$

The answer is the total they will pay at the register with a credit card is $58.90, including the tax and tip.

Example 2

Brandon bought a T-shirt for $8.99 and a pair of shorts for $11.99. If the rate of sales tax is 7%, how much did he spend in total?

First, find the total cost of the items purchased.

$$\$8.99 + \$11.99 = \$20.98$$

Next, find the amount of sales tax. In doing so, first convert the percent to a decimal and then multiply by the total cost. Round your answer to the nearest cent.

$$\$20.98 \times 0.07 = \$1.47 \text{ (Notice that you rounded up to 1.47.)}$$

Then, add this tax to the cost of the items.

$$\$20.98 + \$1.47 = \$22.45$$

The answer is the total cost of the T-shirt and shorts, with tax, is $22.45.

Example 3

Find the total amount on a $65.00 item with 5% sales tax assessed.

First, find the amount of sales tax. In doing so, first convert the percent to a decimal and then multiply by the total cost.

$$\$65.00 \times 0.05 = \$3.25$$

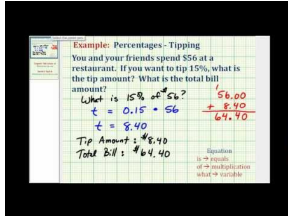Then, add this tax to the cost of the item.

$$\$65.00 + \$3.25 = \$68.25$$

The answer is the total cost of the item, with tax, is $68.25.

Example 4

Find the total amount of a $30.00 bill with a 20% tip.

First, find the amount of the tip.

$$20\% \text{ of } \$30 = 0.20 \times \$30 = \$6.00$$

Then, add the cost of the meal and the tip.

$$\$30 + \$6 = \$36$$

The total amount of the bill, with tip, is $36.

Example 5

Find the total amount of $150.00 item with a 10% sales tax assessed.

First, find the amount of sales tax. In doing so, first convert the percent to a decimal and then multiply by the total cost.

$$\$150.00 \times 0.10 = \$15.00$$

Then, add this tax to the cost of the item.

$$\$150.00 + \$15.00 = \$165.00$$

The answer is the total cost of the item, with tax, is $165.00.

PRACTICE

Calculate the total amount paid, including sales tax and tip, if the sales tax is 4% and a tip of 20% is given.

1. Total: $56.75
2. Total: $43.25
3. Total: $65.00
4. Total: $25.50
5. Total: $18.75
6. Total: $59.00
7. Total: $21.50
8. Total: $44.50
9. Total: $12.50
10. Total: $43.00
11. Total: $89.50
12. Total: $99.35
13. Total: $235.75
14. Total: $300.50
15. Total: $678.25

Resources

> **MEDIA**
>
> Click image to the left or use the URL below.
>
> URL: https://www.ck12.org/flx/render/embeddedobject/183421

6.6 Percent – Discounts (7.4d)

GUIDANCE

Jeremiah works at a local bookstore and convinced his boss to have a 30% off everything sale. The sales tax is 5%. What is the total price on a book whose cover price is $15.99?

In this concept, you will learn to find discount prices given sales.

Sale Prices

In order to reduce inventories or motivate buyers, stores often have sales in which they discount prices by a percent. Instead of a percent increase, this is a percent decrease, which results in a **sale price.**

Let's look at an example.

At the end of the summer, a clothing store puts all swimwear on sale. They offer a discount of 60%. If the regular price is $29.99 on a bikini, what is the sale price?

First, multiply the original price by the percent decrease. Remember that percent means the denominator is 100.

$$29.99 \times \frac{60}{100} = \frac{1799.4}{100}$$
$$= 17.994$$

Next, subtract the amount of decrease from the original price.

$$29.99 - 17.994 = 11.996$$

The answer is 11.996.

The sale price of the bikini is $12.00.

If the sales tax is 6.25%, what is the final price of the discounted bikini?

First, multiply the sale price by the sales tax. Remember that percent means the denominator is 100.

$$12.00 \times \frac{6.25}{100} = \frac{75}{100}$$
$$= 0.75$$

Next, add the sales tax to the sale price.

$$12.00 + 0.75 = 12.75$$

The answer is 12.75.

The total price of the bikini is $12.75.

Calculating a price with tax is the same as increasing by a percent. What if you know the total price including the tax and want to know the original price of a product?

Let's look at an example.

A store clerk charges you $78.75 for a DVD player. The tax in your area is 5%. So how much was the original price of the DVD player?

First, in order to get the total cost, the cash register computes the 5% tax and adds the tax to the original price. In other words:

$$\text{Original Price} + 5\% \text{ of Original Price} = \text{Total Cost}$$

Let p represent the original price and c represent the total cost, the equation becomes:

$$p + 0.05\, p = c \qquad \text{or} \qquad 1.05\, p = c$$

In this case, if you are given p, you can solve for c. Or if you are given c, you can solve for p.

Next, fill in what you know. In this example, you paid a total cost, c, of $78.75. Substitute c in the equation.

$$1.05\, p = c$$
$$1.05\, p = 78.75$$

Then, divide both sides by 1.05 to solve for p.

$$1.05\, p = 78.75$$
$$\frac{1.05\, p}{1.05} = \frac{78.75}{1.05}$$
$$p = 75$$

The answer is 75.

The original price was $75.00

Let's try another one.

You are charged $29.10 for an item with 7% tax included. What was the original price of the item?

First, write an equation.

$$p + 0.07\,p = c \qquad \text{or} \qquad 1.07\,p = c$$

Next, fill in what you know. In this example, you paid a total cost, c, of $29.10. Substitute c in the equation.

$$
\begin{aligned}
1.07\,p &= c \\
1.07\,p &= 29.10
\end{aligned}
$$

Then, divide both sides by 1.07 to solve for p.

$$
\begin{aligned}
1.07\,p &= 29.10 \\
\frac{1.07\,p}{1.07} &= \frac{29.10}{1.07} \\
p &= 27.196
\end{aligned}
$$

The answer is 27.196.

The original price was $27.20.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Jeremiah's bookstore sale.

They have a 30% off any book sale. You bought a book for $15.99 and then had to pay 5% sales tax on the book purchase.

First, calculate the discount on the purchase.

$$
\begin{aligned}
15.99 \times \tfrac{30}{100} &= \tfrac{479.7}{100} \\
&= 4.797
\end{aligned}
$$

Next subtract the discount from the original price to find the price of your book.

$$15.99 - 4.797 = 11.193$$

Then, calculate the sales tax. The price of the book, p, is $11.19. You are looking for the cost, c.

$$
\begin{aligned}
c &= 1.05\,p \\
c &= 1.05(11.19) \\
c &= 11.7495
\end{aligned}
$$

The answer is 11.7495.

The final price is $11.75.

Example 2

Brenda purchases some plants for her gardens. Two trees have a price of $55.00 each. Six tulips cost $2.50 each. The tax on Brenda's purchase is 5.75% but there is an early-bird special of 10% off the entire purchase to those who show up before 10 am (which Brenda did). What is Brenda's total cost?

First, add up all of your purchases before tax and discounts.

$$2 \text{ trees} + 6 \text{ tulips}$$

$$2 \times 55 + 6 \times 2.50 = 125$$

Next, calculate the total cost after sales tax.

$$\begin{aligned} c &= 1.0575\, p \\ c &= 1.0575 \times 125 \\ c &= 132.1875 \end{aligned}$$

Next, calculate the discount on the purchase.

$$\begin{aligned} 132.1875 \times \tfrac{10}{100} &= \tfrac{1321.875}{100} \\ &= 13.21875 \end{aligned}$$

Then, subtract the discount from the total price to find the price paid.

$$132.1875 - 13.21875 = 118.96875$$

The answer is 118.96875.

The final price Brenda paid for the trees and tulips is $118.97.

Example 3

The original price is $27.50. Find the new price given a discount of 30%.

First, calculate the discount on the purchase.

$$\begin{aligned} 27.50 \times \tfrac{30}{100} &= \tfrac{825}{100} \\ &= 8.25 \end{aligned}$$

Next, subtract the discount from the original price to find the price paid.

$$27.50 - 8.25 = 19.25$$

The answer is 19.25.

The final price is $19.25.

Example 4

The original price is $545.00. Find the new price given a discount of 30%.

First, calculate the discount on the purchase.

$$545.00 \times \frac{30}{100} = \frac{16350}{100}$$
$$= 163.5$$

Next, subtract the discount from the original price to find the price paid.

$$545.00 - 163.50 = 381.50$$

The answer is 381.50.

The final price is $381.50.

Example 5

The original price is $75.80. Find the new price given a discount of 30%.

First, calculate the discount on the purchase.

$$75.80 \times \frac{30}{100} = \frac{2274}{100}$$
$$= 22.74$$

Next, subtract the discount from the original price to find the price paid.

$$75.80 - 22.74 = 53.06$$

The answer is 53.06.

The final price is $53.06.

PRACTICE

Your food bill at a restaurant is $85.77. Calculate your total cost after each of the following. Be sure to round when necessary.

1. Service 10%

2. Decent service 15%

3. Great service 20%

4. Outstanding service 25%

5. Poor service 5%

What was the original price given total cost and tax rate? Be sure to round when necessary.

6. Total cost: $1475.68 tax rate: 7%

7. Total cost: $63.80 tax rate: 4.5%

8. Total cost: $55.90 tax rate: 4%

9. Total cost: $80.20 tax rate: 2.5%

10. Total cost: $120.00 tax rate: 5%

11. Total cost: $99.50 tax rate: 2%

12. Total cost: $155.30 tax rate: 3%

13. Total cost: $250.75 tax rate: 3.5%

Solve each problem. There are several steps to solving each problem. Be sure to round when necessary.

14. You take a taxi ride in a foreign country where they add 20% to your total for late night travel. The driver expects an additional 15% tip. How much do you owe for the taxi ride if the meter shows $45?

15. You take your mother out for lobster for Mother's Day. The lobster platters are $24.95 each but include the drink and dessert buffet. Your waitress is a mother, too, so you leave her a 20% tip. However, you did bring a coupon for 25% off. What is your total cost for 2 people?

Resources

MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/167915
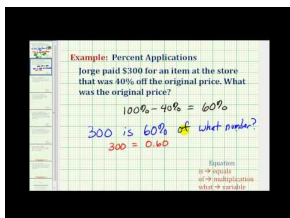
6.7 Simple Interest (7.13e and 8.12d)

GUIDANCE
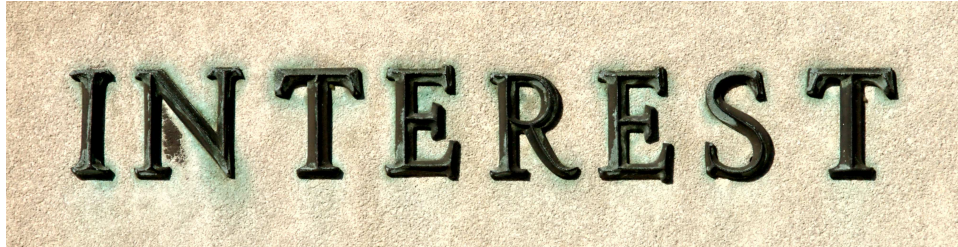
Nisi entered a local triathlon and won the grand prize of $4000. She decides to put all $4000 in the bank and leave it untouched for two years until she finishes high school. The bank offered her a 4% interest rate. After the two years is up, how much money will Nisi have in the bank?

In this concept, you will learn to solve real-world problems involving simple interest.

Interest

Saving money and making wise investments will be an important part of your financial planning. Part of investing is earning interest. When you save money in the bank, the bank uses that money for its own investments. In return for using your money, the bank pays you a certain percent. **Interest** is the percent that a bank pays you for keeping your money in their bank.

All banks have an interest rate r which tells you what percent they will pay you per year t. The principal, p is the amount of money that you borrow or save. In order to calculate how much interest you will make, you can use the formula:

$$I = PRT$$

Now let's look at an example using this formula to calculate interest.

You invest $5,000 in a bank for 2 years at a 3% interest rate. What is the interest you have earned after this time?

First, using the formula for simple interest, fill in all of the numbers you know.

$$p = \$5000$$
$$r = 3\% \text{ or } 0.03$$
$$t = 2 \text{ years}$$

Next, solve for I.

$$I = PRT$$
$$I = (5000)(0.03)(2)$$

Next, solve for I.

$$I = (5000)(0.03)(2)$$
$$I = 300$$

The answer is 300.

You have earned $300 in interest in the two years.

Let's look at another example.

Mrs. Payne has $20,000 to invest. She wants to earn $10,000 in interest. She is considering a savings and loans bank that is offering her 5.6% interest per year. For how long will she have to leave her money in the bank in order to reach her goal of $10,000?

First, using the formula for simple interest, fill in all of the numbers you know.

$$I = 10000$$
$$p = \$20000$$
$$r = 5.6\% \text{ or } 0.056$$

$$
\begin{aligned}
I &= PRT \\
10000 &= (20000)(0.056)(t) \\
10000 &= 1120\,t
\end{aligned}
$$

Next, solve for t by dividing both sides by 1120.

$$
\begin{aligned}
10000 &= 1120\,t \\
\frac{10000}{1120} &= \frac{1120\,t}{1120} \\
t &= 8.93
\end{aligned}
$$

The answer is 8.93.

Mrs. Payne will have to leave her money in the bank for almost 9 years.

You can use the simple interest formula $(I = PRT)$ to find any of the missing variables if you are given values of the others. You have just used it to solve for I and t.

Let's try another example.

Jessica invests $3,000 in a credit union at an interest rate of 3.9%. She leaves the money there for 5 years. What is her balance after that time?

First, using the formula for simple interest, fill in all of the numbers you know.

$$p = \$3000$$
$$r = 3.9\% \text{ or } 0.039$$
$$t = 5 \text{ years}$$

$$
\begin{aligned}
I &= PRT \\
I &= (3000)(0.039)(5)
\end{aligned}
$$

Next, solve for I.

$$
\begin{aligned}
I &= (3000)(0.039)(5) \\
I &= 585
\end{aligned}
$$

Then, to find the bank balance, add the interest earned to the amount invested.

$$
\begin{aligned}
balance &= 3000 + 585 \\
&= 3585
\end{aligned}
$$

The answer is 3585.

Jessica's balance will be $3585 in the five years.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Nisi putting in $4000 in the bank.

Nisi left her $4000 prize money in the bank for two years at 4% interest. She needs to figure out how much money she now has.

First, using the formula for simple interest, fill in all of the numbers you know.

$$
\begin{aligned}
p &= \$4000 \\
r &= 4.0\% \text{ or } 0.04 \\
t &= 2 \text{ years}
\end{aligned}
$$

$$
\begin{aligned}
I &= PRT \\
I &= (4000)(0.09)(2)
\end{aligned}
$$

Next, solve for *I*.

$$
\begin{aligned}
I &= (4000)(0.04)(2) \\
I &= 320
\end{aligned}
$$

Then, to find the total amount in the bank, add the interest earned to the original amount.

$$
\begin{aligned}
\text{balance} &= 4000 + 320 \\
&= 4320
\end{aligned}
$$

The answer is 4320. After two years, Nisi will have $4320.

Example 2

A nurse put $22,000 in the bank 15 years ago. She has earned $21,450 in interest—nearly as much as her initial investment. What was the interest rate that the bank was paying her?

First, using the formula for simple interest, fill in all of the numbers you know.

$$
\begin{aligned}
I &= \$21450 \\
p &= \$22000 \\
t &= 15 \text{ years}
\end{aligned}
$$

$$
\begin{aligned}
I &= PRT \\
21450 &= (22000)(r)(15) \\
21450 &= 330000\,r
\end{aligned}
$$

Next, solve for *r* by dividing by 330000.

$$
\begin{aligned}
21450 &= 330000\,r \\
\frac{21450}{330000} &= \frac{330000\,r}{330000} \\
r &= 0.065
\end{aligned}
$$

Then, multiply the value of r by 100 since interest rates are reported as percentages.

$$
\begin{aligned}
r &= 0.065 \times 100 \\
r &= 65\%
\end{aligned}
$$

The answer is 6.5%.

The nurse invested at 6.5% interest.

Example 3

An investor places $15,000 in a savings account that pays 4.5% interest. She will leave the money there for 6 years. What will her interest be?

First, using the formula for simple interest, fill in all of the numbers you know.

$$
\begin{aligned}
p &= \$15000 \\
r &= 4.5\% \text{ or } 0.045 \\
t &= 6 \text{ years}
\end{aligned}
$$

$$
\begin{aligned}
I &= PRT \\
I &= (15000)(0.045)(6)
\end{aligned}
$$

Next, solve for I.

$$
\begin{aligned}
I &= (15000)(0.045)(6) \\
I &= 4050
\end{aligned}
$$

The answer is 4050.

The investor earned $4050 in interest in the six years.

Example 4

A bank is offering an interest rate of 4.75%. How long would it take to earn $500 if you invested $12,000 in the bank?

First, using the formula for simple interest, fill in all of the numbers you know.

$$
\begin{aligned}
I &= \$500 \\
p &= \$12000 \\
r &= 4.75\% \text{ or } 0.0475
\end{aligned}
$$

$$
\begin{aligned}
I &= PRT \\
500 &= (12000)(0.0475)(t) \\
500 &= 570\,t
\end{aligned}
$$

Next, solve for *t* by dividing both sides by 570.

$$
\begin{aligned}
500 &= 570\,t \\
\frac{500}{570} &= \frac{570\,t}{570} \\
t &= 0.877
\end{aligned}
$$

The answer is 0.88.

It would take 0.88 years or 10.6 months to gain $500 in interest.

Example 5

If you charge $7,000 on a credit card and you bank charges you 15.9%, how much would you owe after a year?

First, using the formula for simple interest, fill in all of the numbers you know.

$$
\begin{aligned}
p &= \$7000 \\
r &= 15.9\% \text{ or } 0.159 \\
t &= 1 \text{ year}
\end{aligned}
$$

$$
\begin{aligned}
I &= PRT \\
I &= (7000)(0.159)(1)
\end{aligned}
$$

Next, solve for *I*.

$$
\begin{aligned}
I &= (7000)(0.159)(1) \\
I &= 1113
\end{aligned}
$$

Then, to find the total owing after one year, add the interest earned to the amount borrowed.

$$
\begin{aligned}
\text{amount owing} &= 7000 + 1113 \\
&= 8113
\end{aligned}
$$

The answer is 8113.

You would owe $8113 after one year.

PRACTICE

Use the simple interest formula $I = PRT$ to solve for the Interest.

1. Find *I* if $p = 62,300$, $r = 0.0525$, $t = 14$.

2. Find *I* if $p = 9800$, $r = 0.028$, $t = 9$.

3. Find *I* if $p = \$600$, $r = 0.05$, $t = 8$

4. Find *I* if $p = \$2300$, $r = 0.06$, $t = 12$

5. Find *I* if $p = \$5500$, $r = 0.08$, $t = 7$

6. Find *I* if $p = \$400$, $r = 0.05$, $t = 57$.

7. Find *I* if $p = \$700$, $r = 0.03$, $t = 9$

8. Find *I* if $p = \$500$, $r = 0.06$, $t = 12$

9. Find *I* if $p = \$800$, $r = 0.09$, $t = 7$

10. Find *I* if $p = \$950$, $r = 0.06$, $t = 4$

Find the new interest and then find the new balance with the given information. There are two steps to solving these problems.

11. $p = 43000$, $r = 0.0365$, $t = 11$

12. $p = 7000$, $r = 0.079$, $t = 4$

13. $p = 8000$, $r = 0.06$, $t = 3$

14. $p = 18000$, $r = 0.04$, $t = 5$

15. $p = 25000$, $r = 0.05$, $t = 3$

16. $p = 3000$, $r = 0.05$, $t = 7$

17. $p = 12000$, $r = 0.04$, $t = 5$

18. $p = 9000$, $r = 0.06$, $t = 10$

19. $p = 7500$, $r = 0.03$, $t = 8$

20. $p = 27500$, $r = 0.04$, $t = 6$

Resources

MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/166535
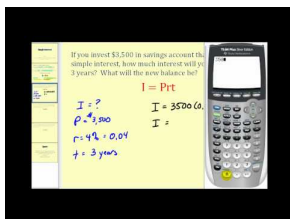
6.8 Compound Interest (7.13e and 8.12d)

GUIDANCE

James is investing $15,000 in the bank. The investment has an interest rate of 6% compounded monthly. After ten years, how much will James have made?

In this concept, you will learn to solve real-world problems involving compound interest.

Compound Interest

Interest is important. Understanding interest helps you to make the best decisions. In most cases, in the real world interest is calculated with the **compound interest** formula. The compound interest formula is:

$$A = P(1 + \frac{r}{n})^{nt}$$

Where A is the amount, P is the principal amount, r is the interest rate, n is the number of times per year the interest is compounded, and t is the number of years.

Basically, this formula accounts for the fact that as you invest and earn interest, your balance grows. You are not only due interest, then, on your original balance, but on the new balance which includes the first installment(s) of interest. You get paid interest on the interest.

Let's look at an example.

You invest $100 for 3 years at 10% interest compounded yearly. How much will you have at the end of the three years?

First, write down what you know.

$$
\begin{aligned}
A &= ? \\
P &= 100 \\
r &= 10\% = 0.10 \\
n &= 1 \\
t &= 3
\end{aligned}
$$

Next, fill in what you know into the compound interest formula.

$$
\begin{aligned}
A &= P\left(1+\tfrac{r}{n}\right)^{nt} \\
A &= 100\left(1+\tfrac{0.10}{1}\right)^{1\times 3} \\
A &= 100(1+0.10)^3
\end{aligned}
$$

Then, solve for your unknown.

$$
\begin{aligned}
A &= 100(1+0.10)^3 \\
A &= 100(1.10)^3 \\
A &= 133.10
\end{aligned}
$$

The answer is 133.10.

In three years, your $100 has grown to $133.10.

Let's see how different it might be using the different interest formulas for the same amount of time at the same rate.

You want to invest $20,000 for the next 20 years. You have two options for your investment. Bank X offers simple interest at a rate of 8%. Bank Y uses compound interest at a rate of 8% compounded yearly. Which should you choose?

First, start with the simple interest formula $(I = PRT)$. Write down what you know.

$$
\begin{aligned}
I &= ? \\
P &= 20000 \\
r &= 8\% = 0.08 \\
t &= 20 \text{ years}
\end{aligned}
$$

Next, fill in what you know into the simple interest formula and solve for I.

$$
\begin{aligned}
I &= PRT \\
I &= (20000)(0.08)(20) \\
I &= 32000
\end{aligned}
$$

Then, add this interest to your original investment.

$$
\begin{aligned}
\text{balance} &= 20000 + 32000 \\
\text{balance} &= 52000
\end{aligned}
$$

The answer is 52000.

Using simple interest, you would have $52,000 in 20 years.

Now let's use the compound interest formula.

First, write down what you know.

$$
\begin{aligned}
A &= ? \\
P &= 20000 \\
r &= 8\% = 0.08 \\
n &= 1 \\
t &= 20
\end{aligned}
$$

Next, fill in what you know into the compound interest formula and solve for A.

$$
\begin{aligned}
A &= P\left(1+\tfrac{r}{n}\right)^{nt} \\
A &= 20000\left(1+\tfrac{0.08}{1}\right)^{1\times 20} \\
A &= 20000(1+0.08)^{20} \\
A &= 93219.14
\end{aligned}
$$

The answer is 93219.14.

Using compound interest, you would have $93,219.14 in 20 years.

You should choose Bank Y to put your money into an investment.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about James and his investment.

James is investing $15,000 for ten years at an interest rate of 6% compounded monthly.

First, write down what you know.

$$
\begin{aligned}
A &= ? \\
P &= 15000 \\
r &= 6\% = 0.06 \\
n &= 12 \\
t &= 10
\end{aligned}
$$

Next, fill in what you know into the compound interest formula and solve for A.

$$
\begin{aligned}
A &= P\left(1+\tfrac{r}{n}\right)^{nt} \\
A &= 15000\left(1+\tfrac{0.06}{12}\right)^{12\times 10} \\
A &= 15000(1.005)^{120} \\
A &= 15000(1.819397) \\
A &= 27290.95
\end{aligned}
$$

The answer is 27290.95.

James will have $27290.95 in 10 years.

Example 2

A fireman invests $40,000 in a retirement account for 2 years. The interest rate is 6%. The interest is compounded monthly. What will his final balance be?

First, write down what you know.

$$
\begin{aligned}
A &= ? \\
P &= 40000 \\
r &= 6\% = 0.06 \\
n &= 12 \\
t &= 2
\end{aligned}
$$

Next, fill in what you know into the compound interest formula and solve for A.

$$
\begin{aligned}
A &= P\left(1+\tfrac{r}{n}\right)^{nt} \\
A &= 40000\left(1+\tfrac{0.06}{12}\right)^{12\times2} \\
A &= 40000(1+0.005)^{24} \\
A &= 40000(1.12716) \\
A &= 45086.39
\end{aligned}
$$

The answer is 45086.39.

Using compound interest, the fireman would have $45,086.39 in 2 years.

Example 3

Calculate the amount of this investment after 5 years with interest compounded yearly.

Principal = $3000

Rate = 4%

First, write down what you know.

$$
\begin{aligned}
A &= ? \\
P &= 3000 \\
r &= 4\% = 0.04 \\
n &= 1 \\
t &= 5
\end{aligned}
$$

Next, fill in what you know into the compound interest formula and solve for A.

$$
\begin{aligned}
A &= P\left(1+\tfrac{r}{n}\right)^{nt} \\
A &= 3000\left(1+\tfrac{0.04}{1}\right)^{1\times5} \\
A &= 3000(1.04)^{5} \\
A &= 3649.96
\end{aligned}
$$

The answer is 3649.96.

Therefore using compound interest, you would have $3649.96 in 5 years.

Example 4

Calculate the amount of this investment after 5 years with interest compounded every two months.

Principal = $5000

Rate = 3%

First, write down what you know.

$$
\begin{aligned}
A &= \text{?} \\
P &= 5000 \\
r &= 3\% = 0.03 \\
n &= 6 \\
t &= 5
\end{aligned}
$$

Next, fill in what you know into the compound interest formula and solve for A.

$$
\begin{aligned}
A &= P\left(1+\frac{r}{n}\right)^{nt} \\
A &= 5000\left(1+\frac{0.03}{6}\right)^{6\times 5} \\
A &= 5000(1.005)^{30} \\
A &= 5000(1.1614) \\
A &= 5807.00
\end{aligned}
$$

The answer is 5807.00.

Using compound interest, you would have $5807.00 in 5 years.

Example 5

Calculate the amount of this investment after 5 years with interest compounded quarterly.

Principal = $12,000

Rate = 9%

First, write down what you know. Note that quarterly is every 3 months (4 times a year).

$$
\begin{aligned}
A &= \text{?} \\
P &= 12000 \\
r &= 9\% = 0.09 \\
n &= 4 \\
t &= 5
\end{aligned}
$$

Next, fill in what you know into the compound interest formula and solve for A.

$$\begin{aligned}
A &= P\left(1+\tfrac{r}{n}\right)^{nt} \\
A &= 12000\left(1+\tfrac{0.09}{4}\right)^{4\times5} \\
A &= 12000(1.0225)^{20} \\
A &= 12000(1.5605) \\
A &= 18726.11
\end{aligned}$$

The answer is 18726.11.

Using compound interest, you would have $18726.11 in 5 years.

PRACTICE

Calculate the simple interest by using I =PRT.

1. Principal = $2000, Rate = 5%, Time = 3 years

2. Principal = $12,000, Rate = 4%, Time = 2 years

3. Principal = $10,000, Rate = 5%, Time = 5 years

4. Principal = $30,000, Rate = 2.5%, Time = 10 years

5. Principal = $12,500, Rate = 3%, Time = 8 years

6. Principal = $34,500, Rate = 4%, Time = 10 years

7. Principal = $16,000, Rate = 3%, Time = 5 years

8. Principal = $120,000, Rate = 5%, Time = 4 years

Calculate the following compound interest calculated yearly.

9. Principal = $3000, Rate = 4%

10. Principal = $5000, Rate = 3%

11. Principal = $12,000, Rate = 2%

12. Principal = $34,000, Rate = 5%

13. Principal = $18,000, Rate = 3%

14. Principal = $7800, Rate = 4%

15. Principal = $8500, Rate = 3%

6.9 Monetary Incentives (7.13f)

GUIDANCE

Look at the signs for the toothpaste below. How much would it cost to buy 2 tubes of toothpaste at each store? What if you use the coupon? Which store has the better buy? How much less is the cost at that store? In this concept, we will learn how to compare prices at stores when we have a coupon to use.

In order to figure out which item is the better buy, like in the question above, we can use the problem solving steps to help.

- Start by **describing** what you see the pictures.
- Next, figure out what **your job** is in this problem. In all of these problems your job will be to figure out which item is the better buy and by how much.
- Then, make a **plan** for how you will solve. In these problems, you will want to figure out the original cost for the number of items you want to buy at each store. Then, calculate how much you will pay after using the coupon. Finally, figure out which store has the better buy and by how much.
- Next, **solve** the problem.
- Finally, **check** to make sure that all your calculations are correct.

Example A

Buy 1 comb. Use the coupon. Which store has the better buy? How much less is the cost at that store?

Solution:

We can use problem solving steps to help.

TABLE 6.1:

Step	Description
Describe:	Two store signs show the cost for combs. The coupon gives 10% off the price.
My Job:	Figure out the store that has the better buy for 1 comb after using the coupon. Then figure out the difference in cost.
Solve:	ABC's: 5 combs are $15 so each comb is $3. Gem's: 6 combs are $12 so each comb is $2. Using the coupon at ABC's, 10% off means the cost after the coupon is 90% of $3.00, or $0.9 \times 3 = \$2.70$. Using the coupon at Gem's, 10% off means the price after the coupon is 90% of $2.00 or $0.9 \times 2 = \$1.80$. $\$2.70 - \$1.80 = \$0.90$ The cost is $0.90 less at Gem's.
Check:	At ABC's 1 comb is $3.00 and 10% off of $3.00 is $0.30. The cost after using the coupon is ($3.00 - $0.30), or $2.70. At Gem's 1 comb is $2.00 and 10% off of $2.00 is $0.20. The cost after using the coupon is ($2.00 - $0.20), or $1.80. $\$2.70 - \$1.80 = \$0.90$.

Example B

Buy 4 boxes of band aids. Use the coupon. Which store has the better buy? How much less is the cost at that store?

Solution:

We can use problem solving steps to help.

Describe :	Two store signs show the cost for boxes of bandaids. The coupon gives 30% off the price.
My Job :	Figure out the store that has the better buy for 4 boxes of band aids after using the coupon. Then figure out the difference in cost.
Plan :	Figure out the cost of 4 boxes of tissue at ABC's. Do the same at Gem's. Take 30% off of each. Subtract to find the difference.
Solve :	ABC's: 6 boxes are $12 so each box is $2 and four boxes are $8.

Gem's: 8 boxes are $24 so 4 boxes are $\frac{1}{2}$ of 24, or $24 \div 2$, or $12.

Using the coupon at ABC's, 30% off means the cost after the coupon is 70% of $8.00, or $0.3 \times 8 = $5.60.

Using the coupon at Gem's, 20% off means the price after the coupon is 70% of $12.00 or $0.3 \times 12 = $8.40.

$8.40 − $5.60 = $2.80

The cost is $2.80 less at ABC's.

Check :	At ABC's 4 boxes are $8.00 and 30% off of $8.00 is $2.40. The cost after using the coupon is $8.00 − $2.40, or $5.60.
	At Gem's 4 boxes are $12.00 and 30% off of $12.00 is $3.60. The cost after using the coupon is $12.00 − $3.60, or $8.40. $8.40 − $5.60 = $2.80.

Example C

Buy 2 boxes of tissue. Use the coupon. Which store has the better buy? How much less is the cost at that store?

Solution:

We can use problem solving steps to help.

Describe :	Two store signs show the cost for boxes of tissue. The coupon gives 20% off the price.
My Job :	Figure out the store that has the better buy for 2 boxes of tissue after using the coupon. Then figure out the difference in cost.
Plan :	Figure out the cost of 2 boxes of tissue at ABC's. Do the same at Gem's. Take 20% off of each. Subtract to find the difference.
Solve :	ABC's: 4 boxes are $12 so 2 boxes are half the price or $6.
	Gem's: 6 boxes are $24 so 2 boxes are $\frac{1}{3}$ of 24, or $24 \div 3$, or $8.
	Using the coupon at ABC's, 20% off means the cost after the coupon is 80% of $6.00, or $0.8 \times 6 = \$4.80$.
	Using the coupon at Gem's, 20% off means the price after the coupon is 80% of $8.20 or $0.8 \times 8 = \$6.40$.
	$6.40 - \$4.80 = \$1.60$
	The cost is $1.60 less at ABC's.
Check :	At ABC's 2 boxes are $6.00 and 20% off of $6.00 is $1.20. The cost after using the coupon is $6.00 - \$1.20$, or $4.80.
	At Gem's 2 boxes are $8.00 and 20% off of $8.00 is $1.60. The cost after using the coupon is $8.00 - \$1.60$, or $6.40. $6.50 - \$4.80 = \$1.60$.

Concept Problem Revisited

ABC's	Gem's	ToothPaste Coupon
Tubes of Toothpaste 4 for $6	Tubes of Toothpaste 10 for $16	20% off

We can use problem solving steps to help.

Describe :   Two store signs show the cost for tubes of toothpaste. The coupon gives
                        20% off the price.

My Job :   Figure out the store that has the better buy for 2 tubes of toothpaste after
                        using the coupon. Then figure out the difference in cost.

Plan :   Figure out the cost of 2 tubes of toothpaste ABC's. Do the same at Gem's.
                        Take 20% off of each. Subtract to find the difference.

Solve :   ABC's: 4 tubes are $6 so 2 tubes are half the price or $3.

                        Gem's: 10 tubes are $16 so 2 tubes are $\frac{1}{5}$ of 16, or $16 \div 5$, or $3.20.

                        Using the coupon at ABC's, 20% off means the cost after the coupon is
                        80% of $3.00, or $0.8 \times 3 = $2.40.

                        Using the coupon at Gem's, 20% off means the price after the coupon is
                        80% of $3.20 or $0.8 \times 3.20 = $2.56.

                        $2.56 - $2.40 = $0.16

                        The cost is $0.16 less at ABC's.

Check :   At ABC's 2 tubes are $3.00 and 20% off of $3.00 is $0.60. The cost after
                        using the coupon is $3.00 - $0.60, or $2.40.

                        At Gem's 2 tubes are $3.20 and 20% off of $3.20 is $0.64. The cost after
                        using the coupon is $3.20 - $0.64, or $2.56. $2.56 - $2.40 = $0.16.

Vocabulary

A *discount* is a savings on an item because of a sale or a coupon. In this concept, we used coupons that had discounts
in the form of *percents* of. *Percent* means parts per 100. The symbol for *percent* is %. For example, 30% means 30
out of every 100. So, 30% of 100 would be 30. 30% of 50 would be 15. When working with percents, it is often
helpful to convert them to decimals when doing calculations, as we did in this concept.

GUIDED PRACTICE

1. Buy 2 toothbrushes. Use the coupon. Which store has the better buy? How much less is the cost at that store?

2. Buy 4 bars of soap. Use the coupon. Which store has the better buy? How much less is the cost at that store?

ABC's — Bars of Soap 8 for $7.20

Gem's — Bars of Soap 10 for $10

Coupon — 30% off

3. Buy 4 bottles of shampoo. Use the coupon. Which store has the better buy? How much less is the cost at that store?

ABC's — Shampoo 5 bottles for $12.50

Gem's — Shampoo 6 bottles for $14.40

Coupon — 40% off

Answers:

1. Gems; $0.25 less.

  ABC's: 1 toothbrush is $5.25 \div 3$ or $1.75, so 2 toothbrushes are $3.50. With the coupon they are $0.5 \times \$3.50$, or $1.75.
  Gem's: 2 toothbrushes is half of $6.00, or $3.00. With the coupon they are $0.5 \times \$3.00$, or $1.50.
  $1.75-$1.50=$0.25

2. ABC's: $0.28 less.

  ABC's: 4 bars of soap are $\frac{1}{2}$ of $7.20, or $3.60. With the coupon, they are $0.7 \times \$3.60$, or $2.52.
  Gem's: 10 bars for $10 is $1 for one bar and $4 for 4 bars. With the coupon they are $0.7 \times \$4.00$ or $2.80.
  $2.80-$2.52=$0.28

3. Gem's: $0.24 less.

  ABC's: One bottle is $12.50 \div 5$ or $2.50, so 4 bottles is $4 \times \$2.50$ or $10. With the coupon, the bottles are $0.6 \times \$10$ or $6.00.
  Gem's: One bottle is $14.40 \div 6$ or $2.40, so 4 bottles is $4 \times \$2.40$ or $9.60. With the coupon, the bottles are $0.6 \times \$9.60$, or $5.76.
  $6.00-$5.76=$0.24

PRACTICE

1. Buy 2 birdies. Use the coupon. Which store has the better buy? How much less is the cost at that store?

Jim's Sports	A-One Athlete	Coupon
Birdies	Birdies	
4 for $4.80	9 for $12.15	50% off
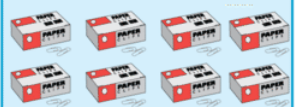
2. Buy 5 water bottles with straws. Use the coupon. Which store has the better buy? How much less is the cost at that store?

Dan's Dishes	KitchenPlace	Coupon
Water Bottles with Straws	Water Bottles with Straws	
7 for $15.75	9 for $19.35	20% off

3. Buy 2 mugs. Use the coupon. Which store has the better buy? How much less is the cost at that store?

Dan's Dishes	KitchenPlace	Coupon
Mugs	Mugs	
4 for $15	6 for $24.90	40% off

4. Buy 4 boxes of paperclips. Use the coupon. Which store has the better buy? How much less is the cost at that store?

Nature's Notes	Sun's Stationery	Coupon
Boxes of Paperclips	Boxes of Paperclips	
8 for $8.80	5 for $6.75	30% off

5. Buy 3 notebooks. Use the coupon. Which store has the better buy? How much less is the cost at that store?
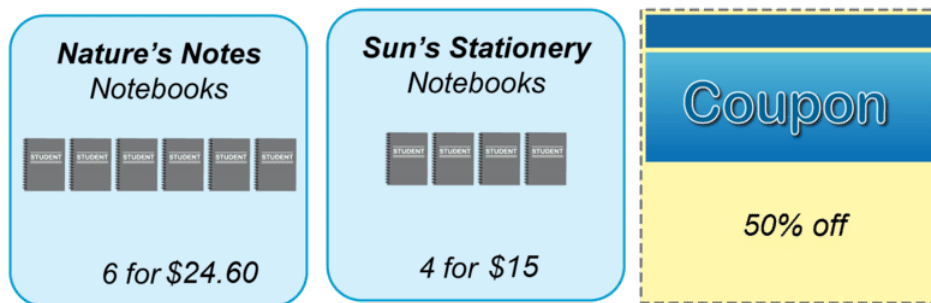
Vocabulary:

- Estimation
- Percent
- Proportions
- Equations
- Percent of change
- Sales tax
- Tips
- Discounts
- Coupon
- Simple Interest
- Compound Interest
- Incentives
- Rates

CHAPTER **7**

Similar Shapes

Chapter Outline

7.5a: generalize the critical attributes of similarity, including ratios within and between similar shapes

7.5c: solve mathematical and real-world problems involving similar shape and scale drawings

7.1 Attributes of Congruency (7.5a)

GUIDANCE

Congruent Polygons

What does it mean when you say that two figures are congruent? The word **congruent** means exactly the same. When you have two figures of any kind that have the same size, shape and measure, you can say that these two figures are congruent.

Let's look at an example.

Are these two angles congruent?
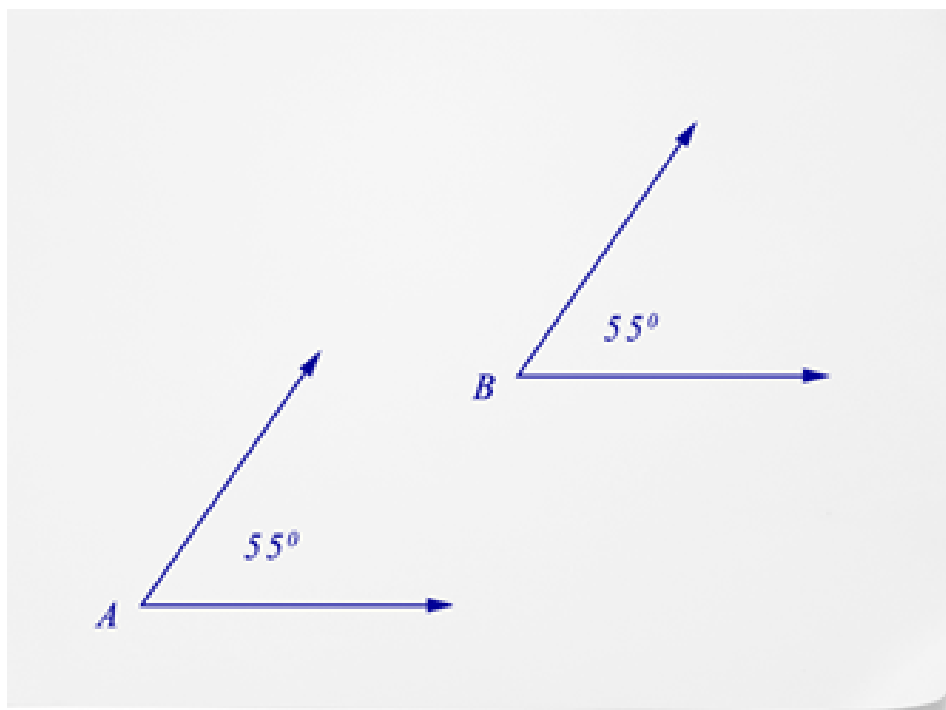
If you look at these two angles, you will see that they are both 55°. The angles are labeled that way, so you can see that they are equal. This means that they are congruent. You can say that angle A is congruent to angle B.

The answer is $\angle A \cong \angle B$.

Just as you said that any two figures can be congruent, you can use this when you look at different types of polygons too. Two polygons can be considered congruent or not congruent.

Let's look at an example involving congruence.

Are these two octagons congruent?
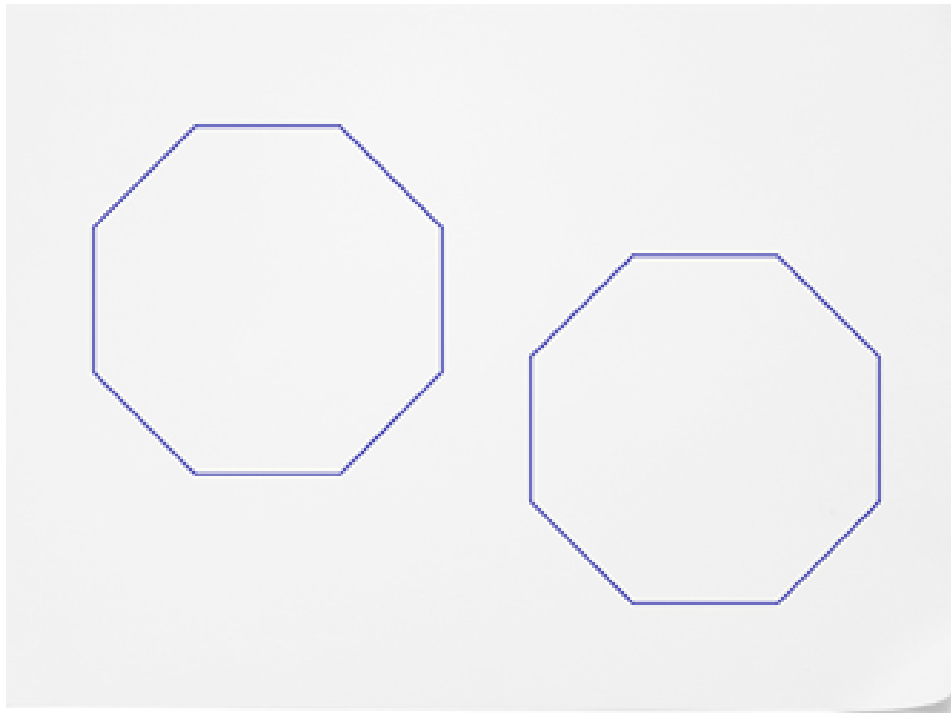
Look at these two octagons. They are exactly the same in every way. You can see that if you put one octagon on top of the other octagon that they would match up perfectly. The side lengths are also congruent and the angle measures are congruent. If two polygons are congruent, then it is a given that the side lengths and the angle measures are also congruent.

Now that you know how to identify whether or not two figures are congruent, you can look at figuring out congruent parts and angles.

First, let's think again about the main characteristics of congruent polygons:

1. Same number of sides
2. Corresponding sides are the same length
3. Corresponding interior angles are the same measure

The last two characteristics can be a bit tricky. Sometimes, you will have two congruent figures, but all of the measures of the corresponding angles won't be the same. For example, if you had two irregular congruent hexagons, this means that there are different angle measures in the two hexagons; however, they are congruent so there are "matching" angles between the two figures.

Take a look at these figures.
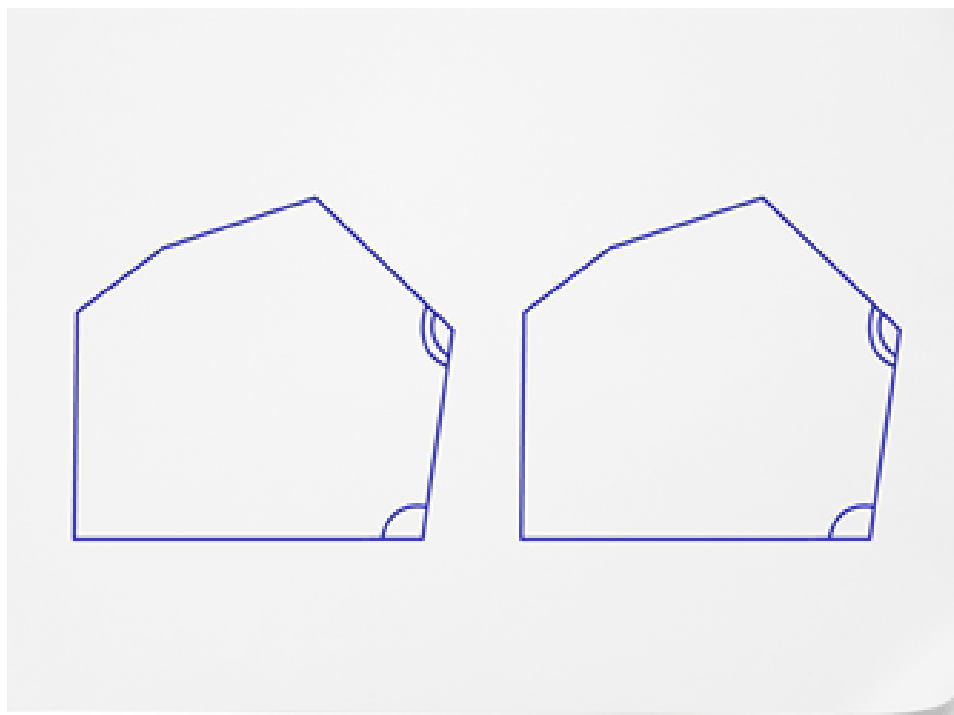
Here you have two hexagons. They are irregular - which means that all of the side lengths and angles are not the same. However, they are congruent. You can see that one matches the other. Because of this, you have corresponding angles that are the same from the first hexagon to the second hexagon.

You can identify corresponding parts of congruent figures. Corresponding parts can include side lengths and angle measures.

Let's look at an example.

Name each pair of corresponding side lengths for these congruent figures.
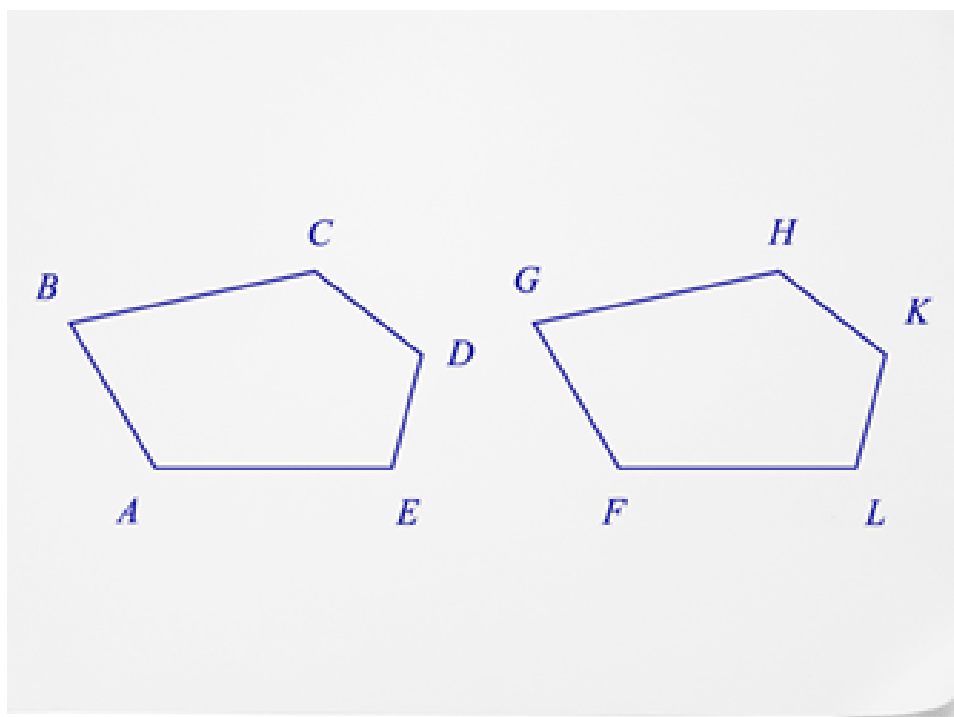
First, look at these two congruent pentagons. To name the corresponding sides, you name the sides that match from one pentagon to another pentagon.

Next, write the pairs of congruent sides. Here are the corresponding sides and how you can write them using mathematical notation.

$$\overline{BA} \cong \overline{GF}$$
$$\overline{BC} \cong \overline{GH}$$
$$\overline{CD} \cong \overline{HK}$$
$$\overline{AE} \cong \overline{FL}$$
$$\overline{ED} \cong \overline{LK}$$

The answer is $\overline{BA} \cong \overline{GF}, \overline{BC} \cong \overline{GH}, \overline{CD} \cong \overline{HK}, \overline{AE} \cong \overline{FL}, \overline{ED} \cong \overline{LK}$.

Then, look at the angles. You can also look at the corresponding angles for two congruent figures. When two figures are congruent, then the matching angles will also be congruent.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about a geodesic dome.

First, let's think about what the word congruent means. Congruent means exactly the same. For an object to be congruent, the side lengths have to be the same. The triangles in the geodesic dome have to be congruent for it to stand up because the triangle is a structure that is well balanced to help with structure and security. Triangles are used in all kinds of construction like roofs and bridges.

The test for congruence of the triangles would be to see that the side lengths are the same.

Example 2

Are these two hexagons congruent?
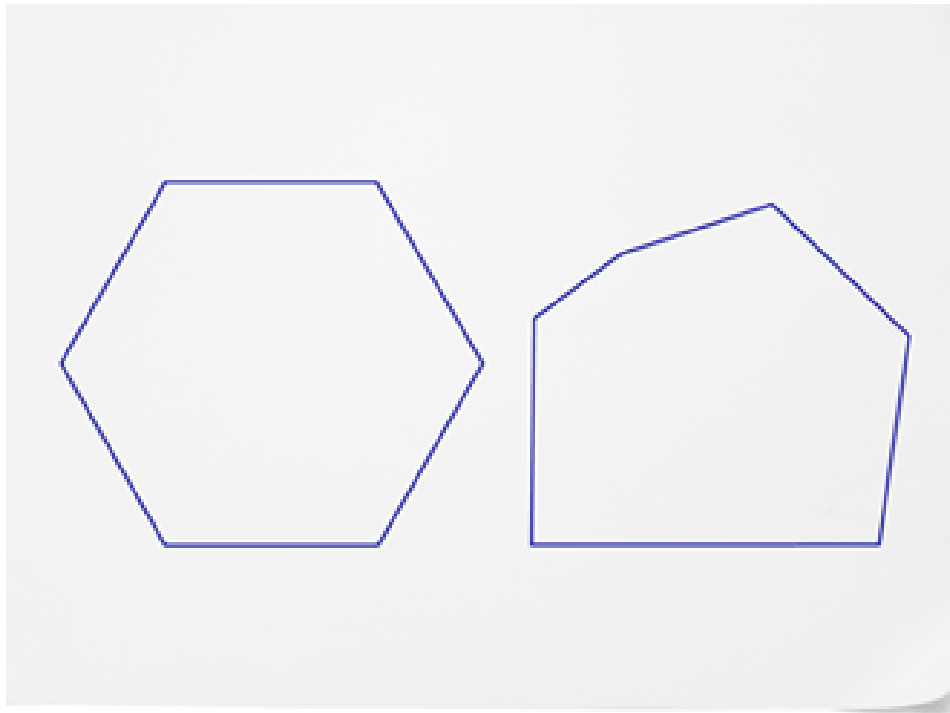
First, look at these two shapes and compare them.

One shape is a regular hexagon and the other is an irregular hexagon.

Next, use the congruent polygon properties to see if the two hexagons are congruent.

In order to be congruent, corresponding sides must be the same length. The regular hexagon has all side lengths with the same measure. The irregular hexagon does not have sides that are the same length.

The answer is no, the two hexagons are not congruent as their corresponding sides are not all the same measure.

Use the following figures to answer each question.
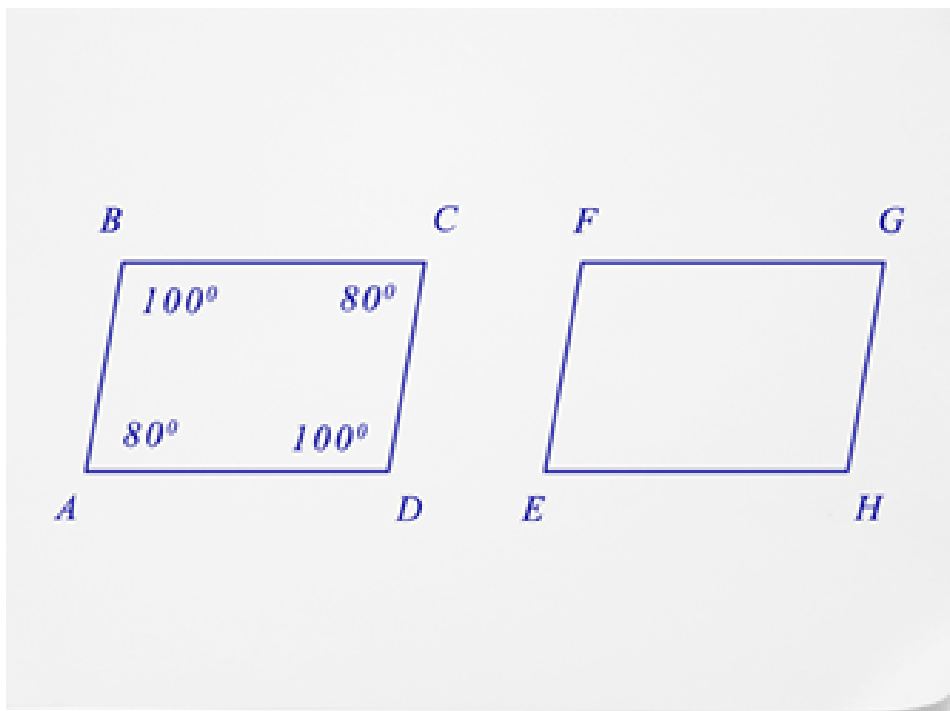
Example 3

What is the angle measure of angle *F*?

First, find the corresponding angle in quadrilateral *ABCD*.

$$\angle F \cong \angle B$$

Next, find the measure of the corresponding angle. This will be the measure of $\angle F$.

The answer is $100°$.

Example 4

Angle *D* is congruent to which other two angles?

First, find the corresponding angle in quadrilateral *EFGH*.

$$\angle D \cong \angle H$$

Next, look at quadrilateral *ABCD*. What are the measures of all of angles?

$$
\begin{aligned}
\angle A &= 80° \\
\angle B &= 100° \\
\angle C &= 80° \\
\angle D &= 100°
\end{aligned}
$$

Then, if $\angle B = \angle D$ and $\angle B \cong \angle F$, then $\angle D \cong \angle H \cong \angle F$.

The answer is $\angle D \cong \angle H \cong \angle F$.

Example 5

What is the measure of angle *G*?

First, find the corresponding angle in quadrilateral *ABCD*.

$$\angle G \cong \angle C$$

Next, find the measure of the corresponding angle. This will be the measure of $\angle G$.

The answer is $80°$.

PRACTICE

Answer each question true or false.

1. Congruent means that a figure has the same side lengths but not the same angle measures.

2. Congruent means exactly the same in every measure.

3. Similar means having the same shape, but not the same size.

4. Two congruent figures would have the same size and shape.

5. Corresponding parts are parts that are in the same figure.

6. You need to understand corresponding parts before you can determine if two figures are congruent.

7. You can determine if two figures are congruent without knowing any of their measurements.

8. Similar figures are also congruent.

9. If two triangles are equilateral triangles, then they are automatically congruent.

10. If two quadrilaterals have measures of 360°, then they are congruent.

The two figures shown are congruent. Use the illustration to answer each question.
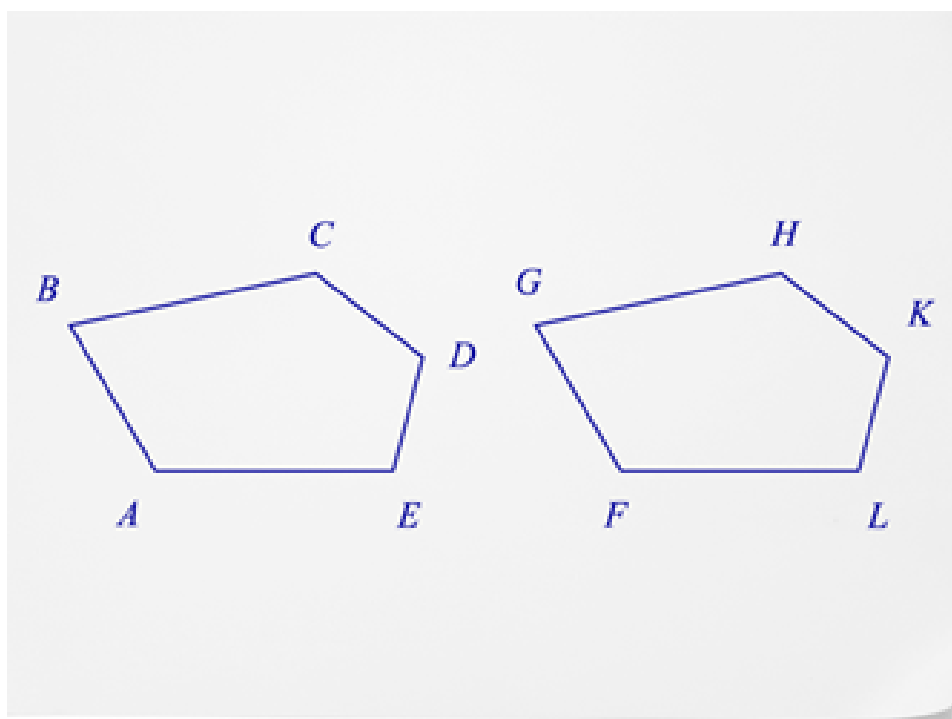
11. If angle *B* has a measure of 75°, which other angle has the same measure?

12. If angle *F* is 120°, which other angle has the same measure?

13. True or false. Angle *E* and angle *K* have the same measure.

14. True or false. Angle *C* and angle *H* have the same measure.

15. Name this figure.

Resources

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/168779
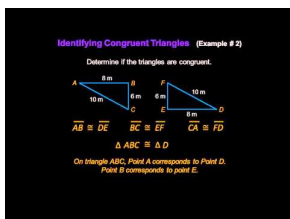
7.2 Attributes of Similarity (7.5a)

GUIDANCE

Similarity of Figures

Some figures look identical except they are different sizes. When you have figures that are proportional to each other, you call these figures **similar** figures. Similar figures have the same angle measures but different side lengths.

Squares are similar shapes because they always have four 90° angles and four equal sides, even if the lengths of their sides differ. Other shapes can be similar too, if their angles are equal.

Let's look at a pair of similar shapes.
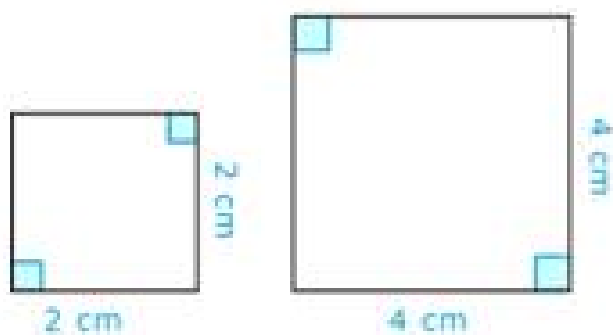
Notice that the figures look the same, but one is smaller than the other. Since they are not the same size, they are not congruent. However, they have the same angles, so they are similar.

Unlike congruent figures, similar figures are not exactly the same. In similar figures, the angles are congruent, even if the sides are not.

Let's look at an example of similarity between figures.

Is the pair of figures below similar?
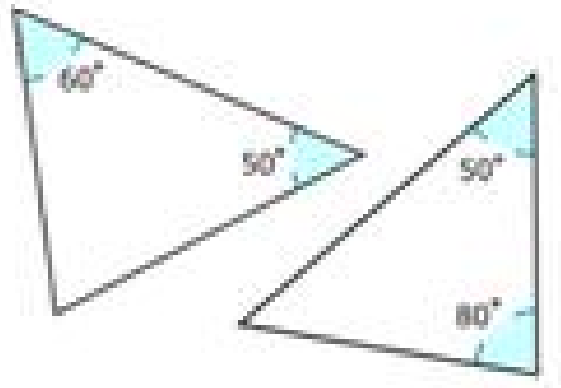
Each triangle has a 50° angle. All three angles must be congruent, however, so let's solve for the missing angle in each angle. Remember, the sum of the three angles is always 180° for a triangle.

Triangle 1

$50 + 60 + \text{angle } 3 = 180$

$110 + \text{angle } 3 = 180$

$\text{angle } 3 = 180 - 110$

$\text{angle } 3 = 70°$

Triangle 2

$50 + 80 + \text{angle } 3 = 180$

$130 + \text{angle } 3 = 180$

$\text{angle } 3 = 180 - 130$

$\text{angle } 3 = 50°$

The angles in the first triangle are 50°, 60°, and 70°. The angles in the second triangle are 50°, 50°, and 80°. These triangles are not similar because their angle measures are different.

There is another way to prove similarity.

List the corresponding angles in the figures below and set up proportions for the side lengths .
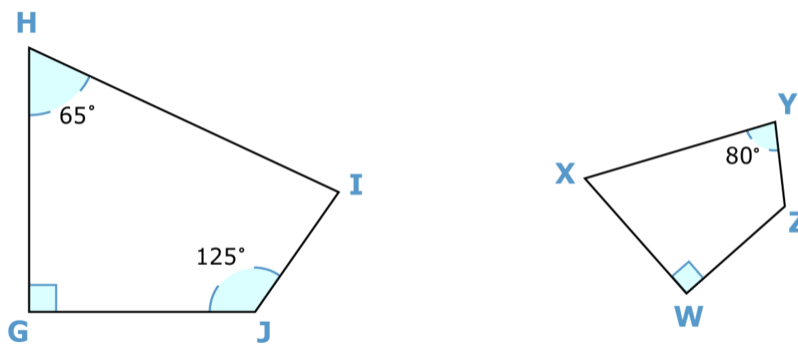
Angles G and W are both right angles, so they correspond to each other. Imagine you can turn the figures to line up the right angles. You might even trace the small figure so that you can place it on top of the larger one.

Angles H and X correspond to each other. So do angles I and Y and angles J and Z. Now let's name these two quadrilaterals: $GHIJ$ is similar to $WXYZ$.

The sides in similar figures are proportional. Proportions have the same ratio. Look at $GHIJ$ and $WXYZ$ again. Write each pair of sides as a proportion.

$\frac{GH}{WX}, \frac{HI}{XY}, \frac{IJ}{YZ}, \frac{GJ}{WZ}$

The sides from one figure are on the top, and the corresponding sides of the other figure are on the bottom. If the proportions each have the same quotient, the figure is similar.

Here is an example.

Is the pair of figures below similar?
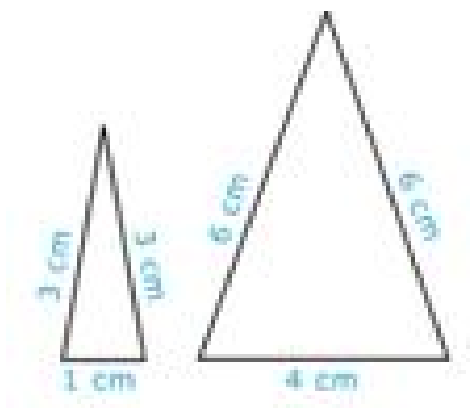
First, let's write out the pairs of proportional corresponding sides:

$\frac{6}{3}, \frac{6}{3}, \frac{4}{1}$

The proportions show side lengths from the large triangle on the top and its corresponding side in the small triangle on the bottom. The pairs of sides must have the same proportion in order for the triangles to be similar.

Next, test whether the three proportions are the same by dividing each. If the quotient is the same, the pairs of sides must exist in the same proportion to each other.

$$\frac{6}{3} = 2$$
$$\frac{6}{3} = 2$$
$$\frac{4}{1} = 4$$

Then, evaluate the results.

Only two pairs of sides have the same proportion. The third pair of sides does not exist in the same proportion as the other two, so these triangles cannot be similar.

The answer is that the figures are not similar.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Helga and her quilt.

She wants the pattern to be uniform and only includes similar squares. She sets up proportions that compare the medium sized squares to the small squares. If one side of the medium square is 3 inches and the proportional corresponding sides have a quotient of 3, what is the side length of the smaller squares?

First, determine which side length is the numerator.

3

Next, set up a proportion to match the given information.

$\frac{3}{x} = 3$

Then, solve for the missing side.

$x = 1$

The answer is that the side length of the smaller square is 1 inch.

Example 2

Is the pair of figures below similar?
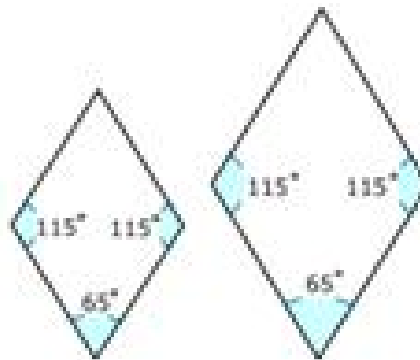
First, note the given information.

The measures of three of the angles.

Next, check the relationship between corresponding angles.

You know the measure of three angles in each figure. In fact, they are all corresponding angles. As you know, the four angles in a quadrilateral must have a sum of $360°$. Because the three known angles are the same for both figures, you don't even need to solve for the fourth to know that it will be the same in both figures.

Then, determine if the pair is similar.

These two figures are similar because their angle measures are all congruent.

The answer is that the pair is similar.

Example 3

Is the pair below similar?
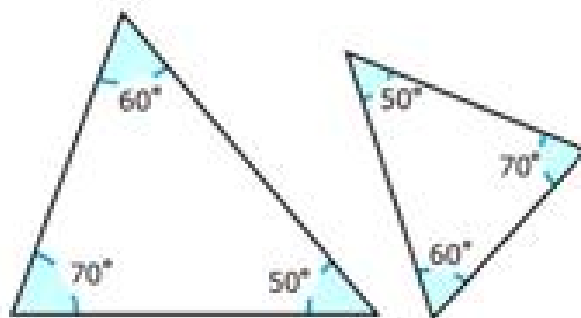
First, look at the given information.

All of the angles are given.

Next, compare the angles to see if the corresponding angles are equal.

Yes.

Then, determine if the pair is similar.

These two figures are similar because their angle measures are all congruent.

The answer is that the pair is similar.

Example 4

The two figures below are similar. What is the measure of angle *Z*?
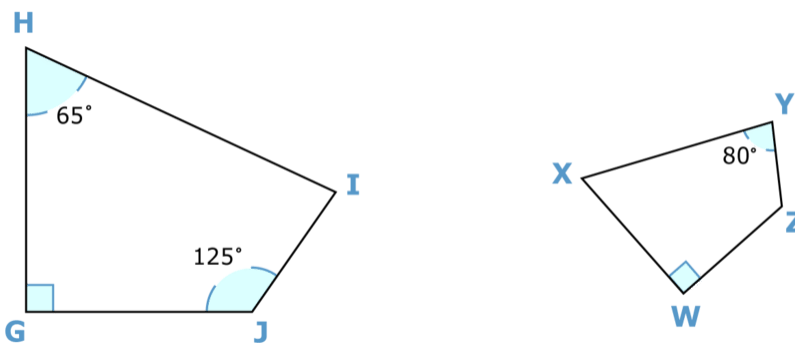
First, look at the given information.

Three angles are provided for the first figure and one angle for the second figure.

Next, identify the angle that corresponds to angle *Z*.

Angle *J* is the corresponding angle.

Then, determine the value of angle *Z*.

Angle $Z = 125^o$.

The answer is that the measure of angle *Z* is 125°.

Example 5

List all of the pairs of corresponding sides in the figures below as proportions.
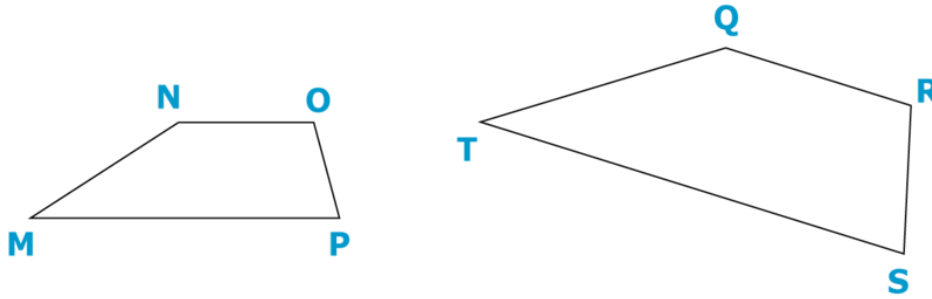
First, line up the figures by their angles.

Trace one figure and rotate it until it matches the other.

Next, choose a side to begin.

OP and *RS* are the shortest sides in each figure. They are proportional, so you can write

$\frac{OP}{RS}$

Then, list the rest of the pairs.

$\frac{NO}{QR}, \frac{MP}{TS}, \frac{MN}{TQ}$

The answer is $\frac{OP}{RS}, \frac{NO}{QR}, \frac{MP}{TS}, \frac{MN}{TQ}$.

PRACTICE

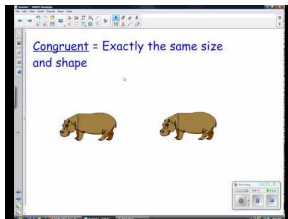Tell whether the pairs of figures below are congruent, similar, or neither.
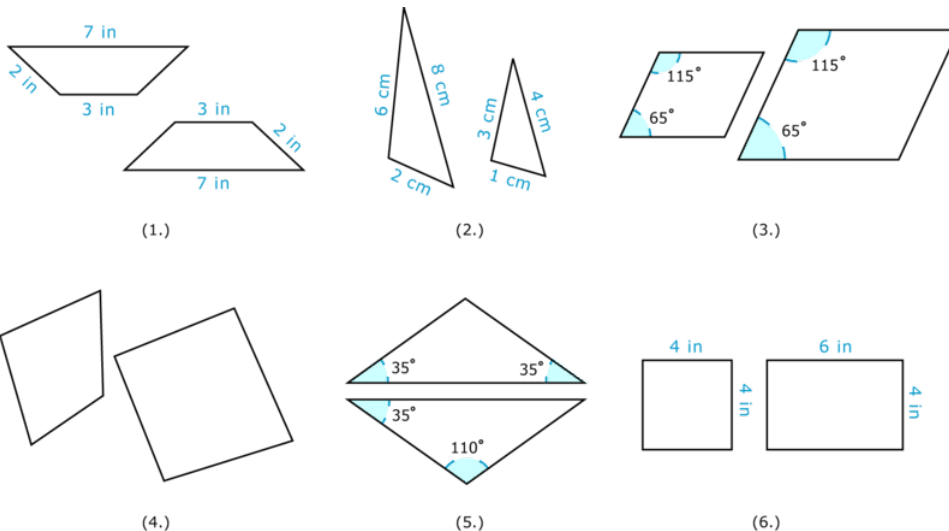
7. True or false. If triangles *DEF* and *GHI* are similar, then the side lengths are different but the angle measures are the same.
8. True or false. Similar figures have exactly the same size and shape.
9. Triangles *LMN* and *HIJ* are similar. If this is true, then the side lengths are the same, true or false.
10. What is a proportion?
11. True or false. To figure out if two figures are similar, then their side lengths form a proportion.
12. Define similar figures
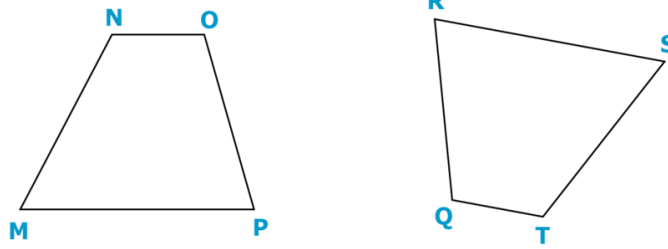
13. Are the two figures similar or congruent?
14. Define congruent figures.
15. True or false. A proportion is formed by a pair of equal ratios.

Resources



MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/183652

7.3 Finding the Missing Sides of Similar Figures (7.5a)

GUIDANCE

Jurnee wants to make a new sail for her father's sailboat. She finds the old sketch that she used to make the first sail and she redraws the sketch so that she can make the new sail. She uses inches as her scale for the figures, but she plans to use feet when she makes the actual sail. Her two figures are similar.
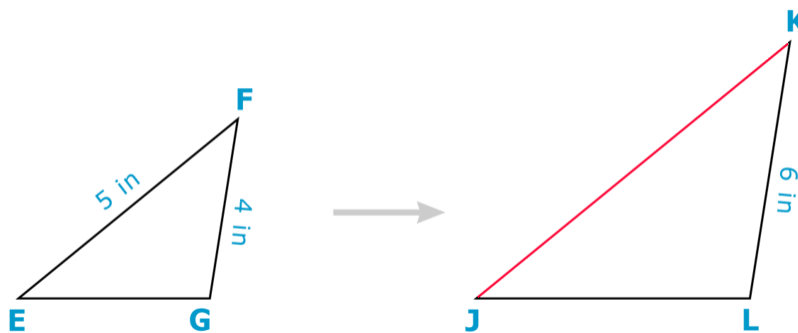
How can Jurnee use a proportion to decide the length of $\overline{KJ}$ in inches?

In this concept, you will learn how to use proportions to figure out the length of a missing side.

Calculating Unknown Measures in Similar Figures

You can write ratios to compare the lengths of sides.
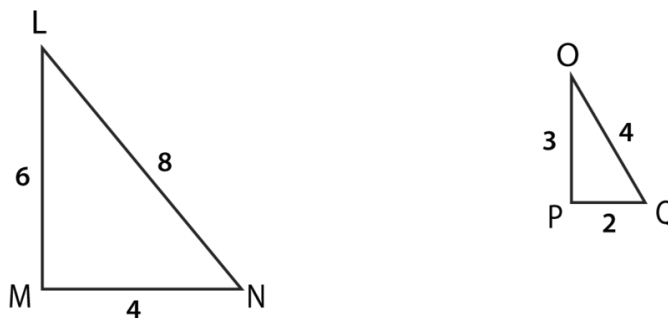
First, identify the corresponding sides of these two similar triangles, then place the first side in the numerator and the corresponding side in the denominator.

$$\frac{LM}{OP} = \frac{LN}{OQ} = \frac{MN}{PQ}$$

These ratios are written in a proportion or a set of three equal ratios. Remember that there is a relationship between the corresponding sides because they are parts of similar triangles. The side lengths of the similar triangles form a proportion.

Let's substitute the given measurements into the formula.

$$\frac{6}{3} = \frac{8}{4} = \frac{4}{2}$$

There is a pattern with the ratios of corresponding sides. You can see that the measurement of each side of the first triangle divided by two is the measure of the corresponding side of the second triangle.

Use patterns like this to problem solve the length of missing sides of similar triangles.
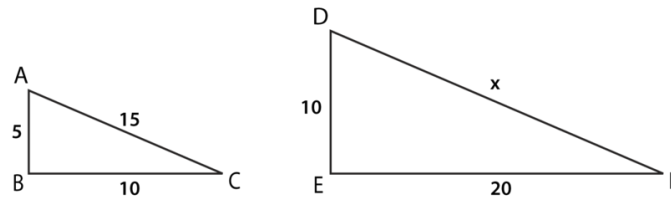
These are two similar triangles because they have the same shape but a different size. Therefore, the corresponding sides are similar.

If you look at the side lengths, you should see that there is one variable. That is the missing side length. You can figure out the missing side length by using proportions because the corresponding side lengths form a proportion. Let's write ratios that form a proportion and find the pattern to figure out the length of the missing side.

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$
$$\frac{5}{10} = \frac{15}{x} = \frac{10}{20}$$

Looking at this you can see the pattern. The side lengths of the second triangle are double the length of the corresponding side of the first triangle.

Using this pattern, you can see that the length of *DF* in the second triangle will be twice the length of *AC*. The length of *AC* is 15.
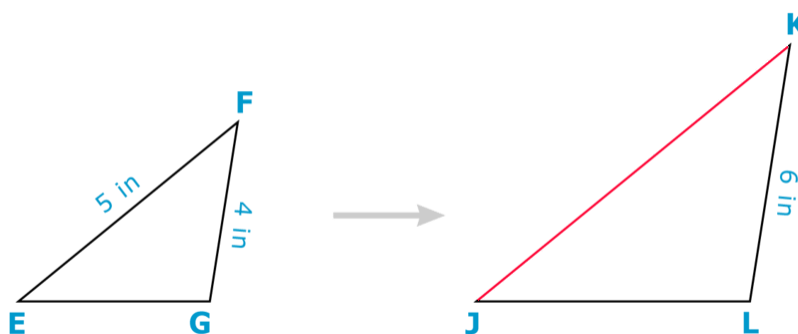
$15 \times 2 = 30$

The length of *DF* is 30.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Jurnee and the sail.

She wants to make a new sail for her father. She uses an old sketch and makes a new sketch that is similar.

How can Jurnee decide the length of $\overline{KJ}$ in inches?

First, use the corresponding sides to set up a proportion.

$\frac{\overline{KJ}}{5} = \frac{6}{4}$

Next, use cross products.

$$\overline{KJ} \times 4 = 4\overline{KJ}$$
$$5 \times 6 = 30$$
$$4\overline{KJ} = 30$$

Then, solve for $\overline{KJ}$.

$$30 \div 4 = 7.5$$
$$\overline{KJ} = 7.5$$

The answer is that $\overline{KJ} = 7.5$ cm.

Example 2

Solve for the missing value.

$\frac{8}{10} = \frac{4}{5} = \frac{2}{x}$

First, identify the pattern.

The denominator is the numerator divided by 0.8.

Next, set up an equation to solve for x.

$\frac{2}{0.8} = x$

Then, solve for x.

$x = 2.5$

The answer is that $x = 2.5$.

Example 3

Solve for the missing value.

$\frac{6}{12} = \frac{x}{24} = \frac{3}{6}$

First, identify the pattern.

The denominator is twice the size of the numerator.

Next, set up an equation to solve for x.

$24 = 2 \times x$

Then, solve for x.

$\frac{24}{2} = x = 12$

The answer is that $x = 12$.

Example 4

Solve for the missing value.

$\frac{12}{x} = \frac{16}{4} = \frac{20}{5}$

First, identify the pattern.

The denominator is the result of dividing the numerator by 4.

Next, set up an equation to solve for x.

$\frac{12}{4} = x$

Then, solve for x.

$x = 3$

The answer is that $x = 3$.

Example 5

Solve for the missing value.

$\frac{8}{2} = \frac{16}{4} = \frac{x}{1}$

First, identify the pattern.

The denominator is the result of dividing the numerator by 4.

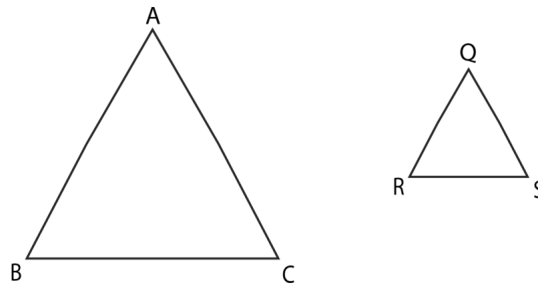Next, set up an equation to solve for x.

$\frac{x}{4} = 1$

Then, solve for x.

$x = 4 \times 1 = 4$

The answer is that $x = 4$.

PRACTICE

Use the figures to answer the following questions.

1. Are these two triangles similar or congruent?
2. How do you know?
3. Which side is congruent to AB?
4. Which side is congruent to AC?
5. Which side is congruent to RS?
6. Look at the following proportion and solve for missing side length x.

$$\frac{7}{3.5} = \frac{x}{3.5} = \frac{6}{y}$$

$$x = \underline{\qquad}$$

7. What is the side length for y?
8. How did you figure these out?

Figure out the missing value in each pair of ratios.

9. $\frac{6}{12} = \frac{x}{24}$
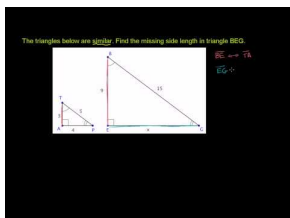10. $\frac{8}{12} = \frac{x}{3}$
11. $\frac{9}{10} = \frac{18}{y}$
12. $\frac{4}{5} = \frac{x}{2.5}$
13. $\frac{16}{20} = \frac{4}{y}$
14. $\frac{19}{21} = \frac{x}{42}$
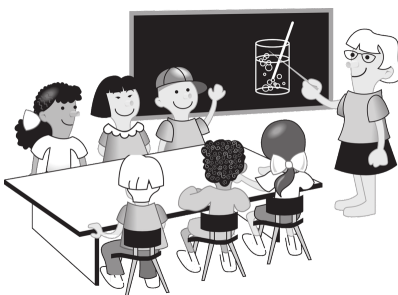15. $\frac{9}{54} = \frac{6}{y}$

Resources

MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/181948

7.4 Scale Drawings (7.5c)

GUIDANCE

The Supermarket Display

Jessica is in charge of creating a new project display at the supermarket. Her manager noticed her doodling in a notebook and was impressed with her artistic ability.

He called Jessica into his office and handed her a design on a sheet of paper.

"We want to create a special poster to focus on a key product each week. This is the sketch of the design. That product will be on sale and hopefully this new display will help people notice it and buy it," Jessica's manager explains.

The design on the page is 8" $\times$ 5". It is a rectangle and the scale at the bottom of the page says 1" = 6".

Jessica takes out some foam core and an exacto knife. She knows that she needs the scale to figure out the exact measurements of the display. The problem is that she can't remember how to do the math.

"If it were 1" to 1 ft," she thinks, "Then the poster would be 8 ft $\times$ 5 ft, but that isn't it. 6" is only 1/2 of 1 ft, so this scale is 1" to 1/2 ft."

Jessica is puzzled. What are the dimensions for the display?

This lesson works on scale and proportions. You can use a scale to figure out the change in measurements from a picture to the actual thing.

Be ready so that you can help Jessica at the end of the lesson.

What You Will Learn

In this lesson you will learn how to do the following skills.

- Given scale ratios, find actual or scale dimensions using proportions.
- Given actual and scale dimensions, find scale ratios using proportions.
- Solve real-world problems involving scale drawings and maps using proportions.

Teaching Time

I. **Given Scale Ratios, Find Actual or Scale Dimensions Using Proportions**

In our last lesson we talked about how you can find *proportions* in everyday life. This lesson focuses on scale and scale drawings. This is a key place where we use proportions in everyday life.

What is a scale drawing?

A *scale drawing* is a drawing that is used to represent an object that is too large to be drawn in its actual dimensions. For example, if you had a very tall building, you couldn't make a drawing of the building that is as large as the building itself. Think about how many sheets of paper you would need to draw a 25 foot tall building.

This is an example where a scale drawing would be very useful. We can use a *scale* to represent measurements and then draw the building in a size that makes more sense.

What is a scale?

When we talk about *scale,* we aren't talking about the object that we use to weigh things.

The *scale* that we are talking about is a fraction that shows the relationship between the measurement in a drawing and the measurement of a real object.

Let's look at an example.

$$\frac{1 \; inch}{4 \; feet}$$

This scale says that we would use one inch to represent every four feet. The top number is the scale that we would use in the drawing. The bottom number represents the measurement of the actual building.

Let's say that we wanted to draw a building that is sixteen feet tall using this scale. We could set up a proportion to solve for the number of inches that we would need to draw.

$$\frac{1 \; inch}{4 \; feet} = \frac{x \; inches}{16 \; feet}$$

Now we can use what we learned in our last lesson about cross-multiplying to solve proportions. This will help us to figure out the scale dimension of the building.

$$16 = 4x$$

Our answer is four inches. The two ratios now form a proportion.

What about if we have more than one dimension?

Let's say that we have a room that is 8' × 12' and we want to use a scale of 1" = 2 feet. How many inches long and wide would this drawing be?

First, let's look at the width of the room. It is eight feet wide. We can set up a proportion using the scale and the actual width to figure out the measurement of the drawing.

$$\frac{1 \; in}{2 \; ft} = \frac{x}{8 \; ft}$$

Next, we solve the proportion.

$$2x = 8$$
$$x = 4 \; inches$$

On the drawing, the width will be four inches.

Now we need to look at the length. The room is 12 feet long. We can set up a proportion using the scale and the actual length to figure out the length of the drawing.

$$\frac{1 \; in}{2 \; ft} = \frac{x}{12 \; ft}$$

Next, we solve the proportion by cross-multiplying.

$$2x = 12$$
$$x = 6$$

On the drawing, the length of the room will be six inches.

Next, we can do a scale drawing of this room. If one unit on the drawing is equal to one inch, here is our room.
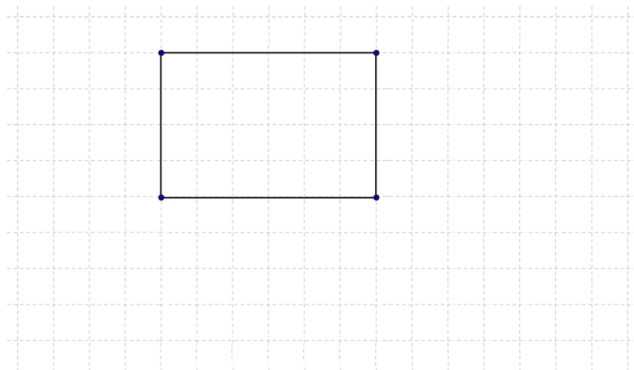
We can also use scale dimensions to figure out the actual dimensions of something. We will use proportions to do this as well. Here is an example.

Example

The flower bed design shows that the width of the garden on the drawing is six inches. If the scale is 1" = 5 feet, how wide is the actual flower garden?

To solve this problem, we need to set up a proportion. Let's start by writing the scale in the form of a ratio.

$$\frac{1''}{5\ ft}$$

Next, we can write the actual dimensions that we know with a variable as our unknown and make this the second ratio in this proportion.

$$\frac{1''}{5'} = \frac{6''}{x}$$

The drawing of our flower bed is six inches. We can solve the proportion for the actual dimensions of the flower bed by cross-multiplying.

$$1x = 30$$
$$x = 30\ ft$$

The actual flower bed is 30 feet wide.

Practice a few of these on your own. Solve each proportion for the scale measurement or the actual measurement.

1. $\frac{1''}{3\ ft} = \frac{x}{21\ ft}$
2. $\frac{3\ in}{6\ ft} = \frac{9\ in}{x}$
3. $\frac{2\ in}{10\ ft} = \frac{x}{120\ ft}$

Check the accuracy of your work with a friend. Did you remember to label your work with the appropriate units?

II. **Given Actual and Scale Dimensions, Find Scale Ratios Using Proportions**

Now that you have figured out how to use proportions to figure out actual and scale dimensions, we can look at figuring out *scale factors*.

What is a scale factor?

A *scale factor* **is another name for a scale ratio. When looking for a scale factor, you can look at the relationship between the scale measurement and the actual measurement to determine what scale was used. This scale is called the scale factor.**

Let's look at an example.

Example

A fence is actually 16 feet long. If the fence is drawn as four inches, what is the scale factor?

To figure this out, we need to write a ratio to compare the drawing of the fence to the actual measurement.

$$\frac{4''}{16\ ft}$$

Now we want to figure out the scale factor. To do this, we simplify the ratio using the greatest common factor. The greatest common factor of 4 and 16 is 4.

$$4 \div 4 = 1$$
$$16 \div 4 = 4$$

The scale factor is $\frac{1''}{4\ ft}$.

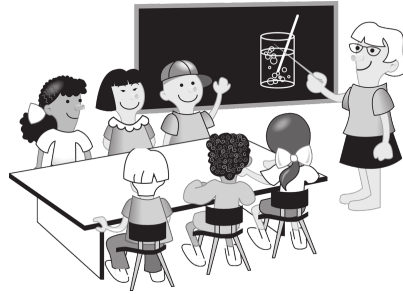**Use this information to simplify and find the following scale factors.**

1. $\frac{5''}{25'}$
2. $\frac{2''}{50'}$
3. $\frac{12.5''}{25'}$

Check your work with a partner. Did you figure out the scale factor of the lst problem?

GUIDED PRACTICE

The Supermarket Display

Now it is time to use what you have learned to help Jessica. Here is the problem again. Begin by underlining all of the important information.

Jessica is in charge of creating a new project display at the supermarket. Her manager noticed her doodling in a notebook and was impressed with her artistic ability.

He called Jessica into his office and handed her a design on a sheet of paper.

"We want to create a special poster to focus on a key product each week. This is the sketch of the design. The key product will be on sale and hopefully this new display will help people notice it and buy it," Jessica's manager explains.

The design on the page is 8" × 5". It is a rectangle and the scale at the bottom of the page says 1" = 6".

Jessica takes out some foam core and an exacto knife. She knows that she needs the scale to figure out the exact measurements of the display. The problem is that she can't remember how to do the math.

"If it were 1" to 1 ft," she thinks, "Then the poster would be 8 ft × 5 ft, but that isn't it. 6" is only 1/2 of 1 ft, so this scale is 1" to 1/2 ft."

Now that you have underlined the important information, write the scale factor for the poster.

Scale factor is 1" : 6"

Or $\frac{1}{6}$

The drawing shows that the length of the rectangular sign is 8" and the width is 5". If the one inch is equal to one half of a foot, then the length is 4 ft and width is 2.5 ft.

Jessica is amazed at how simple it actually was to figure that out once she knew how to use the scale factor. Her poster is 4 ft × 2.5 ft. She gets right to work on the poster and design!

Vocabulary

Here are the vocabulary words that are found in this lesson.

Proportion

two equal ratios

Scale Drawing

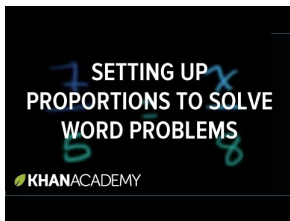a drawing used when a real life object is too big to draw with its actual dimensions.

Scale

the relationship of the size of a drawing to the size of the real object

Scale Factor

the relationship between the measurement of the drawing and the measurement of the real object.
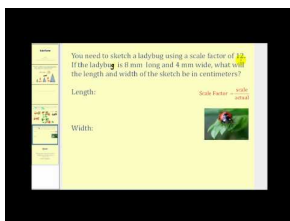
Technology Integration

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/5419

Khan Academy, Solving Proportions

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/1347

James Sousa, Scale Factor

PRACTICE

Directions: Find actual dimensions using proportions.

1. The scale of the drawing shows that 1" = 5 feet. If the drawing shows the height of the building as 5 inches, how tall is the actual building?

2. The scale of the drawing shows that 2" = 10 feet. If the drawing shows the height of the building as 8 inches, how tall is the actual building?

3. The scale of the drawing shows that 1" = 3 feet. If the drawing shows the height of the tree as 9 inches, how tall is the tree?

4. The scale of the drawing shows that 2" = 7 feet. If the drawing shows that the height of the tree is 6 inches, how tall is the tree?

5. The scale of the drawing shows that 1" = 3 feet. If the drawing shows that the height of the tree house is 3", how high is the actual tree house?

Directions: Find the scale dimensions using proportions.

6. The scale of the map shows that 1" = 50 miles. If the map shows that there is 5" between the two cities, what is the actual distance?

7. The scale of the map shows that 2" = 100 km. If the map shows that there are 3" between the two cities, what is the actual distance between them?

8. The scale of the map shows that 4" = 200 km. If the map shows that there are 5 inches between the two cities, what is the actual distance between them?

9. The scale of the garden design shows that 2" = 3 feet. How big is the garden if the rectangular plot is 4" × 6"?

10. The scale of the room design shows that 1" = 2 feet. How big is the actual room if the design shows a square that is 5 inches wide?

Directions: Simplify each ratio to find the scale factor.

11. $\frac{4''}{6\ ft}$

12. $\frac{12''}{24\ ft}$

13. $\frac{6''}{18\ ft}$

14. $\frac{9''}{27\ ft}$

15. $\frac{4''}{16\ ft}$

16. $\frac{5''}{30\ ft}$

17. $\frac{3''}{30\ ft}$

18. $\frac{3''}{60\ miles}$

19. $\frac{4''}{100\ miles}$

20. $\frac{5''}{1000\ km}$

Vocabulary:

- Similarity
- Scale Drawings
- Scale Factor
- Proportional
- Similar Figures
- Ratios

CHAPTER **8** **Measurement Conversions**

Chapter Outline

7.4e: convert between measurement systems, including the use of proportions and the use of unit rates

Write and Solve Proportions by Using Equivalent Rates (7.4e)

GUIDANCE

Equivalent Rates

A **ratio** is a comparison between two quantities or numbers. Ratios can be written in fraction form, with a colon or by using the word "to". Sometimes, you will compare ratios. Sometimes one ratio will be greater than another, and other times they can be equal or equivalent. When you have two equal ratios, you have a proportion. A **proportion** is created when two ratios are equal, or we can say that two equal ratios form a proportion.

You can write a proportion when we know that two ratios are equivalent.

$1:2 = 2:4$

These two ratios are equivalent. You can say that the two ratios form a proportion.

Let's look at an example.

Do these two ratios, $\frac{3}{4}$ and $4:24$ form a proportion?

First, put the ratio $4:24$ into fraction form.

$$4:24 = \frac{4}{24}$$

Next, reduce the fraction.

$$\frac{4}{24} = \frac{1}{6}$$

Then, compare the two fractions.

$$\frac{1}{6} \neq \frac{3}{4}$$

The answer is $\frac{1}{6} \neq \frac{3}{4}$.

If the ratios are equivalent, they form a proportion. Since the ratios are not equivalent, the ratios do not form a proportion.

To write a proportion, set two equivalent fractions equal to each other, using the information in the problem.

Let's do another example.

If you know the ratio of girls to boys in a class is $2:3$, and you know there are 24 boys in the class, you can write a proportion in order to find the number of girls in the class.

First, write the ratio of the girls to boys.

$$\frac{\text{girls}}{\text{boys}} = \frac{2}{3}$$

Next, write the proportion statement knowing there are 24 boys in the class.

$$\frac{2}{3} = \frac{x}{24}$$

Then, cross multiply to solve for *x*.

$$
\begin{array}{rcl}
\frac{2}{3} & = & \frac{x}{24} \\
3x & = & 2 \times 24 \\
3x & = & 48 \\
x & = & 19
\end{array}
$$

The answer is 19.

The class has 19 girls and 24 boys in the class.

Let's use equivalent rates to solve a proportion.

The ratio of teachers to students in a certain school is 2 : 25. If there are 400 students in the eighth-grade class, how many teachers are there?

First, write the ratio of the teachers to students.

$$\frac{\text{teacher}}{\text{students}} = \frac{2}{25}$$

Next, write the proportion statement knowing there are 400 students in the 8th grade.

$$\frac{2}{25} = \frac{x}{400}$$

Then, cross multiply to solve for *x* .

$$
\begin{array}{rcl}
\frac{2}{25} & = & \frac{x}{400} \\
25x & = & 2 \times 400 \\
25x & = & 800 \\
x & = & 32
\end{array}
$$

The answer is 32.

There are 32 8th grade teachers.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Jamie's robust reading challenge.

Jamie reads 15 pages in 25 minutes and wants to know how many pages she can read in 65 minutes.

First, write a proportion to represent this problem.

$$\frac{15}{25} = \frac{x}{65}$$

Next, cross multiply.

$$
\begin{aligned}
\frac{15}{25} &= \frac{x}{65} \\
25x &= 15 \times 65 \\
25x &= 975
\end{aligned}
$$

Then, divide by 25 to solve for x.

$$
\begin{aligned}
25x &= 975 \\
\frac{25x}{25} &= \frac{975}{25} \\
x &= 39
\end{aligned}
$$

The answer is 39.

Therefore Jamie can read 39 pages in 65 minutes.

Example 2

Write a proportion to describe this situation. The proportion of red paper to white paper in a stack is 2 to 7. If there are 32 red pieces of paper, what proportion could be used to find the number of pieces of white paper?

First, write the ratio of the teachers to students.

$$\frac{\text{red paper}}{\text{white paper}} = \frac{2}{7}$$

Next, write the proportion statement knowing there are 32 pieces of red paper.

$$\frac{2}{7} = \frac{32}{x}$$

Then, cross multiply to solve for x .

$$
\begin{aligned}
\frac{2}{7} &= \frac{32}{x} \\
2x &= 7 \times 32 \\
2x &= 224 \\
x &= 112
\end{aligned}
$$

The answer is 112.

There are 112 white pieces of paper.

Example 3

Solve for x in the proportion $\frac{3}{4} = \frac{6}{x}$ by using equal ratios.

First, cross multiply.

$$
\begin{aligned}
\frac{3}{4} &= \frac{6}{x} \\
3x &= 4 \times 6 \\
3x &= 24
\end{aligned}
$$

Next, divide by 3 x .

$$
\begin{aligned}
3x &= 24 \\
\frac{3x}{3} &= \frac{24}{3} \\
x &= 8
\end{aligned}
$$

The answer is 8.

Therefore $\frac{3}{4} = \frac{6}{8}$.

Example 4

Solve for x in the proportion $\frac{9}{50} = \frac{x}{100}$ by using equal ratios.

First, cross multiply.

$$
\begin{aligned}
\frac{9}{50} &= \frac{x}{100} \\
50x &= 9 \times 100 \\
50x &= 900
\end{aligned}
$$

Next, divide by 50 to solve for x.

$$
\begin{aligned}
50x &= 900 \\
\frac{50x}{50} &= \frac{900}{50} \\
x &= 18
\end{aligned}
$$

The answer is 18.

Therefore $\frac{9}{50} = \frac{18}{100}$.

Example 5

Solve for x in the proportion $\frac{3.5}{7} = \frac{x}{35}$ by using equal ratios.

First, cross multiply.

$$
\begin{aligned}
\frac{3.5}{7} &= \frac{x}{35} \\
7x &= 3.5 \times 35 \\
7x &= 122.5
\end{aligned}
$$

Next, divide by 7 to solve for x.

$$
\begin{aligned}
7x &= 122.5 \\
\frac{7x}{7} &= \frac{122.5}{7} \\
x &= 17.5
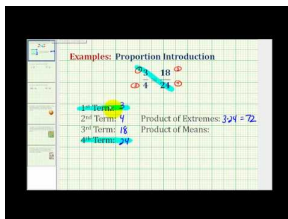\end{aligned}
$$

The answer is 17.5.

Therefore $\frac{3.5}{7} = \frac{175}{35}$.

PRACTICE

Solve each proportion using equal ratios.

1. $\frac{3}{4} = \frac{x}{12}$
2. $\frac{5}{6} = \frac{x}{12}$
3. $\frac{4}{7} = \frac{8}{y}$
4. $\frac{2}{3} = \frac{12}{y}$
5. $\frac{4}{5} = \frac{44}{y}$
6. $\frac{12}{13} = \frac{x}{26}$
7. $\frac{9}{10} = \frac{81}{y}$
8. $\frac{6}{7} = \frac{18}{y}$
9. $\frac{7}{8} = \frac{x}{56}$
10. $\frac{12}{14} = \frac{36}{x}$
11. $\frac{6}{4} = \frac{x}{12}$
12. $\frac{12}{14} = \frac{24}{x}$
13. $\frac{13}{14} = \frac{x}{42}$
14. $\frac{1.5}{4} = \frac{x}{8}$
15. $\frac{3.5}{4.5} = \frac{x}{9}$
16. $\frac{9}{14} = \frac{108}{x}$

Resources

MEDIA

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8.2 Write and Solve Proportions by Using Cross-Products (7.4e)

GUIDANCE

Cross Products

A proportion is created when two ratios are equal. Sometimes, you will know three parts of a proportion and there will be one missing part. When this happens, you will need to solve a proportion.

A way of solving a proportion is called **cross-multiplying**, and this involves algebra. The rule for cross multiplying is:

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

This is also called "the product of the means is equal to the product of the extremes." The values in the 'b' and 'c' positions are called the **means**, and the values in the 'a' and 'd' positions are called the **extremes**.

Let's look at an example.

Solve for x.

$$\frac{x}{5} = \frac{9}{10}$$

First, cross multiply.

$$\begin{aligned} \frac{x}{5} &= \frac{9}{10} \\ 10x &= 9 \times 5 \\ 10x &= 45 \end{aligned}$$

Next, solve for x by dividing both sides by 10.

$$\begin{aligned} 10x &= 45 \\ \frac{10x}{10} &= \frac{45}{10} \\ x &= 4.5 \end{aligned}$$

The answer is 4.5.

Here is another example.

Solve for x.

$$\frac{4}{5} = \frac{16}{x}$$

First, cross multiply.

347

$$
\begin{array}{rcl}
\frac{4}{5} & = & \frac{16}{x} \\
4x & = & 5 \times 16 \\
4x & = & 80
\end{array}
$$

Next, solve for *x* by dividing both sides by 4.

$$
\begin{array}{rcl}
4x & = & 80 \\
\frac{4x}{4} & = & \frac{80}{4} \\
x & = & 20
\end{array}
$$

The answer is 20.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Sandra's science fiction study.

Sandra wants to figure out how many of the 460 students will choose science fiction, given that 7 out of 20 students already selected science fiction as their favorite movie genre.

First, write your proportion.

$$
\frac{7}{20} = \frac{x}{460}
$$

Next, cross multiply.

$$
\begin{array}{rcl}
\frac{7}{20} & = & \frac{x}{460} \\
20x & = & 7 \times 460 \\
20x & = & 3220
\end{array}
$$

Then, solve for *x* by dividing both sides by 20.

$$
\begin{array}{rcl}
20x & = & 3220 \\
\frac{20x}{20} & = & \frac{3220}{20} \\
x & = & 161
\end{array}
$$

The answer is 161.

Sandra, using the ratio of 7 : 20 would find 161 of the 46- students choose science fiction as their favorite type of movie.

Example 2

The ratio of apples to bananas at a store is 3 to 8. If there are 90 apples, how many bananas are there?

First, write your proportion.

$$\frac{3}{8} = \frac{90}{x}$$

Next, cross multiply.

$$
\begin{aligned}
\frac{3}{8} &= \frac{90}{x} \\
3x &= 8 \times 90 \\
3x &= 720
\end{aligned}
$$

Then, solve for x by dividing both sides by 3.

$$
\begin{aligned}
3x &= 720 \\
\frac{3x}{3} &= \frac{720}{3} \\
x &= 240
\end{aligned}
$$

The answer is 240.

There were 240 bananas.

Example 3

Solve for x.

$$\frac{x}{9} = \frac{18}{27}$$

First, cross multiply.

$$
\begin{aligned}
\frac{x}{9} &= \frac{18}{27} \\
27x &= 9 \times 18 \\
27x &= 162
\end{aligned}
$$

Next, solve for x by dividing both sides by 27.

$$
\begin{aligned}
27x &= 162 \\
\frac{27x}{27} &= \frac{162}{27} \\
x &= 6
\end{aligned}
$$

The answer is 6.

Example 4

Solve for y.

$$\frac{3}{7} = \frac{33}{y}$$

First, cross multiply.

$$\begin{aligned}
\tfrac{3}{7} &= \tfrac{33}{y} \\
3y &= 7 \times 33 \\
3y &= 231
\end{aligned}$$

Next, solve for y by dividing both sides by 3.

$$\begin{aligned}
3y &= 231 \\
\tfrac{3y}{3} &= \tfrac{231}{3} \\
y &= 77
\end{aligned}$$

The answer is 77.

Example 5

Solve for x.

$$\frac{x}{2} = \frac{49.5}{99}$$

First, cross multiply.

$$\begin{aligned}
\tfrac{x}{2} &= \tfrac{49.5}{99} \\
99x &= 2 \times 49.5 \\
99x &= 99
\end{aligned}$$

Next, solve for x by dividing both sides by 99.

$$\begin{aligned}
99x &= 99 \\
\tfrac{99x}{99} &= \tfrac{99}{99} \\
x &= 1
\end{aligned}$$

The answer is 1.

PRACTICE

Solve each proportion by using cross-multiplying with algebra. You may round to the nearest tenth when necessary.

1. $\frac{3}{5} = \frac{y}{2.5}$

2. $\frac{6}{7} = \frac{2.5}{y}$

3. $\frac{4}{5} = \frac{2}{x}$

4. $\frac{9}{11} = \frac{14}{x}$

5. $\frac{2}{3} = \frac{5}{y}$

6. $\frac{12}{3} = \frac{4}{y}$

7. $\frac{22}{40} = \frac{11}{x}$

8. $\frac{60}{x} = \frac{5}{10}$

9. $\frac{12}{50} = \frac{3}{y}$

10. $\frac{42}{36} = \frac{7}{y}$

11. $\frac{56}{63} = \frac{x}{9}$

12. $\frac{120}{130} = \frac{1.2}{y}$

Solve each problem.

13. The ratio of fiction to nonfiction books at a library is 5 to 3. If there are 480 nonfiction books, write a proportion that could be used to find f, the number of fiction books.

14. The ratio of cherry trees to apple trees at an orchard is 4 to 9. If there are 184 cherry trees, write a proportion that could be used to find a, the number of apple trees.

15. The ratio of cars to SUVs in a parking lot is 10 to 7. If there are 84 SUVs, write a proportion that could be used to find c, the number of cars in the lot.

Resources

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/167919
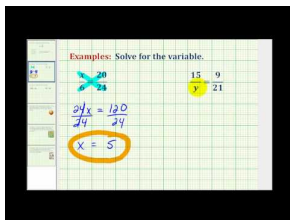
8.3 Convert Customary Units of Measurement (7.4e)

GUIDANCE

Karen is making cookies for a school fundraiser. The recipe she uses calls for 20 ounces of liquid. How many cups does this represent?

In this concept, you will learn to convert customary units of measurement.

Customary and Metric Systems

In the United States, you often use the **customary system** of measurement. The customary system is made up of units such as inches, feet, cups, gallons and pounds and is often referred to as Imperial measure.

You may have also heard of the **Metric System** of measurement. The metric system is often used in countries outside of the United States, and in science measurements. Here, you will learn how to convert between different customary units of measurement.

First, let's look at some of the units in the customary system of measurement.

Customary Units of Measurement

Length	Weight	Capacity
12 inches = 1 foot		1 cup = 8 ounces
3 feet = 1 yard	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 ton = 2000 pounds	1 quart = 2 pints
		1 gallon = 4 quarts

Now let's look at how you convert among customary units of measurement. Because there is a relationship between different units of measure, you can use proportions to help you convert between customary units of measurement.

Let's look at an example.

How many feet are in 5 yards?

First, set up a proportion.

$$\frac{1 \text{ yard}}{3 \text{ feet}} = \frac{5 \text{ yard}}{x \text{ feet}}$$

Next, cross multiply to solve for x.

$$\begin{aligned} \frac{1}{3} &= \frac{5}{x} \\ 1x &= 3 \times 5 \\ x &= 15 \end{aligned}$$

The answer is 15.

Therefore, 5 yards is equal to 15 feet.

Here is another example.

How many ounces are there in 20 pounds?

First, set up a proportion.

$$\frac{1 \text{ pound}}{6 \text{ ounces}} = \frac{20 \text{ pounds}}{x \text{ ounces}}$$

Next, cross multiply to solve for x.

$$\begin{aligned} \frac{1}{16} &= \frac{20}{x} \\ 1x &= 16 \times 20 \\ x &= 320 \end{aligned}$$
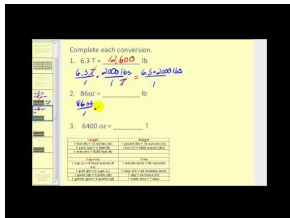
The answer is 320.

Therefore, 20 pounds is equal to 320 ounces.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Karen's cooking measurements.

Karen needs to convert 20 ounces into cups.

First, set up a proportion.

$$\frac{1 \text{ cup}}{8 \text{ ounces}} = \frac{x \text{ cups}}{20 \text{ ounces}}$$

Next, cross multiply.

$$\begin{aligned}
\frac{1}{8} &= \frac{x}{20} \\
8x &= 1 \times 20 \\
8x &= 20
\end{aligned}$$

Then, cross multiply to solve for x.

$$\begin{aligned}
8x &= 20 \\
\frac{8x}{8} &= \frac{20}{8} \\
x &= 2.5
\end{aligned}$$

The answer is 2.5.

Therefore, 20 ounces is equal to 2.5 cups.

Example 2

Eight pints is equal to how many gallons?

Set up a proportion to convert pints to gallons.

$$\frac{1 \text{ gallon}}{4 \text{ quarts}} \times \frac{1 \text{ quart}}{2 \text{ pints}} = \frac{1 \text{ gallon}}{8 \text{ pints}}$$

The answer is 1.

Therefore, 8 pints is equal to 1 gallon.

Example 3

Convert 6 tons to pounds.

First, set up a proportion.

$$\frac{1 \text{ ton}}{2000 \text{ pounds}} = \frac{6 \text{ tons}}{x \text{ pounds}}$$

Next, cross multiply to solve for *x*.

$$\begin{aligned} \frac{1}{2000} &= \frac{6}{x} \\ 1x &= 6 \times 2000 \\ x &= 12000 \end{aligned}$$

The answer is 12000.

Therefore, 6 tons is equal to 12,000 pounds.

Example 4

Convert 3 yards to feet.

First, set up a proportion.

$$\frac{1 \text{ yard}}{3 \text{ feet}} = \frac{3 \text{ yards}}{x \text{ feet}}$$

Next, cross multiply to solve for *x*.

$$\begin{aligned} \frac{1}{3} &= \frac{3}{x} \\ 1x &= 3 \times 3 \\ x &= 9 \end{aligned}$$

The answer is 9.

Therefore, 3 yards is equal to 9 feet.

Example 5

Convert 18 gallons to quarts.

First, set up a proportion.

$$\frac{1 \text{ gallon}}{4 \text{ quarts}} = \frac{18 \text{ gallons}}{x \text{ quarts}}$$

Next, cross multiply to solve for *x*.

$$\begin{aligned} \frac{1}{4} &= \frac{18}{x} \\ 1x &= 4 \times 18 \\ x &= 72 \end{aligned}$$

The answer is 72.

Therefore, 18 gallons is equal to 72 quarts.

PRACTICE

Solve each problem by converting among customary units of measurement.

1. 102 inches = ____ feet
2. 25 pounds = ____ ounces
3. 160 cups = ____ gallons
4. 150 pounds = ____ tons
5. 6 feet = ____ inches
6. 360 inches = ____ feet
7. 5.5 feet = ____ inches
8. 900 inches = ____ feet
9. 32 ounces = ____ pounds
10. 320 ounces = ____ pounds
11. 6 pounds = ____ ounces
12. 15 pounds = ____ ounces
13. 6 cups = ____ pints
14. 3 gallons = ____ quarts
15. 8 quarts = ____ pints
16. 24 pints = ____ quarts

Resources

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8.4 Convert Customary Units of Measurement in Real-World Situations (7.4e)

GUIDANCE

Evana is making a recipe for fruit punch that uses 3 cups of pineapple juice. If she makes 5 batches of the recipe, how many quarts of pineapple juice will she need?

In this concept, you will learn to convert customary units of measurement in real-world situations.

Customary System

The **customary system**, also known as the Imperial System, is made up of units such as inches, feet, cups, gallons and pounds. Let's look at conversions within the customary system of measurement.

Customary Units of Measurement

Length	Weight	Capacity
12 inches = 1 foot		1 cup = 8 ounces
3 feet = 1 yard	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 ton = 2000 pounds	1 quart = 2 pints
		1 gallon = 4 quarts

Let's look at an example.

The distance from John's house to Mike's house on a map is 4.5 inches. The scale of the map is 1.5 inches $= 2$ miles. What is the actual distance from John's house to Mike's house in feet?

First, set up a proportion.

$$\frac{1.5 \text{ inches}}{2 \text{ miles}} = \frac{4.5 \text{ inches}}{x \text{ miles}}$$

Next, cross multiply.

$$\begin{aligned} \frac{1.5}{2} &= \frac{4.5}{x} \\ 1.5x &= 2 \times 4.5 \\ 1.5x &= 9 \end{aligned}$$

Then, divide both sides by 1.5 to solve for x.

$$\begin{aligned} 1.5x &= 9 \\ \frac{1.5x}{1.5} &= \frac{9}{1.5} \\ x &= 6 \end{aligned}$$

Then, if the actual distance is 6 miles, what is this distance in feet.

$$\frac{1 \text{ mile}}{5280 \text{ feet}} = \frac{6 \text{ miles}}{x \text{ feet}}$$

Then, cross multiply to solve for x .

$$\begin{aligned} \frac{1}{5280} &= \frac{6}{x} \\ 1x &= 6 \times 5280 \\ x &= 31680 \end{aligned}$$

The answer is 31680.

The distance between the two houses is 31,680 feet.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Evana and her thirsty conversion.

Evana needs to make 5 batches with each batch needing 3 cups of juice. She needs to find the total amount of juice in quarts.

First, find the total number of cups she needs. If there are 3 cups in one batch, and she is making 5 batches, then she will need:

$$3 \times 5 = 15 \text{ cups}$$

Next, set up a proportion.

$$\frac{4 \text{ cups}}{1 \text{ quart}} = \frac{15 \text{ cups}}{x \text{ quarts}}$$

Then, cross multiply.

$$
\begin{aligned}
\frac{4}{1} &= \frac{15}{x} \\
4x &= 1 \times 15 \\
4x &= 15
\end{aligned}
$$

Then, divide both sides by 4 to solve for x.

$$
\begin{aligned}
4x &= 15 \\
\frac{4x}{4} &= \frac{15}{4} \\
x &= 3.75
\end{aligned}
$$

The answer is 3.75.

Evana needs to make 3.75 quarts of punch.

Example 2

A scale model of a building has a height of 3.5 feet. The scale of the model is $1\frac{1}{2}$ inch $= 10$ feet. What is the actual height of the building?

First, set up a proportion to find the height in inches.

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{x \text{ inches}}{3.5 \text{ feet}}$$

Next, cross multiply to solve for x.

$$\begin{aligned}
\frac{12}{1} &= \frac{x}{3.5} \\
1x &= 12 \times 3.5 \\
x &= 42
\end{aligned}$$

The answer is 42.

The scale model is 42 inches high.

Then, set up a proportion to solve for the actual height of the building.

$$\frac{1.5 \text{ inches}}{10 \text{ feet}} = \frac{42 \text{ inches}}{x \text{ feet}}$$

Then, cross multiply.

$$\begin{aligned}
\frac{1.5}{10} &= \frac{42}{x} \\
1.5x &= 10 \times 42 \\
1.5x &= 420
\end{aligned}$$

Then, divide both sides by 1.5 in order to solve for x.

$$\begin{aligned}
1.5x &= 420 \\
\frac{1.5x}{1.5} &= \frac{420}{1.5} \\
x &= 280
\end{aligned}$$

The answer is 280.

The building is 280 feet tall.

Example 3

Karin has a recipe that calls for 3 gallons of cider. How many quarts will she need?

First, set up a proportion.

$$\frac{1 \text{ gallon}}{4 \text{ quarts}} = \frac{3 \text{ gallons}}{x \text{ quarts}}$$

Next, cross multiply to solve for x.

$$\begin{aligned}
\frac{1}{4} &= \frac{3}{x} \\
1x &= 3 \times 4 \\
x &= 12
\end{aligned}$$

The answer is 12.

Karin will need 12 quarts of cider.

Example 4

Jack threw the ball 12 feet. How many inches did he throw the ball?

First, set up a proportion.

$$\frac{1 \text{ foot}}{12 \text{ inches}} = \frac{12 \text{ feet}}{x \text{ inches}}$$

Next, cross multiply to solve for x.

$$\begin{array}{rcl} \frac{1}{12} & = & \frac{12}{x} \\ 1x & = & 12 \times 12 \\ x & = & 144 \end{array}$$

The answer is 144.

Jack threw the ball 144 inches.

Example 5

Carl drank 3 pints of lemonade. How many ounces did he drink?

First, set up a proportion.

$$\frac{1 \text{ pint}}{16 \text{ ounces}} = \frac{3 \text{ pints}}{x \text{ ounces}}$$

Next, cross multiply to solve for x.

$$\begin{array}{rcl} \frac{1}{16} & = & \frac{3}{x} \\ 1x & = & 16 \times 3 \\ x & = & 48 \end{array}$$

The answer is 48.

Carl drank 48 ounces of lemonade.

PRACTICE

Solve each problem.

1. Justin ran 3 miles. How many feet did he run?
2. If the flour weighed four pounds, how many ounces did it weigh?
3. How many pounds is equal to 4 tons?
4. Mary needs 3 cups of juice for a recipe. How many ounces does she need?

5. Jess bought 3 quarts of pineapple juice. How many pints did she purchase?
6. If Karen bought 16 quarts of ice cream, how many gallons did she buy?
7. The length of the garden is four yards. How many feet is that?
8. If the width of the garden is 4 yards, how many inches is that?
9. Will eight cups of water fit in a two quart saucepan?
10. A recipe calls for 2 pints of milk. If Jorge cuts the recipe in half, how many cups of milk will he need?
11. Audrey is making brownies for a bake sale. The recipe calls for 8 ounces of flour for every 24 brownies. If she makes 96 brownies, how many pounds of flour will she need?
12. Two buildings are 5 inches apart on a map. The scale on the map is $\frac{1}{4}$ inch $= 1$ mile. What is the actual distance between the two buildings?
13. The length of a classroom on a floor plan is 2.5 inches. The scale of the map is $\frac{1}{2}$ inches $= 5$ feet What is the actual length of the classroom in inches?
14. A scale model of a mountain is 2.75 feet tall. The scale of the model is $\frac{1}{4}$ inch $= 50$ feet What is the actual height of the mountain in feet?
15. A scale drawing of a town includes a park that measures 0.5 inch by 1.5 inches. If the scale of the map is 0.5 inches $= 1$ mile, what is the area of the park in square feet?

8.5 Convert Metric Units of Measurement (7.4e)

GUIDANCE

Veera drives big rigs all over the country. She knows that the semitrailer she's driving is 3.5 m high. The bridge she's about to go over is 352.5 cm high. Will Veera's truck fit under the bridge?

In this concept, you will learn to convert metric units of measurement.

Metric System

The **metric system** of measurement is the primary measurement system in many countries; it contains units such as meters, kilometers and liters. You can remember the conversions by learning the prefixes: milli-means thousandth, centi-means hundredth, and kilo-means thousand. So a millimeter is one-thousandth of a meter, and a kilometer is one thousand meters.

Metric Units of Measurement

Length	Mass	Capacity
1 millimeter (mm) = 0.1 cm	1 milligram (mg) = 0.001 g	1 milliliter (mL) = about 20 drops of water
1 centimeter (cm) = 10 mm	1 gram (g) = 1000 m (about the weight of a paperclip)	1 liter (L) = 1000 mL
1 meter (m) = 100 cm		
1kilometer (km) = 1000 m	1 kilogram (kg) = 1000 g	

Now that you have reviewed these units of measurement, you can look at converting among the different units of measurement.

Let's look at an example.

How many centimeters are in 5 meters?

First, set up a proportion.

$$\frac{1\ m}{100\ cm} = \frac{5\ m}{x\ cm}$$

Next, cross multiply to solve for x.

$$
\begin{aligned}
\frac{1}{100} &= \frac{5}{x} \\
1x &= 5 \times 100 \\
x &= 500
\end{aligned}
$$

The answer is 500.

Therefore 5 meters is equal to 500 centimeters.

Let's look at another example.

Henry is making a recipe for lemonade that uses 2 liters of water. If he makes 3 batches of the recipe, how many milliliters of water will he need?

First find the total number of liters he needs.

If there are 2 liters in one batch, and he is making 3 batches, then he will need $2 \times 3 = 6$ liters.

Next, set up a proportion.

$$\frac{1\ L}{1000\ mL} = \frac{6\ L}{x\ mL}$$

Then, cross multiply to solve for x.

$$
\begin{aligned}
\frac{1}{1000} &= \frac{6}{x} \\
1x &= 6 \times 1000 \\
x &= 6000
\end{aligned}
$$

The answer is 6000.

Therefore 6 liters is equal to 6000 milliliters.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Veera and her tall bridge.

Veera needs to determine the height of the 352.2 cm bridge in meters in order to compare it to the truck. If the bridge height is greater than 3.5 m, the truck will fit under the bridge.

First, set up a proportion.

$$\frac{1\ m}{100\ cm} = \frac{x\ m}{352.5\ cm}$$

Next, cross multiply.

$$
\begin{aligned}
\frac{1}{100} &= \frac{x}{352.5} \\
100x &= 1 \times 352.5 \\
100x &= 352.5
\end{aligned}
$$

Then, divide both sides by 100 to solve for x.

$$
\begin{aligned}
100x &= 352.5 \\
\frac{100x}{100} &= \frac{352.5}{100} \\
x &= 3.525
\end{aligned}
$$

The answer is 3.525.

The bridge clearance is 3.525 m. This means the truck has 0.025 m or 2.5 cm clearance.

Example 2

Convert 4500 ml into liters.

First, set up a proportion.

$$
\frac{1\ L}{1000\ mL} = \frac{x\ L}{4500\ mL}
$$

Next, cross multiply.

$$
\begin{aligned}
\frac{1}{1000} &= \frac{x}{4500} \\
1000x &= 1 \times 4500 \\
1000x &= 4500
\end{aligned}
$$

Then divide both sides by 1000 to solve for x.

$$
\begin{aligned}
1000x &= 4500 \\
\frac{1000x}{1000} &= \frac{4500}{1000} \\
x &= 4.5
\end{aligned}
$$

The answer is 4.5.

Therefore 4.5 liters is equal to 4500 milliliters.

Example 3

Convert 5.5 grams into milligrams.

First, set up a proportion.

$$\frac{1\ g}{1000\ mg} = \frac{5.5\ g}{x\ mg}$$

Next, cross multiply to solve for x .

$$
\begin{aligned}
\frac{1}{1000} &= \frac{5.5}{x} \\
1x &= 5.5 \times 1000 \\
x &= 5500
\end{aligned}
$$

The answer is 5500.

Therefore 5.5 grams is equal to 5500 milligrams.

Example 4

Convert 40 mm into centimeters.

First, set up a proportion.

$$\frac{1\ cm}{10\ mm} = \frac{x\ cm}{40\ mm}$$

Next, cross multiply.

$$
\begin{aligned}
\frac{1}{10} &= \frac{x}{40} \\
10x &= 1 \times 40 \\
10x &= 40
\end{aligned}
$$

Then, divide both sides by 10 to solve for x.

$$
\begin{aligned}
10x &= 40 \\
\frac{10x}{10} &= \frac{40}{10} \\
x &= 4
\end{aligned}
$$

The answer is 4.

Therefore 4 cm is equal to 40 mm.

PRACTICE

Solve each problem.

1. P3 $km =$ _____ m

2. 2000 $m =$ _____ km

3. 5.5 $km =$ _____ m

4. 2500 $m =$ _____ km

5. $12000\ m =$ _____ km

6. $500\ cm =$ _____ m

7. $6000\ cm =$ _____ m

8. $4\ m =$ _____ cm

9. $11\ m =$ _____ cm

10. $50\ mm =$ _____ cm

11. $3\ cm =$ _____ mm

12. $15\ cm =$ _____ mm

13. $2000\ g =$ _____ kg

14. $35000\ g =$ _____ kg

15. $7\ kg =$ _____ g

Resources

MEDIA

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URL: https://www.ck12.org/flx/render/embeddedobject/167979
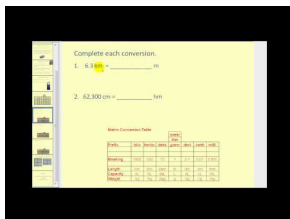
8.6 Convert Metric Units of Measurement in Real-World Situations (7.4e)

GUIDANCE

Jessica works in a science lab. She needs to convert the liquid measure that she is working with from liters to milliliters. She has been given 3.5 liters to convert. If each container that Jessica has holds 100 milliliters, how many containers will she need?

In this concept, you will learn to convert metric units of measurement in real-world situations.

Converting Metric Units of Measurements

The metric system of measurement is the primary measurement system in many countries; it contains units such as meters, kilometers and liters. Let's review the metric units of measurement.

Metric Units of Measurement

Length	Weight	Capacity
12 inches = 1 foot		1 cup = 8 ounces
3 feet = 1 yard	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 ton = 2000 pounds	1 quart = 2 pints
		1 gallon = 4 quarts

Now, let's look at a real-world problem involving converting metric units.

A scale model of a building has a height of 1.5 meters. The scale of the model is $1\ cm = 0.5\ m$. What is the actual height of the building?

First, set up a proportion to change 1.5 m to cm.

$$\frac{100\ cm}{1\ m} = \frac{x\ cm}{1.5\ m}$$

Next, cross multiply to solve for the number of centimeters for the height of the scale model.

$$\begin{aligned} \frac{100}{1} &= \frac{x}{1.5} \\ 1x &= 100 \times 1.5 \\ x &= 150 \end{aligned}$$

Next, set up a proportion to find the height of the actual model.

$$\frac{1\ cm}{0.5\ m} = \frac{150\ cm}{x\ m}$$

Then, cross multiply to solve for x.

$$\begin{aligned} \frac{1}{0.5} &= \frac{150}{x} \\ 1x &= 0.5 \times 150 \\ x &= 75 \end{aligned}$$

The answer is 75.

The actual height of the building is 75 meters.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Jessica's scientific measurements.

Jessica is given 3.5 L of liquid and needs to fill containers with 100 mL each. Jessica needs to know how many containers she will need.

First, set up a proportion to change 3.5 L to mL.

$$\frac{1000 \ mL}{1 \ L} = \frac{x \ mL}{3.5 \ L}$$

Next, cross multiply to solve for the number of milliliters Jessica has.

$$\begin{array}{rcl} \frac{1000}{1} & = & \frac{x}{3.5} \\ 1x & = & 1000 \times 3.5 \\ x & = & 3500 \end{array}$$

Next, divide by 100 to find the number of containers.

$$\frac{3500 \ mL}{100 \ mL} = 35$$

The answer is 35.

Jessica will need 35 containers.

Example 2

Marcy is making beef stew. Her recipe calls for 900 grams of beef. She looks in the refrigerator and sees that she has 1.5 kilograms of beef wrapped in a package. Marcy isn't sure how much of the beef she should use. Does Marcy have enough beef for her recipe?

First, set up a proportion to change 1.5 kg to g.

$$\frac{1000 \ g}{1 \ kg} = \frac{x \ g}{1.5 \ kg}$$

Next, cross multiply to solve for the number of grams of beef Marcy has in her refrigerator.

$$\begin{array}{rcl} \frac{1000}{1} & = & \frac{x}{1.5} \\ 1x & = & 1000 \times 1.5 \\ x & = & 1500 \end{array}$$

The answer is 1500 g.

Marcy has 1500 grams of meat, but only needs 900 grams for her recipe. She will have 600 grams of meat left over.

Example 3

If the scale used for a model is 1 *cm* = 0.5 *m* and the scale measurement is 3.2 meters, what is the actual measurement?

First, set up a proportion to change 3.2 m to cm.

$$\frac{100\ cm}{1\ m} = \frac{x\ cm}{3.2\ m}$$

Next, cross multiply to solve for the number of centimeters for the height of the scale model.

$$
\begin{aligned}
\frac{100}{1} &= \frac{x}{3.2} \\
1x &= 100 \times 3.2 \\
x &= 320
\end{aligned}
$$

Next, set up a proportion to find the height of the actual model.

$$\frac{1\ cm}{0.5\ m} = \frac{320\ cm}{x\ m}$$

Then, cross multiply to solve for x.

$$
\begin{aligned}
\frac{1}{0.5} &= \frac{320}{x} \\
1x &= 0.5 \times 320 \\
x &= 160
\end{aligned}
$$

The answer is 160.

The actual height of the building is 160 meters.

Example 4

If the scale used for a model is $1\ cm = 0.5\ m$ and the scale measurement is 0.75 meters, what is the actual measurement?

First, set up a proportion to change 0.75 m to cm.

$$\frac{100\ cm}{1\ m} = \frac{x\ cm}{0.75\ m}$$

Next, cross multiply to solve for the number of centimeters for the height of the scale model.

$$
\begin{aligned}
\frac{100}{1} &= \frac{x}{0.75} \\
1x &= 100 \times 0.75 \\
x &= 75
\end{aligned}
$$

Next, set up a proportion to find the height of the actual model.

$$\frac{1\ cm}{0.5\ m} = \frac{75\ cm}{x\ m}$$

Then, cross multiply to solve for *x*.

$$
\begin{aligned}
\frac{1}{0.5} &= \frac{75}{x} \\
1x &= 0.5 \times 75 \\
x &= 37.5
\end{aligned}
$$

The answer is 37.5.

The actual height of the building is 37.5 meters.

Example 5

If the scale used for a model is $1\ cm = 0.5\ m$ and the scale measurement is 0.25 m, what is the actual measurement?

First, set up a proportion to change 0.25 m to cm.

$$
\frac{100\ cm}{1\ m} = \frac{x\ cm}{0.25\ m}
$$

Next, cross multiply to solve for the number of centimeters for the height of the scale model.

$$
\begin{aligned}
\frac{100}{1} &= \frac{x}{0.25} \\
1x &= 100 \times 0.25 \\
x &= 25
\end{aligned}
$$

Next, set up a proportion to find the height of the actual model.

$$
\frac{1\ cm}{0.5\ m} = \frac{25\ cm}{x\ m}
$$

Then, cross multiply to solve for *x*.

$$
\begin{aligned}
\frac{1}{0.5} &= \frac{25}{x} \\
1x &= 0.5 \times 25 \\
x &= 12.5
\end{aligned}
$$

The answer is 12.5.

The actual height of the building is 12.5 meters.

PRACTICE

Figure out the measurements if the scale is $1\ cm = 0.5\ m$.

1. 3.5 m

2. 10 m

3. 6.5 m

4. 0.5 m

5. 2.5 m

6. 2.2 m

7. 4.5 m

8. 4 m

9. 3 m

10. 11 m

Solve each problem.

11. A recipe calls for 400 grams of flour. If Leena makes one quarter of the recipe, how many kilograms of flour will she need?

12. Two buildings are 9 centimeters apart on a map. The scale of the map is 0.5 centimeter = 2 kilometers.. What is the actual distance between the two buildings in meters?

13. A scale model of a tower is 1.25 meters tall. The scale of the model is 0.5 $cm = 5$ $meters$. What is the actual height of the tower in meters?

14. A scale drawing of a conference center includes a meeting room that measures 1.5 centimeters by 2.5 centimeters. If the scale of the drawing is 1 centimeter = 2 meters, what is the area of the meeting room in square centimeters?

15. Samir ran a race that was 10 kilometers long. About how many meters did Samir run?

Resources

MEDIA

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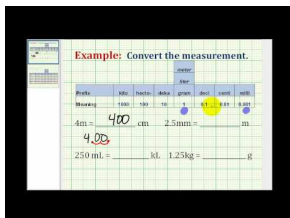
8.7 Conversion Applications Involving Rates and Unit Analysis (7.4e)

Unit Rate

A **rate** often refers to speed or a rate could refer to the amount of money someone makes per hour.

When you talk about a **unit rate**, you look at comparing a rate to 1. An example of a unit rate would be 65 miles per hour. A key word when working with unit rate is the word "per".

Let's look at an example.

Jeff makes $150.00 an hour as a consultant. What is his rate per minute?

First, use units to convert hours to minutes.

$$\begin{aligned} \# \text{ min} &= 1 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \\ \# \text{ min} &= 60 \text{ min} \end{aligned}$$

Next, convert $\frac{\$150}{\text{hr}}$ into $\frac{\$}{\text{min}}$.

$$\begin{aligned} \frac{\$150.00}{1 \text{ hr}} &= \frac{\$150.00}{60 \text{ min}} \\ \frac{\$150.00}{60 \text{ min}} &= \$2.5 \end{aligned}$$

The answer is 2.5.

Jeff makes $\frac{\$2.50}{\text{min}}$.

Unit analysis is when you look at how to measure individual units in different measurement amounts and it is used to convert units of measurement.

Let's look at an example.

Juanita worked for 18 hours last week. She made $116.00 at the end of the week. Juanita was sure that her manager had made a mistake and that she should have made more money. Juanita makes $9.00 per hour. Did Juanita make the correct amount of money or was there a mistake?

First, find out how much Juanita should have made if she made.

$$\begin{aligned} \text{pay} &= \frac{\$9.00}{1 \text{ hr}} \times 18 \text{ hr} \\ \text{pay} &= \$162 \end{aligned}$$

Next, determine if Juanita was not paid correctly.

Juanita was paid $116 but should have been paid $162.

There was a mistake in her pay by $46.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Joanne's marathon training.

Joanne can run 30 km in 5 hours but wants to know her rate per minute.

First, use units to convert hours to minutes.

$$\begin{aligned} \# \min &= 5 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \\ \# \min &= 300 \text{ min} \end{aligned}$$

Next, convert $30 \frac{\text{km}}{\text{hr}}$ into $\frac{\text{km}}{\text{min}}$.

$$\begin{aligned} \frac{30 \text{ km}}{1 \text{ hr}} &= \frac{30 \text{ km}}{300 \text{ min}} \\ \frac{30 \text{ km}}{300 \text{ min}} &= 0.10 \end{aligned}$$

The answer is 0.10.

Joanne runs at a speed of $0.1 \frac{\text{km}}{\text{min}}$.

Example 2

Jesse has a car that holds 14 gallons of gasoline. During the first week of the month, gasoline cost $2.75 per gallon. During the second week of the month, gasoline cost $2.50 per gallon. How much was the total cost for the 28 gallons of gasoline?

First, start by writing a variable expression to work on this problem. You know that the number of gallons of gasoline does not change. Therefore, let x represent the number of gallons of gasoline.

$$\text{Total cost} = 2.75x + 2.50x$$

Next, substitute 14 for our variable x.

$$\begin{aligned} \text{Total cost} &= 2.75(14) + 2.50(14) \\ \text{Total cost} &= 38.5 + 35 \\ \text{Total cost} &= 73.5 \end{aligned}$$

The answer is 73.5.

Therefore, Jess paid $73.50 in gasoline for the two weeks.

Example 3

Ten apples cost $3.99. What is the cost for one apple?

First, set up a proportion.

$$\frac{\$3.99}{10 \text{ apples}} = \frac{x}{1 \text{ apple}}$$

Next, cross multiply.

$$\begin{array}{rcl}\frac{3.99}{10} & = & \frac{x}{1} \\ 10x & = & 1 \times 3.99 \\ 10x & = & 3.99\end{array}$$

Then, divide both sides by 10 to solve for x.

$$\begin{array}{rcl}10x & = & 3.99 \\ \frac{10x}{10} & = & \frac{3.99}{10} \\ x & = & 0.399\end{array}$$

The answer is 0.399.

One apple costs $0.40 or 40¢.

Example 4

Fifteen gallons of gasoline costs $45.00. How much is it per gallon?

First, set up a proportion.

$$\frac{\$45.00}{15 \text{ gallons}} = \frac{x}{1 \text{ gallon}}$$

Next, cross multiply.

$$\begin{array}{rcl}\frac{45}{15} & = & \frac{x}{1} \\ 15x & = & 1 \times 45 \\ 15x & = & 45\end{array}$$

Then, divide both sides by 15 to solve for x.

$$\begin{array}{rcl}15x & = & 45 \\ \frac{15x}{15} & = & \frac{45}{15} \\ x & = & 3\end{array}$$

The answer is 3.

One gallon of gas costs $3.00.

Example 5

Two tickets to a ballgame costs $111.50. What is the cost for one ticket?

First, set up a proportion.

$$\frac{\$111.50}{2 \text{ tickets}} = \frac{x}{1 \text{ ticket}}$$

Next, cross multiply.

$$\begin{aligned}
\frac{111.50}{2} &= \frac{x}{1} \\
2x &= 1 \times 111.50 \\
2x &= 111.50
\end{aligned}$$

Then, divide both sides by 2 to solve for x.

$$\begin{aligned}
2x &= 111.50 \\
\frac{2x}{2} &= \frac{111.50}{2} \\
x &= 55.75
\end{aligned}$$

The answer is 55.75.

One ticket to the game is $55.75

PRACTICE

Use what you have learned to solve each problem.

1. Peter runs at a rate of 10 kilometers per hour. How many kilometers will he cover in 8 hours?
2. A cheetah can run at a speed of 60 miles per hour. What is his distance after 6 hours?
3. What is the distance formula?
4. If a car travels at a rate of 65 miles per hour for 30 minutes, how far will it travel?
5. A train travels at a rate of 50 miles per hour. If it needs to travel 320 miles, how many minutes will it take?
6. A car travels 65 mph for 12 hours. How many miles will it travel?
7. A bus traveled 300 miles at an average speed of 50 miles per hour. How long did this trip take the bus?
8. A car traveled at an average speed of 40 miles per hour through a construction zone. If the car traveled 20 miles at this rate, how many hours did it take to travel the 20 miles?
9. What is velocity?
10. What is the formula for velocity?
11. What is the velocity of an object that travels 500 miles in 2.5 hours?
12. If an object has a velocity of 125 miles per hour, how long will it take to travel 4,375 miles?
13. If an object has a velocity of 7 kilometers per minute, how far will it travel in 2 hours?
14. If an object has a velocity of 4 meters per second, how many kilometers will it travel in 2 days?
15. The formula for density is $D = \frac{m}{v}$ where D represents the density of an object, m represents the mass of the object, and v represents the volume of the object. What is the density of a brick that weighs 9 pounds and has a volume of 36 $cu. in.$?

Resources
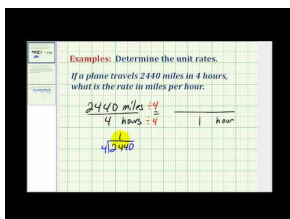
Vocabulary:

- Unit rate
- Proportionality

CHAPTER 9 Circumference, Area, and Surface Area

Chapter Outline

7.9b: determine the circumference and area of circles

7.9c: determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles

7.9d: solve problems involving the lateral and total surface area of a rectangular prism, rectangular pyramid, triangular prism, and triangular pyramid by determining the area of the shape's net

9.1 Circumference of a Circle (7.9b)

GUIDANCE

Circumference

Circumference is the distance around a circle. The circumference can also be called the perimeter of a circle. However, we use the term circumference for circles because they are round.

Circumference Formula: $C = \pi d$ where the diameter $d = 2r$, or twice the radius. So $C = 2\pi r$ as well.
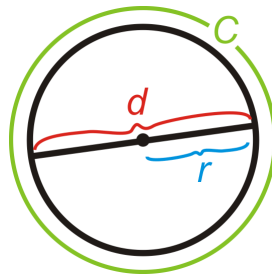
π, or **"pi"** is the ratio of the circumference of a circle to its diameter. It is approximately equal to 3.14159265358979323846...

You should have a π button on your calculator. If you don't, you can use 3.14 as an approximation for π. You can also leave your answers in terms of π for many problems.

What if you were given the radius or diameter of a circle? How could you find the distance around that circle?

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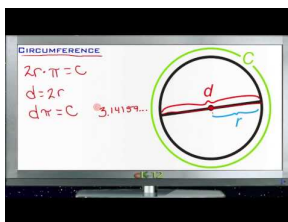
GUIDED PRACTICE

Example 1

Find the circumference of a circle with a radius of 7 cm.

Plug the radius into the formula.

$$C = 2\pi(7) = 14\pi \approx 44 \ cm$$

Example 2

The circumference of a circle is 64π units. Find the diameter.

Again, you can plug in what you know into the circumference formula and solve for d.

$$64\pi = \pi d$$
$$64 \ units = d$$

Example 3

A circle is inscribed in a square with 10 in. sides. What is the circumference of the circle? Leave your answer in terms of π.
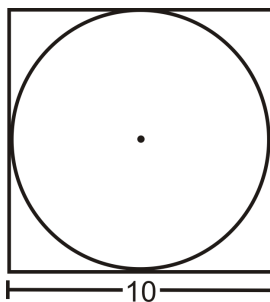
From the picture, we can see that the diameter of the circle is equal to the length of a side. $C = 10\pi \ in.$

Example 4

Find the perimeter of the square in Example 3. Is it more or less than the circumference of the circle? Why?

The perimeter is $P = 4(10) = 40 \ in.$ In order to compare the perimeter with the circumference we should change the circumference into a decimal.

$C = 10\pi \approx 31.42 \ in.$ This is less than the perimeter of the square, which makes sense because the circle is inside the square.

Example 5

The tires on a compact car are 18 inches in diameter. How far does the car travel after the tires turn once? How far does the car travel after 2500 rotations of the tires?

One turn of the tire is the circumference. This would be $C = 18\pi \approx 56.55$ *in*. 2500 rotations would be $2500 \cdot$ 56.55 *in approx* 141,375 *in*, 11,781 ft, or 2.23 miles.

PRACTICE

Fill in the following table. Leave all answers in terms of π.

TABLE 9.1:

	diameter	*radius*	*circumference*
1.	15		
2.		4	
3.	6		
4.			84π
5.		9	
6.			25π
7.			2π
8.	36		

9. Find the circumference of a circle with $d = \frac{20}{\pi}$ *cm*.

Square *PQSR* is inscribed in $\odot T$. $RS = 8\sqrt{2}$.
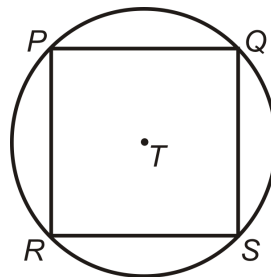
10. Find the length of the diameter of $\odot T$.
11. How does the diameter relate to *PQSR*?
12. Find the perimeter of *PQSR*.
13. Find the circumference of $\odot T$.

For questions 14-17, a truck has tires with a 26 in diameter.

14. How far does the truck travel every time a tire turns exactly once? What is this the same as?
15. How many times will the tire turn after the truck travels 1 mile? (1 mile = 5280 feet)
16. The truck has travelled 4072 tire rotations. How many miles is this?
17. The average recommendation for the life of a tire is 30,000 miles. How many rotations is this?

9.2 Area of a Circle (7.9b)

GUIDANCE

Area of a Circle

To find the area of a circle, all you need to know is its radius. If *r* is the radius of a circle, then its **area** is $A = \pi r^2$.
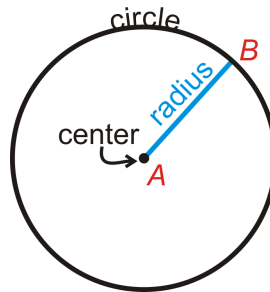
We will leave our answers in terms of π, unless otherwise specified.

What if you were given the radius or diameter of a circle? How could you find the amount of space the circle takes up?

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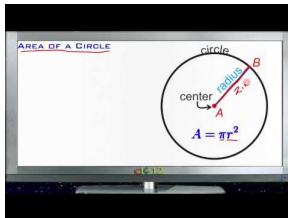
GUIDED PRACTICE

Example 1

Find the area of a circle with a diameter of 12 cm.

If $d = 12\ cm$, then $r = 6\ cm$. The area is $A = \pi \left(6^2\right) = 36\pi\ cm^2$.

Example 2

If the area of a circle is 20π units, what is the radius?

Plug in the area and solve for the radius.

$$20\pi = \pi r^2$$
$$20 = r^2$$
$$r = \sqrt{20} = 2\sqrt{5} \, units$$

Example 3

A circle is inscribed in a square. Each side of the square is 10 cm long. What is the area of the circle?
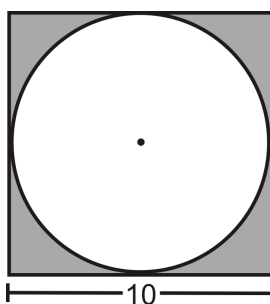
The diameter of the circle is the same as the length of a side of the square. Therefore, the radius is 5 cm.

$$A = \pi 5^2 = 25\pi \, cm^2$$

Example 4

Find the area of the shaded region from Example 3.

The area of the shaded region would be the area of the square minus the area of the circle.

$$A = 10^2 - 25\pi = 100 - 25\pi \approx 21.46 \, cm^2$$

Example 5

Find the diameter of a circle with area 36π.

First, use the formula for the area of a circle to solve for the radius of the circle.

$$A = \pi r^2$$
$$36\pi = \pi r^2$$
$$36 = r^2$$
$$r = 6$$

If the radius is 6 units, then the diameter is 12 units.

PRACTICE

Fill in the following table. Leave all answers in terms of π.

TABLE 9.2:

	radius	*Area*	*circumference*
1.	2		
2.		16π	
3.			10π
4.			24π
5.	9		
6.		90π	
7.			35π
8.	$\frac{7}{\pi}$		
9.			60
10.		36	

Find the area of the shaded region. Round your answer to the nearest hundredth.

11.

12.

13.
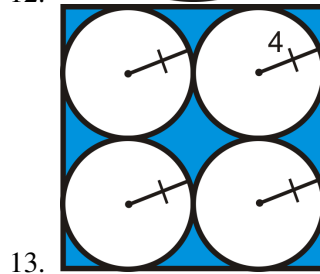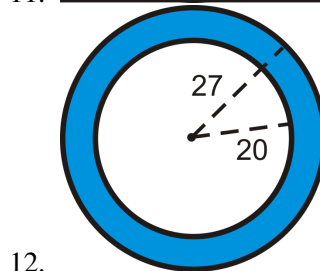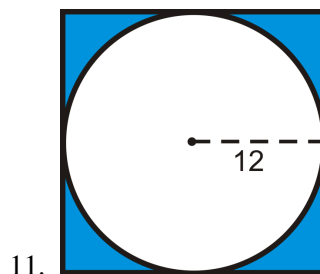
9.3 Area of Composite Figures (7.9c)

GUIDANCE

What if you wanted to find the area of a shape that was made up of other shapes? How could you use your knowledge of the area of rectangles, parallelograms, and triangles to help you? After completing this Concept, you'll be able to answer questions like these.

Watch This
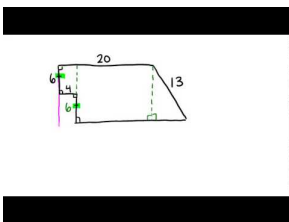
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CK-12 Foundation: Chapter10AreaofCompositeShapesA

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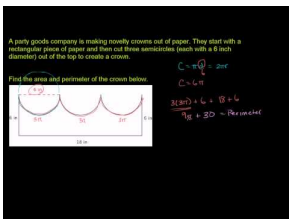
Khan Academy: Area and Perimeter of Composite Figures

Guidance Continued

Perimeter is the distance around a shape. The perimeter of any figure must have a unit of measurement attached to it. If no specific units are given (feet, inches, centimeters, etc), write "units." **Area** is the amount of space inside a figure. If two figures are congruent, they have the same area. This is the **congruent areas postulate.** This postulate needs no proof because congruent figures have the same amount of space inside them. Keep in mind that two figures with the same area are not necessarily congruent.

A **composite shape** is a shape made up of other shapes. To find the area of such a shape, simply find the area of each part and add them up. The **area addition postulate** states that if a figure is composed of two or more parts that do not overlap each other, then the area of the figure is the sum of the areas of the parts.

Example A

Find the area of the figure below. You may assume all sides are perpendicular.
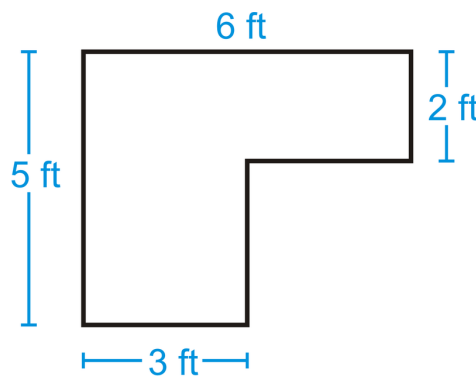
Split the shape into two rectangles and find the area of each.
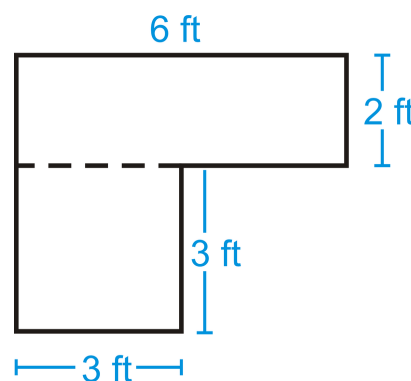
$$A_{top\ rectangle} = 6 \cdot 2 = 12\ ft^2$$
$$A_{bottom\ square} = 3 \cdot 3 = 9\ ft^2$$

The total area is $12 + 9 = 21\ ft^2$.

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Example B
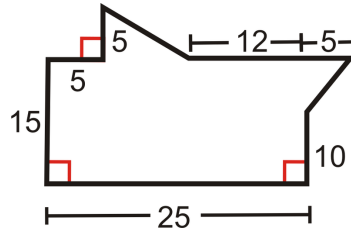
- Divide the shape into two triangles and one rectangle.
- Find the area of the two triangles and rectangle.
- Find the area of the entire shape.

Solution:

- One triangle on the top and one on the right. Rectangle is the rest.
- Area of triangle on top is $\frac{8(5)}{2} = 20\ units^2$. Area of triangle on right is $\frac{5(5)}{2} = 12.5\ units^2$. Area of rectangle is $375\ units^2$.
- Total area is $407.5\ units^2$.

Example C

Find the area of the figure below.
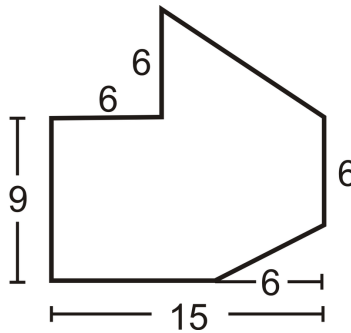
Divide the figure into a triangle and a rectangle with a small triangle cut out of the lower right-hand corner.
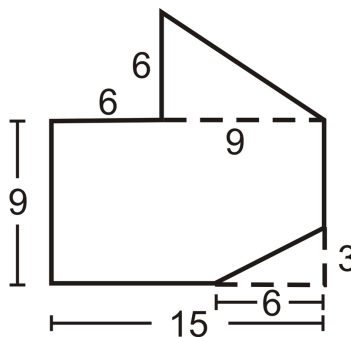
$$A = A_{top\ triangle} + A_{rectangle} - A_{small\ triangle}$$

$$A = \left(\frac{1}{2} \cdot 6 \cdot 9\right) + (9 \cdot 15)\Big) - (\frac{1}{2} \cdot 3 \cdot 6\Big)$$

$$A = 27 + 135 - 9$$

$$A = 153\ units^2$$

Watch this video for help with the Examples above.
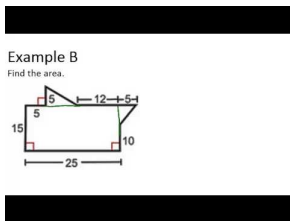
Example B
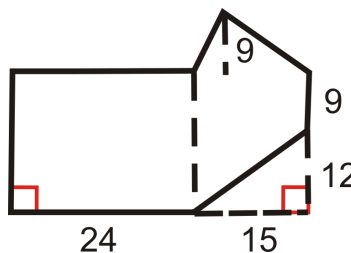Find the area.

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/137565

CK-12 Foundation: Chapter10AreaofCompositeShapesB

GUIDED PRACTICE

1. Find the area of the rectangles and triangle.

2. Find the area of the whole shape.

Answers:

1. Rectangle #1: Area $= 24(9 + 12) = 504\ units^2$. Rectangle #2: Area $= 15(9 + 12) = 315\ units^2$. Triangle: Area $= \frac{15(9)}{2} = 67.5\ units^2$.

2. You need to subtract the area of the triangle from the bottom right corner. Total Area $= 504 + 315 + 67.5 - \frac{15(12)}{2} = 796.5\ units^2$
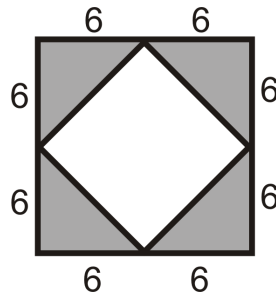
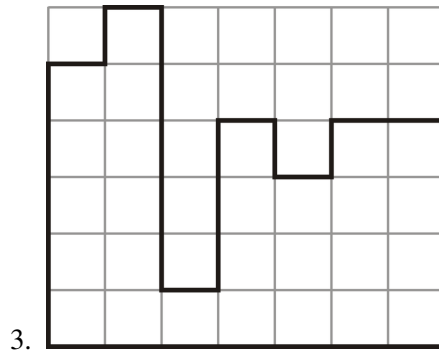## Interactive Practice

PRACTICE

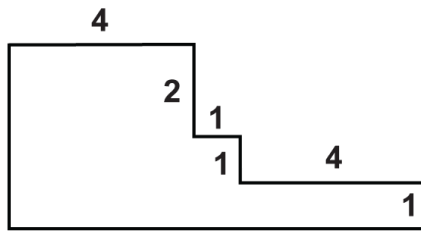Use the picture below for questions 1-2. Both figures are squares.

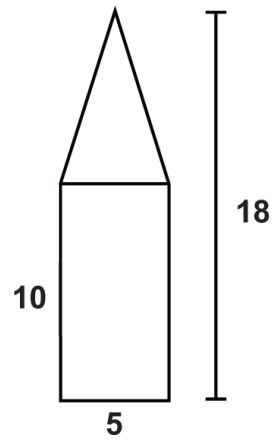1. Find the area of the unshaded region.
2. Find the area of the shaded region.

Find the area of the figure below. You may assume all sides are perpendicular.
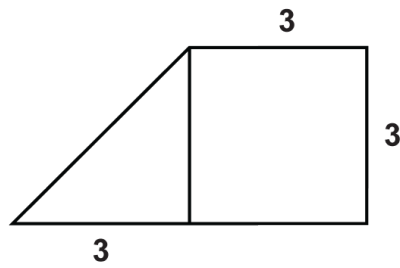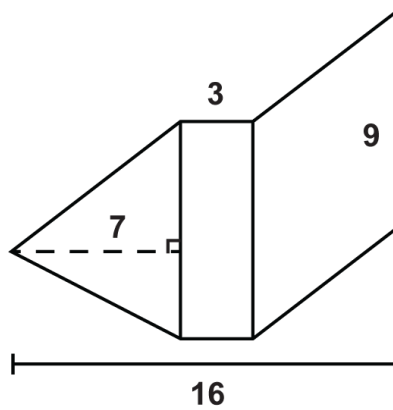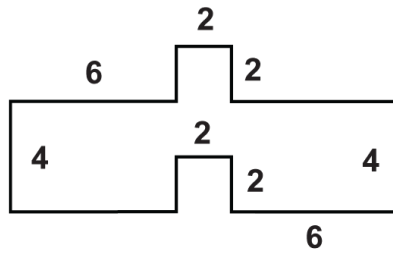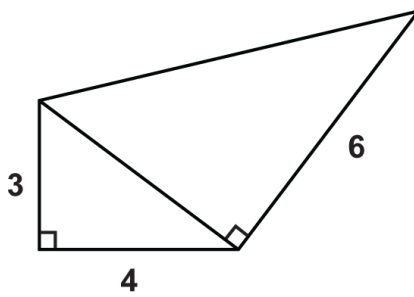

3.

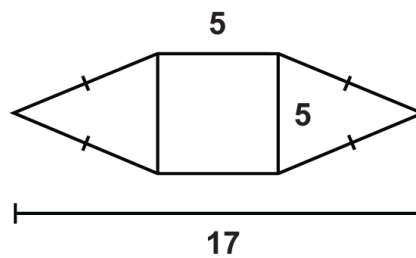Find the areas of the composite figures.

4.

5.

6.

7.

8.

9.

Use the figure to answer the questions.

10. What is the area of the square?
11. What is the area of the triangle on the left?
12. What is the area of the composite figure?

Find the area of the following figures.

13. Find the area of the unshaded region.
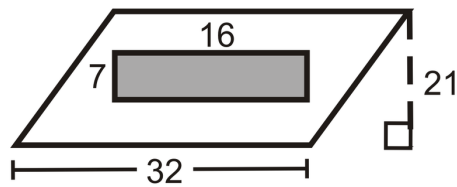
14. Lin bought a tract of land for a new apartment complex. The drawing below shows the measurements of the sides of the tract. Approximately how many acres of land did Lin buy? You may assume any angles that look like right angles are 90°. (1 acre $\approx$ 40,000 square feet)
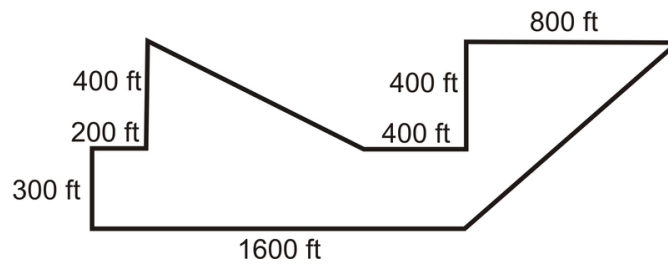
15. Linus has 100 ft of fencing to use in order to enclose a 1200 square foot rectangular pig pen. The pig pen is adjacent to the barn so he only needs to form three sides of the rectangular area as shown below. What dimensions should the pen be?
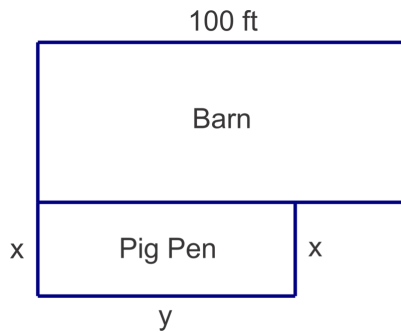
9.4 Nets of Prisms and Pyramids (7.9d)

GUIDANCE

Nets of Solid Figures
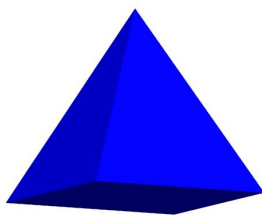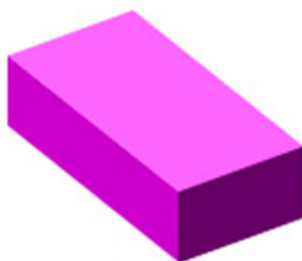
If you look at the solid figures above, you can see the parts of the figures that are toward you; however, you cannot see the parts that are on the backside and underneath or above. If you want to see the entire figure, then you need to draw a representation of the figure that is two-dimensional, or planar. You can think of this as unfolding or taking apart the solid so that you can see all of the parts.

Look back at the three figures. The first one is a rectangular prism. If you want to draw this as a two-dimensional figure, then you have to think about all of the parts of the rectangular prism. There are four edges of the base of a rectangular prism, so you know that there are six faces.
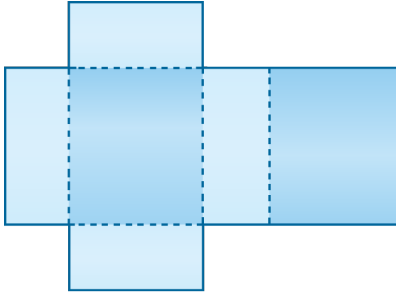
Look at how this box has been unfolded. You can see the pattern that it makes. If you picture it in your mind, you can see how you could fold up the box again. The edges form the lines and you fold it along the dotted lines to create a rectangular prism once again.

The second figure is a square pyramid. Let's think about its parts and how it might look if it was unfolded. The base is a square and there are four triangles on the sides. You can unfold it from the vertex down. Here is what it looks like.
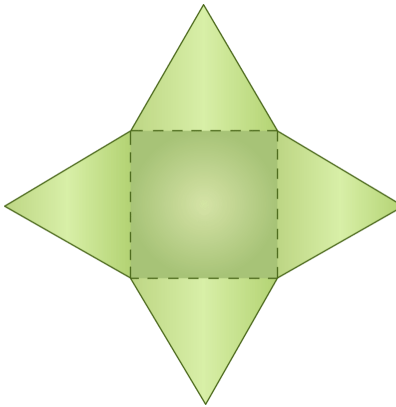
You can see that if you fold on the dotted lines, the faces are rejoined, forming a vertex at the top of the figure.

The last figure is a cylinder. You know that the base and the top of a cylinder are circles. Think about the middle as a rectangle that is wound around the edges of the circles.
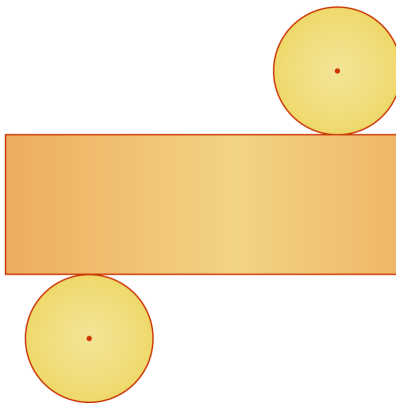
Now let's look at a cone.

IMAGE NOT AVAILABLE

When looking at a cone from the front, you see a triangle. When you look from the bottom, you see a circle. When looking at the cone from above, you see a circle with a point in the center. The point represents the top of the cone. The circle represents the base.
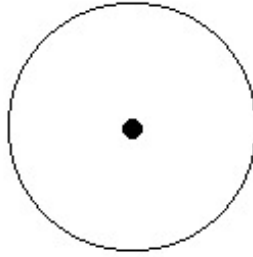
Imagine the triangle that you see from the front wrapping around the circular base, forming the vertex at the top. The net of a cone looks like this:

IMAGE NOT AVAILABLE

Nets let you see the shape of the base, as well as the side or faces, of a solid figure all at once.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Michael and Amy's unknown boxes.

With regard to the first box:

First, all of the faces are rectangles.

Next, there are four rectangular side faces.

Then, there are two rectangular end faces. When folded, these represent the base and top.

The answer for the first box is it is a rectangular prism.

With regard to the second box:

First, all of the faces are squares.

Next, there are four square side faces.

Then, there are two square end faces. When folded, these represent the base and the top.

The answer for the second box is it is a cube.

Example 2

Draw the net of a pentagonal prism.

IMAGE NOT AVAILABLE

First, draw the top and base of the pentagonal prism, remembering that the top and base of any prism is the same shape.
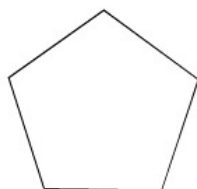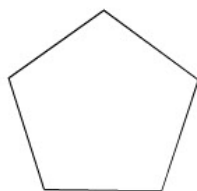
Next, draw the rectangular side faces of the pentagonal prism, remembering that there are five edges on the base (and top) and that this is the same as the number of faces on the side.
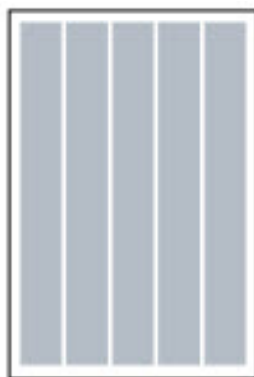
The answer is that the net of a pentagonal prism looks like this:
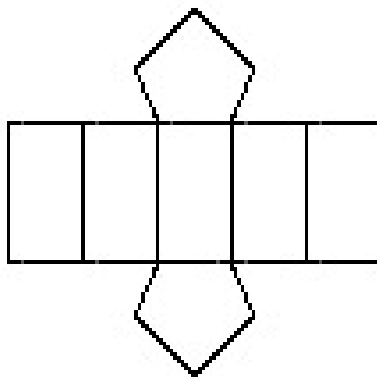
Example 3

What solid figure is represented by the following net?

IMAGE NOT AVAILABLE

First, observe that there is one six-sided figure - a hexagon - representing the base of the figure.

Next, observe that there are six triangles that make up the side faces of the figure.

Then, observe that the six triangles meet in the center which will form a vertex at the top when folded.

The answer is the solid figure represented by the net above is a hexagonal pyramid.

Example 4

Draw the net of a triangular prism.
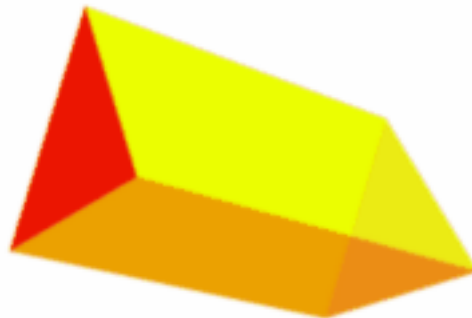
First, draw the top and base of the triangular prism, remembering that the top and base of any prism is the same shape.
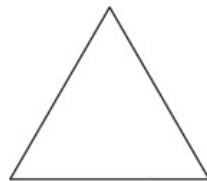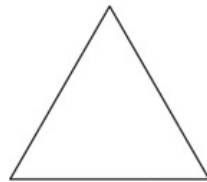
Next, draw the rectangular side faces of the triangular prism, remembering that there are three edges on the base (and top) and that this is the same as the number of faces on the side.
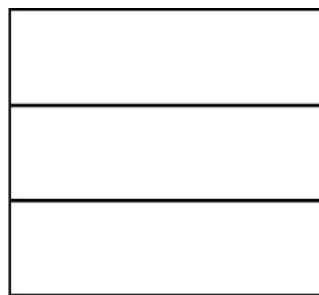
Then, draw each of the different sections as one picture, with the base and top triangles in the appropriate positions around the side rectangles.

The answer is that the net of a triangular prism looks like this:

IMAGE NOT AVAILABLE

PRACTICE

Identify each solid by its net.

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.
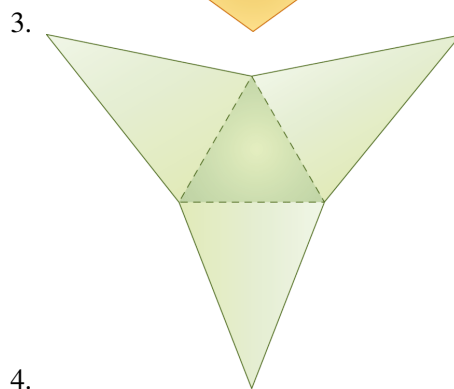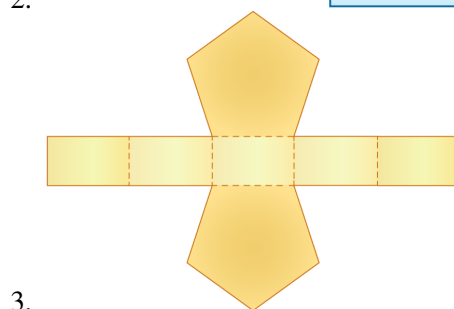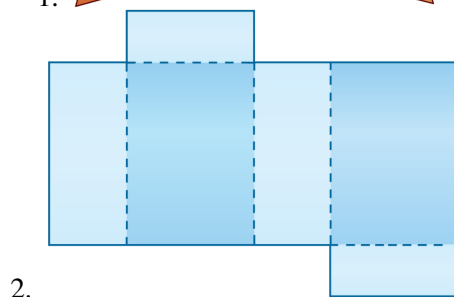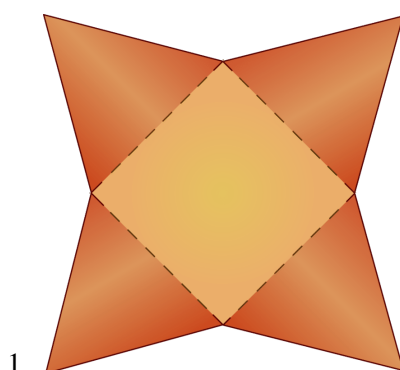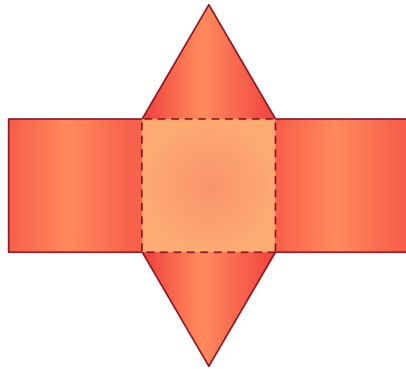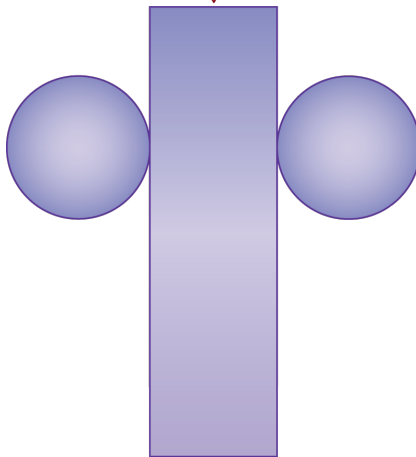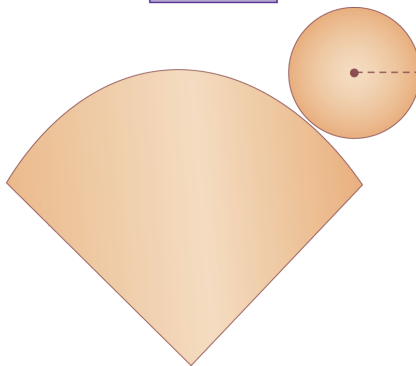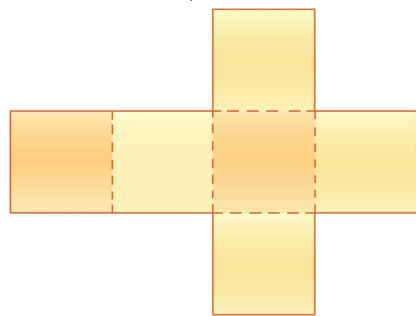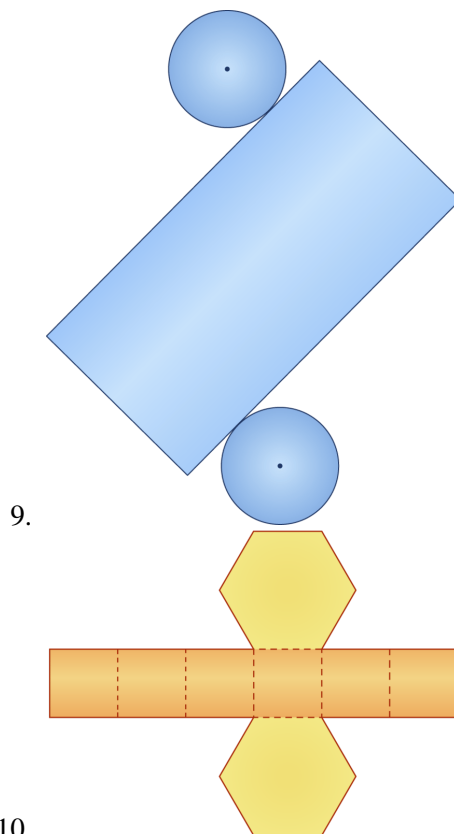
11. Draw an example of a new net for each of the figures in numbers 1-10.

Resources

MEDIA

Click image to the left or use the URL below.

URL: https://www.ck12.org/flx/render/embeddedobject/183869
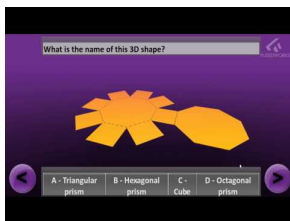
9.5 Lateral and Total Surface Area of Prisms (7.9d)
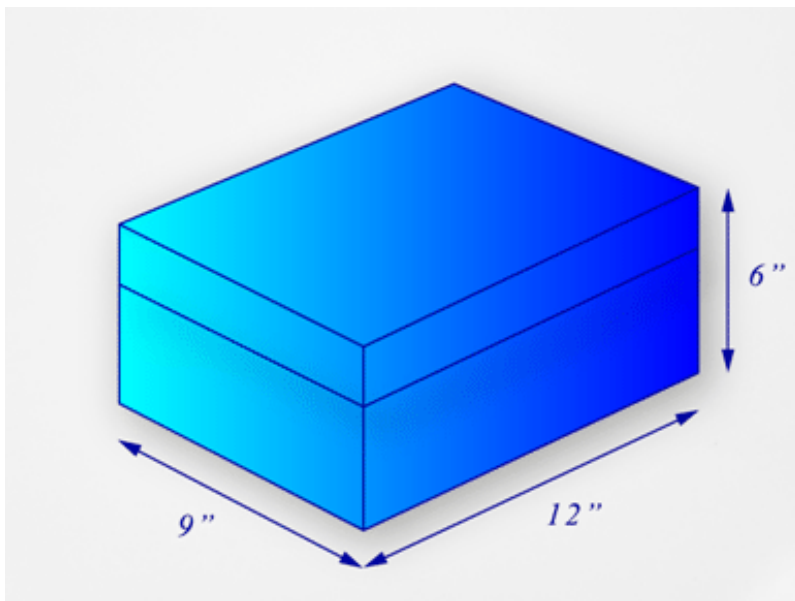
Crystal has a fragile gift that she is packing in a box before wrapping it. To figure out how much wrapping paper she will need she has drawn the box with all its dimensions. How can Crystal calculate how much gift-wrap she will need using the above dimensions?

In this concept, you will learn to find the surface area of prisms.

Surface Area

A **prism** is a three-dimensional figure with two parallel congruent polygons as bases. The side faces of a prism are rectangular in shape.

One measure when working with three-dimensional figures is called **surface area**. **Surface area** is the total of the areas of each face of a solid figure. Imagine you could wrap one of these figures in wrapping paper, like a present. The amount of wrapping paper needed to cover the figure represents its surface area. To find the surface area, you must be able to calculate the area of each face and then add these areas together.

There are several different ways to calculate surface area. One way is to use a **net**. A net is a two-dimensional diagram of a three-dimensional figure. Imagine you could unfold a box so that it is completely flat. You would have something that looks like this.
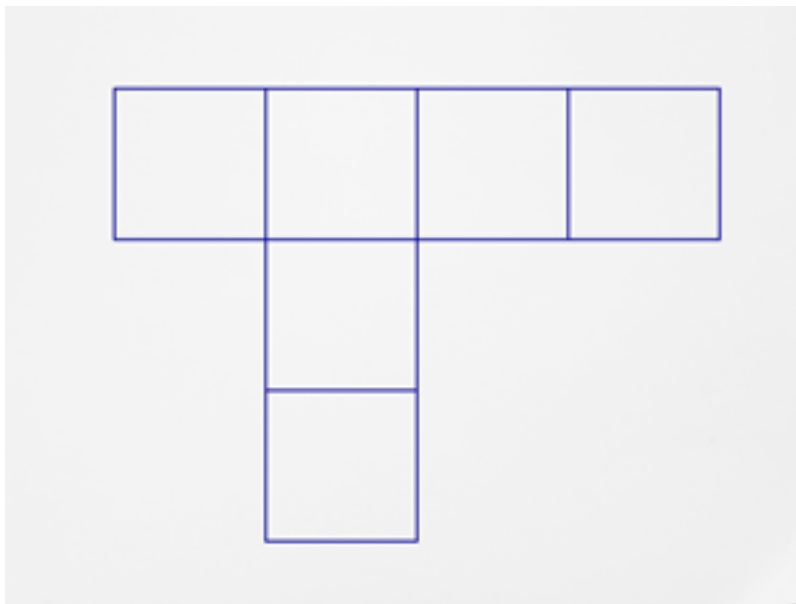
If you folded this up, you could see that it would form a cube. A cube is made up of faces that are squares. If you wanted to figure out the surface area or measurement of the outer covering of this cube, then you could find the area of each surface of the cube and then add the products together.

You could also look at a net of a rectangular prism.
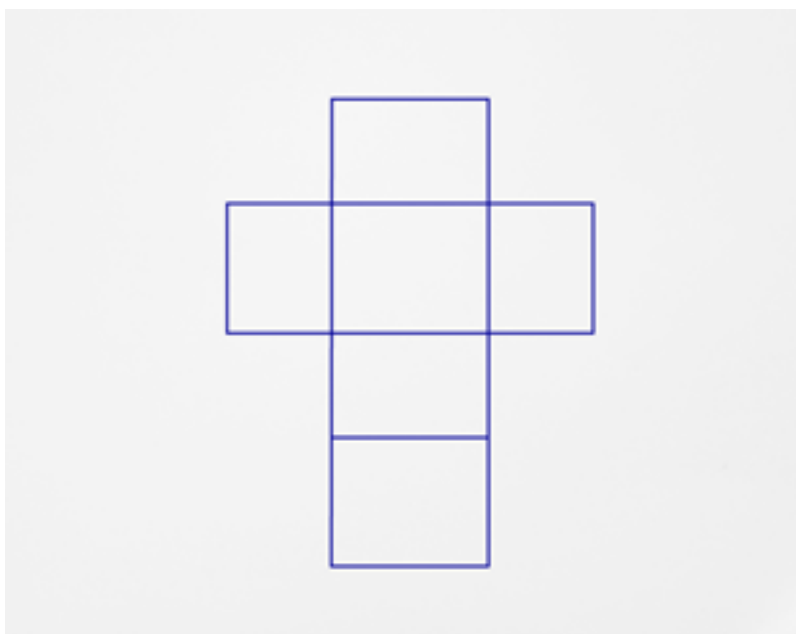
A rectangular prism is made up of rectangles. To find the surface area of a prism, you would need to calculate the area of each of the faces and then add them together.

Let's begin by calculating the surface area of a rectangular prism.
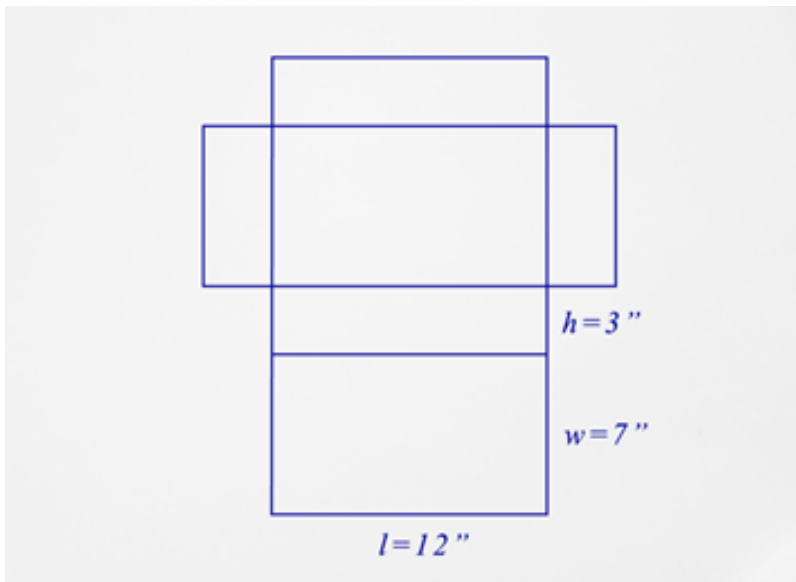
First, let's break apart the shape so you can organize the six area calculations.

Next, calculate the area of each face.
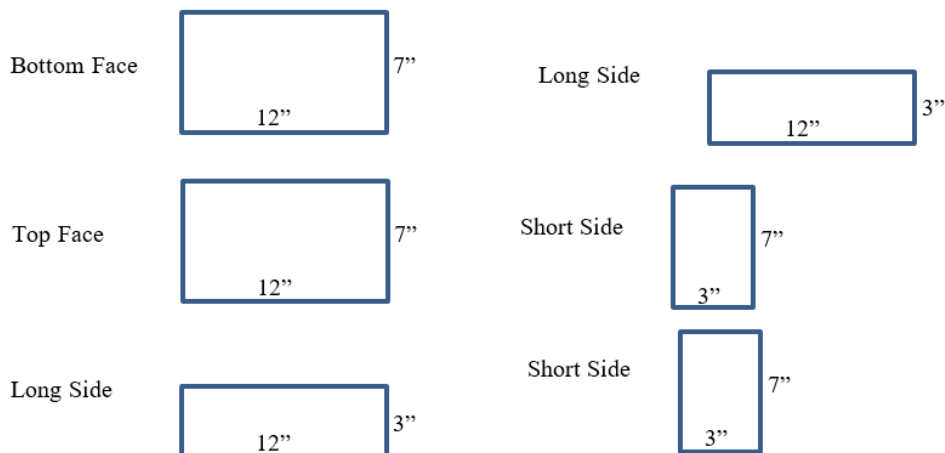
$$A_{\text{bottom face}} = l \times w$$
$$= 12 \times 7$$
$$= 84$$

$$A_{\text{longer side}} = l \times w$$
$$= 12 \times 3$$
$$= 36$$

$$A_{\text{top face}} = l \times w$$
$$= 12 \times 7$$
$$= 84$$

$$
\begin{aligned}
A_{\text{shorter side}} &= l \times w \\
&= 7 \times 3 \\
&= 21
\end{aligned}
$$

$$
\begin{aligned}
A_{\text{longer side}} &= l \times w \\
&= 12 \times 3 \\
&= 36
\end{aligned}
$$

$$
\begin{aligned}
A_{\text{shorter side}} &= l \times w \\
&= 7 \times 3 \\
&= 21
\end{aligned}
$$

Then, add the areas together.

$$
\begin{aligned}
\text{Surface Area} &= 84 + 84 + 36 + 36 + 21 + 21 \\
\text{Surface Area} &= 282
\end{aligned}
$$

The answer is 282.

The surface area of the prism is 282 in^2.

Now you can see each face so that you can calculate their area and then add them together. However, you can also use a formula to represent the faces as you find their area. The formula gives a nice short cut that you can use for any kind of prism, no matter what shape its base is.

Take a look at the formula below.

$$SA = Ph + 2B$$

Let's look at the first part of the formula. *P* represents the perimeter of the base, and *h* represents the height of the prism. By multiplying the perimeter and height, you are finding the area of all of the side faces at once. This will be very useful if the prism that you are working with isn't just a cube or a rectangular prism.

The second part of the formula represents the area of the top and bottom faces. *B* represents the area of one base, which you find using whichever area formula is appropriate for the shape of the base. Then you multiply it by 2 to show the area of the top and bottom faces at once.

Let's look at an example.

Find the surface area of this figure using a formula.
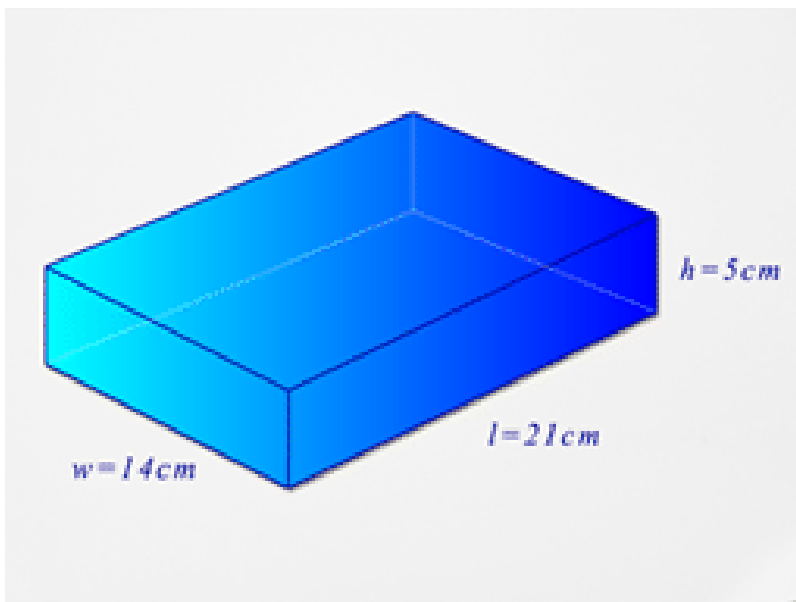
First, find the perimeter of the base.

$$
\begin{aligned}
P &= 2l + 2w \\
P &= 2 \times 21 + 2 \times 14 \\
P &= 70
\end{aligned}
$$

Next, calculate the area of the base.

$$
\begin{aligned}
A &= l \times w \\
A &= 21 \times 14 \\
A &= 294
\end{aligned}
$$

Then, knowing the height is 5 cm, determine the surface area.

$$
\begin{aligned}
SA &= Ph + 2B \\
SA &= 70 \times 5 + 2 \times 294 \\
SA &= 350 + 588 \\
SA &= 938
\end{aligned}
$$

The answer is 938.

The surface area of the prism is 938 cm^2.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Crystal and her gift box.

The box to be wrapped has a length of $12''$, a wide of $9''$ and a height of $6''$.

First, find the perimeter of the base.

$$
\begin{aligned}
P &= 2l + 2w \\
P &= 2 \times 12 + 2 \times 9 \\
P &= 42
\end{aligned}
$$

Next, calculate the area of the base.

$$
\begin{aligned}
A &= l \times w \\
A &= 12 \times 9 \\
A &= 108
\end{aligned}
$$

Then, knowing the height is 6 in., determine the surface area.

$$
\begin{aligned}
SA &= Ph + 2B \\
SA &= 42 \times 6 + 2 \times 108 \\
SA &= 252 + 216 \\
SA &= 468
\end{aligned}
$$

The answer is 468.

Crystal needs 468 $in.^2$ of wrapping paper.

Example 2

What is the surface area of the figure below?

$H = 15''$
$h = 3''$
$a = 5''$
$b = 8''$
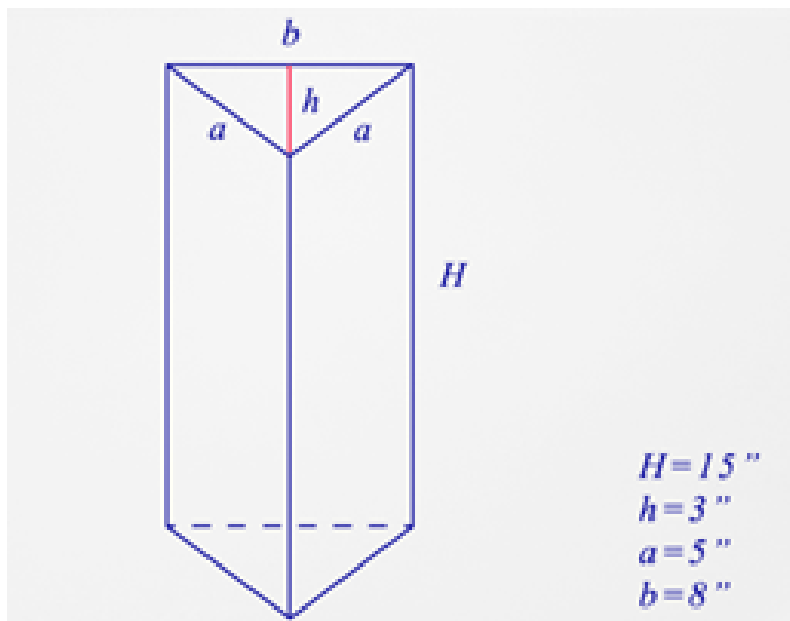
First, find the perimeter of the base.

$$
\begin{aligned}
P &= 2a + b \\
P &= 2 \times 5 + 8 \\
P &= 18
\end{aligned}
$$

Next, calculate the area of the base.

$$\begin{aligned} A &= \tfrac{1}{2}bh \\ A &= \tfrac{1}{2}8 \times 3 \\ A &= 12 \end{aligned}$$

Then, knowing the height is 15 in, determine the surface area.

$$\begin{aligned} SA &= Ph + 2B \\ SA &= 18 \times 15 + 2 \times 12 \\ SA &= 270 + 24 \\ SA &= 294 \end{aligned}$$

The answer is 294.

The surface area of the prism is 294 in^2.

Example 3

True or false: The surface area includes the inside of a prism.

The surface area is the measurement of the outer covering of a prism.

The answer is false.

Example 4

True or false: A net shows all three dimensions of a prism.

A net does show all dimensions of a prism.

The answer is true.

Example 5

True or false: You know a figure is a prism because the faces are rectangles.

A figure with all rectangular faces is a prism.

The answer is true.

PRACTICE

Look at each figure and then answer the following questions.
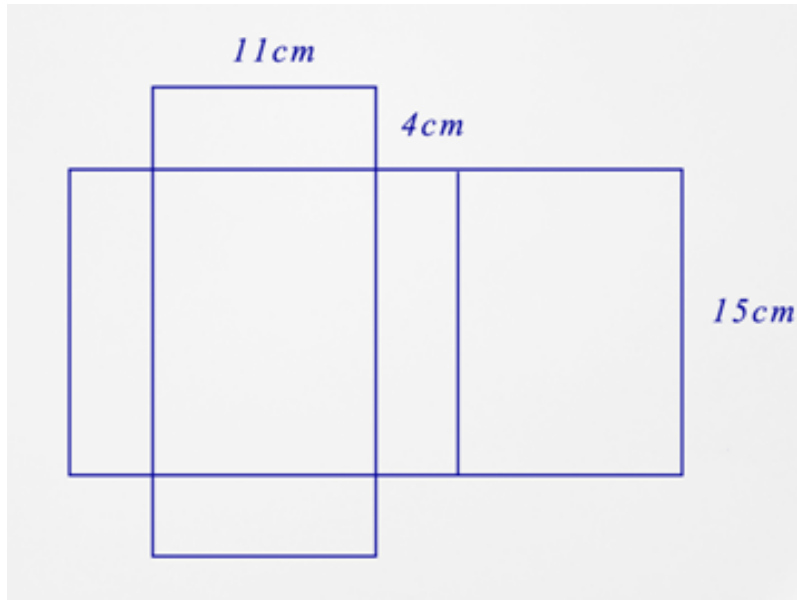
1. What is the name of the figure pictured above?

2. What is the surface area of this figure?

3. What is the shape of it's base?

4. What is the height of this figure?

5. What is the area of this figure's base?
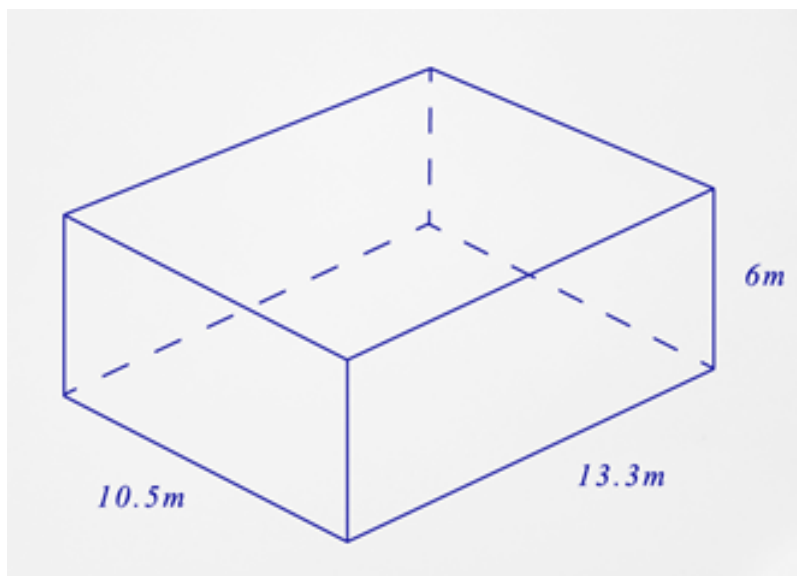
6. What is the name of this figure?

7. What is the surface area of this figure?

8. What is the shape of it's base?

9. What is the height of this figure?

10. What is the area of this figure's base?
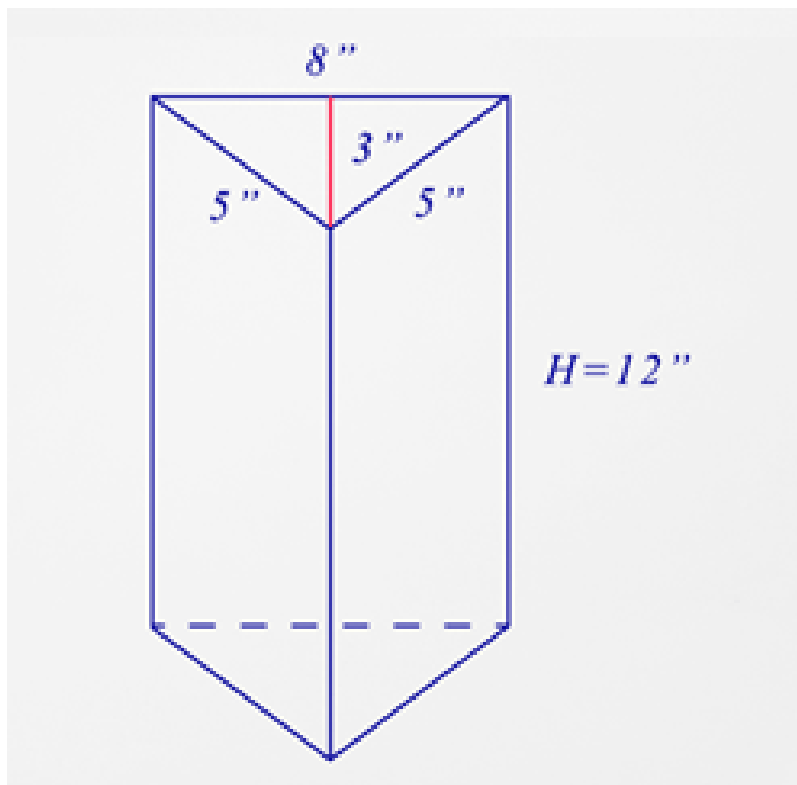
11. What is the name of this figure?

12. What is the shape of the base?

13. How many bases does this figure have?

14. How many side faces are there?

15. What is the surface area of this figure?

Resources
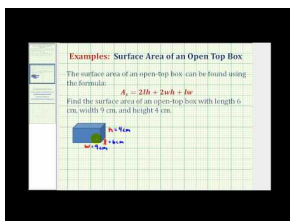
9.6 Lateral and Total Surface Area of Pyramids (7.9d)

GUIDANCE

Finding the Surface Area of a Pyramid

A **pyramid** has sides that are triangular faces and a base. The base can be any shape. Here are three examples:
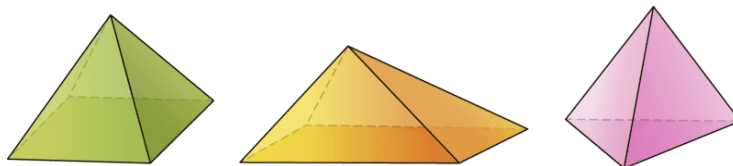
The **surface area** is the sum of all of the areas of the faces in a solid figure. Imagine wrapping a pyramid in wrapping paper, like a present. The amount of wrapping paper needed to cover the figure represents its surface area. To find the surface area, you must be able to calculate the area of each face and then add these areas together.

One way to find the surface area of a three-dimensional figure is by using a **net**. A **net** is a two-dimensional diagram of a three-dimensional figure. Imagine unfolding a pyramid so that it is completely flat. It would look something like this net.
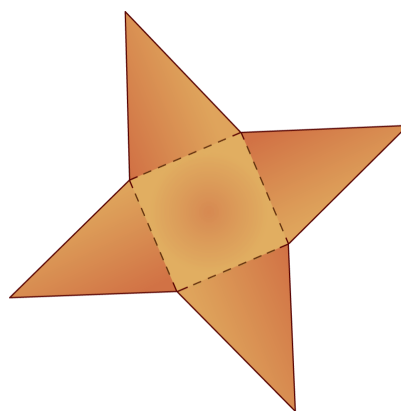
With a net, you can see each face of the pyramid more clearly. To find the surface area, you need to calculate the area for each face of the net, the sides and the base.

The side faces of a pyramid are always triangles, so the area formula for triangles is used to calculate their areas: $A = \frac{1}{2}bh$.

The area of the base depends on what shape it is. Remember, pyramids can have bases in the shape of a triangle, square, rectangle, or any other polygon. Use whichever area formula is appropriate for the shape. Here are some common formulas to find area.

$$\text{rectangle} : A = lw$$
$$\text{square} : A = s^2$$
$$\text{triangle} : A = \frac{1}{2}bh$$

Take a look at the pyramid below. Which formula should you use to find the area of the base?
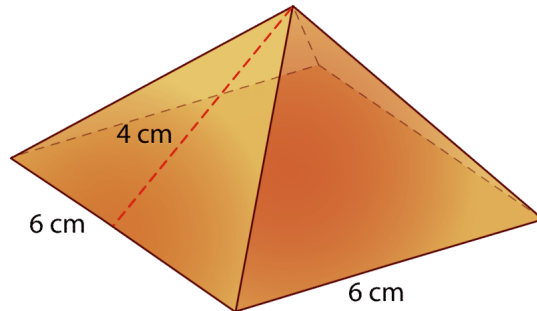
The base of the pyramid is a square with sides of 6 centimeters, so you should use the area formula for a square: $A = s^2$. Now find the area of the base.

$$A = 6^2$$
$$A = 36 \ sq.cm$$

Next, find the area of one of the triangles. You use the formula $A = \frac{1}{2}bh$. The base width is 6 cm and the slant height, the height of the side is 4 cm.

$$A = \frac{1}{2}(6)(4)$$
$$A = \frac{1}{2}(24)$$
$$A = 12 \ sq.cm$$

There are four triangles, so multiply this number by four and then add it to the area of the square.

$$SA = 4(12) + 36$$
$$SA = 48 + 36$$
$$SA = 84 \ sq.cm$$

Nets let you see each face of a pyramid so that you can calculate its area. However, you can also use a **formula** to represent the surface area of a pyramid. The formula is like a short cut because you can put all of the measurements in for the appropriate variable in the formula and solve for *SA*, surface area.

Here is the formula for finding the surface area of a pyramid.

$$SA = \frac{1}{2} \ \text{perimeter} \times \text{slant height} + B$$

The first part of the formula, $\frac{1}{2}$ perimeter $\times$ slant height, is a quick way of finding the area of all of the triangular sides of the pyramid at once. Remember, the area formula for a triangle is $A = \frac{1}{2}bh$. In the formula, b stands for base. The perimeter of the pyramid's bottom face represents all of the bases of the triangular faces at once, because it's their sum. The height of each triangle is always the same; this is called the slant height of the pyramid. Therefore "$\frac{1}{2}$ perimeter $\times$ slant height" is really the same as $\frac{1}{2}bh$.

The *B* in the formula represents the area of the base. Remember, pyramids can have bases of different shapes, so the area formula is used to find *B* varies. You find the area of the base first and then put it into the formula in place of *B*.

Let's look at an example.

What is the surface area of the pyramid below?
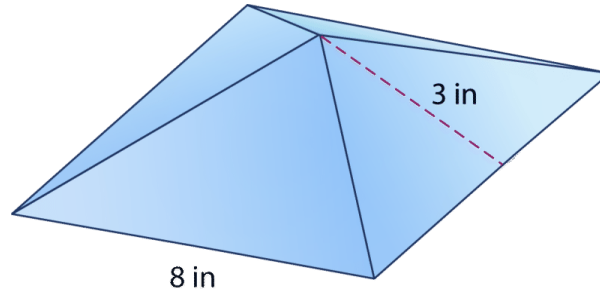
This is a square pyramid. The four sides of the base are all 8 inches, so the perimeter of the base is $8 \times 4 = 32$ *inches*. You will need to use the area formula for a square to find *B*.

$$B = s^2$$
$$B = (8)^2$$
$$B = 64 \ in.^2$$

Now you have all the information that you need. Put it into the formula and solve for *SA*, surface area.

$$SA = \frac{1}{2} \ \text{perimeter} \times \text{slant height} + B$$
$$SA = \left[\frac{1}{2}(32) \times 3\right] + 64$$
$$SA = (16 \times 3) + 64$$
$$SA = 48 + 64$$
$$SA = 112 \ in.^2$$

The answer is the surface area of the pyramid is 112 square inches.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Janice and Julian and their square pyramid tent.

They are planning for their tent to have a base of 6 feet wide and a slant height of 8 feet. How much canvas will they need?

First, calculate the area and perimeter of the square base:

$$B = s^2$$
$$B = 6^2$$
$$B = 36 \ ft^2$$

$$P = 4 \times 6$$
$$P = 24 \ ft$$

Next, plug the values into the formula for surface area of a pyramid and multiply the values together inside the brackets:

$$SA = \frac{1}{2} \ \text{perimeter} \times \text{slant height} + B$$
$$SA = \left[\frac{1}{2}(24) \times 8 \right] + 36$$

$$SA = (12 \times 8) + 36$$
$$SA = 96 + 36$$

Then, add the values together for the answer, making sure to include the appropriate unit of measurement:

$$SA = 96 + 36$$
$$SA = 132 \ ft^2$$

The answer is the surface area of the tent is 132 square feet. Janice and Julian will require 132 square feet of canvas to make their tent.

Example 2

Find the surface area of the figure below.
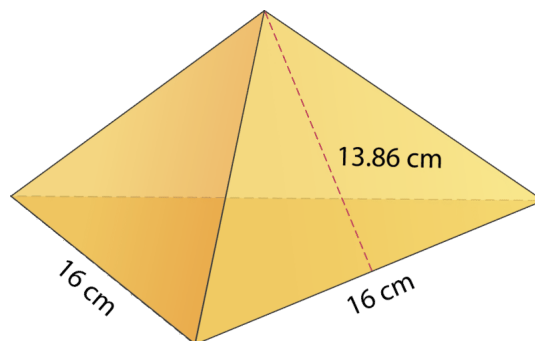
This is a triangular pyramid because its base is a triangle. That means you need to use the area formula for triangles to find B. First, calculate the area of the base and the perimeter. To do this, identify the shape of the base and use the appropriate formula for B:

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(16)(13.86)$$
$$B = 8(13.86)$$
$$B = 110.88 \ cm^2$$

The sides of the base are all the same length, so you can calculate the perimeter by multiplying

$16 \times 3 = 48$.

Next, plug the values into the formula for surface area of a pyramid and multiply the values together inside the brackets:

$$SA = \frac{1}{2} \text{ perimeter} \times \text{slant height} + B$$
$$SA = \left[\frac{1}{2}(48) \times 6\right] + 110.88$$

$$SA = (24 \times 6) + 110.88$$
$$SA = 144 + 110.88$$

Then, add the values together, making sure to include the appropriate unit of measurement:

$$SA = 144 + 110.88$$
$$SA = 254.88 \ cm^2$$

The answer is the surface area of this triangular pyramid is 254.88 square centimeters.

Example 3

Find the surface area of a square pyramid with side of 6 cm, slant height of 5 cm.

First, calculate the area and perimeter of the square base:

$$B = s^2$$
$$B = 6^2$$
$$B = 36 cm^2$$

$$P = 4 \times 6$$
$$P = 24 \ cm$$

Next, plug the values into the formula for surface area of a pyramid and multiply the values together inside the brackets:

$$SA = \frac{1}{2} \text{ perimeter} \times \text{slant height} + B$$
$$SA = \left[\frac{1}{2}(24) \times 5 \right] + 36$$

$$SA = (12 \times 5) + 36$$
$$SA = 60 + 36$$

Then, add the values together for the answer, remembering the appropriate unit of measurement.

$$SA = 60 + 36$$
$$SA = 96 \ cm^2$$

The answer is the surface area of the square pyramid is 96 cm^2.

Example 4

Find the surface area of a rectangular pyramid with a length of 6 in, a width of 4 in and a slant height of 3 in.

First, calculate the area and perimeter of the rectangular base:

$$B = lw$$
$$B = 6 \times 4$$
$$B = 24 \ in^2$$

$$P = 2l + 2w$$
$$P = 2(6) + 2(4)$$
$$P = 12 + 8$$
$$P = 20 \ in$$

Next, plug the values into the formula for surface area of a pyramid and multiply the values together inside the brackets:

$$SA = \frac{1}{2} \text{ perimeter} \times \text{slant height} + B$$

$$SA = \left[\frac{1}{2}(20) \times 3\right] + 24$$

$$SA = (10 \times 3) + 24$$
$$SA = 30 + 24$$

Then, add the values together for the answer. Remember the appropriate unit of measurement.

$$SA = 30 + 24$$
$$SA = 54 \text{ } in^2$$

The answer is 54 *in²*.

Example 5

Find the surface area of a square pyramid with side of 8 in and slant height of 9 in.

First, calculate the area and perimeter of the square base.

$$B = s^2$$
$$B = 8^2$$
$$B = 64 in^2$$

$$P = 4 \times 8$$
$$P = 32 \text{ } in$$

Next, plug the values into the formula for surface area of a pyramid and multiply the values together inside the brackets:

$$SA = \frac{1}{2} \text{ perimeter} \times \text{slant height} + B$$

$$SA = \left[\frac{1}{2}(32) \times 9\right] + 64$$

$$SA = (16 \times 9) + 64$$
$$SA = 144 + 64$$

Then, add the values together for the answer, remembering the appropriate unit of measurement:

$$SA = 144 + 64$$
$$SA = 208 \ in^2$$

The answer is the surface area of the square pyramid is 208 in^2.

PRACTICE

Find the surface area of each square pyramid. Remember that b means base and sh means slant height.

1. $b = 4 \ in, \ sh = 5 \ in$
2. $b = 4 \ in, \ sh = 6 \ in$
3. $b = 6 \ in, \ sh = 8 \ in$
4. $b = 5 \ in, \ sh = 7 \ in$
5. $b = 7 \ m, \ sh = 9 \ m$
6. $b = 8 \ m, \ sh = 10 \ m$
7. $b = 9 \ cm, \ sh = 11 \ cm$
8. $b = 11 \ m, \ sh = 13 \ m$
9. $b = 6 \ in, \ sh = 9 \ in$
10. $b = 10 \ cm, \ sh = 12 \ cm$

Find the surface area of each rectangular pyramid.

11. $l = 10 \ in, w = 8 \ in, sh = 6 \ in$
12. $l = 12 \ ft, w = 8 \ ft, sh = 10 \ ft$
13. $l = 4 \ in, w = 2 \ in, sh = 3 \ in$
14. $l = 16 \ m, w = 12 \ m, sh = 11 \ m$
15. $l = 15 \ in, w = 10 \ in, sh = 12 \ in$

9.7 Circumference, Area, and Surface Area Application Situations (7.9d)

1.) A cosmetics company that makes small cylindrical bars of soap wraps the bars in plastic prior to shipping. Find the surface area of a bar of soap if the diameter is 5 cm and the height is 2 cm. Use 3.14 for π.	2.) Chloe wants to wrap a present in a box for Sarah. The top and bottom of the box is 8 in. by 3 in., the sides are both 3 in. by 2 in. and the front and back are 8 in. by 2 in. How much wrapping paper will Chloe need to wrap the present?
3.) The lateral area of a cylinder is 94.2 cm². The height is 6 cm. What is the radius?	4.) Tara made a rectangular fence around her rose so it would not get hurt by her dog. She put a fence around the rose that was 4 ft long, 2 ft wide, and 3 ft high. How much material did she use to make her fence?

FIGURE 9.1

5.) A company is deciding which box to use for their merchandise. The first box measures 8 inches by 6.25 inches by 10.5 inches. The second box measures 9 inches by 5.5 inches by 11.75 inches. Which box required more material to make?	6.) If each box (from #5) used material that cost $0.03 per square inch to make, how much does a company save by choosing to make fifty boxes of the smaller box in comparison to fifty boxes of the larger box?

FIGURE 9.2
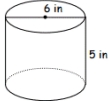
7.) Mary wants to make several cans like the one below. She is going to cut them from a sheet of metal that has an area of 3,750 in². How many can she make? Use 3.14 for π. 6 in 5 in	8.) Trey and Matt each have a rectangular prism. The base of Trey's prism is 4 cm by 7 cm and has a height of 12 cm. The base of Matt's prism is 6 cm by 10 cm and has a height of 3 cm. Whose rectangular prism will require more material to construct?
9.) A library has an aquarium in the shape of a rectangular prism. The base is 6 ft by 2.5 ft. The height is 4 ft. How many square feet of glass was used to build the aquarium?	10.) Marley bakes a rectangular cake that is 9 inches wide, 13 inches long and 2 inches high. She removes the cake from the pan to frost it. How many square inches of frosting does she need?

FIGURE 9.3

11.) Campbell Soup is creating a new soup label. If a can has a height of 6 in and a diameter of 4 in, how much material does Campbell need for each soup label?	12.) Diana wants to make several cans with a diameter of 6 inches and a height of 8 inches. She is going to cut the cans from a sheet of metal that has an area of 2,675 in². How many can she make?

13.) Mrs. Amrhein made a cylindrical pencil holder. It has a height of 5 in and a diameter of 3 in. How much material does she need to make her pencil holder?	14.) Grete opens and lays a shoe box out flat. She sees that the top and the bottom of the box both measure 10 inches by 6 inches, the sides of the box both measure 10 inches by 2 inches, and the front and back of the box both measure 6 inches by 2 inches. What is the surface area of the shoe box?
15a.) You are painting a room that is 18 ft long, 14 ft wide and 8 ft high. Find the area of the four walls that you are going to paint.	15b.) If the paint costs $6.50 a gallon and each gallon covers 128 ft² of wall, how much will it cost to paint the room?
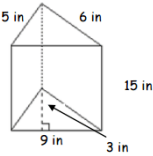
16.) Kaitlyn bakes two rectangular cakes to put on top of each other. Each cake is 6 inches wide, 12 inches long and 3 inches high. She removes the cake from the pan to frost it. How many square inches of frosting does she need for both cakes?	17.) Solve: 5 in 6 in 15 in 9 in 3 in

18.) Nam and Brandon each have a cylindrical canister of Pringles. Nam's cylinder has a diameter of 7 inches and a height of 13 inches. Brandon's cylinder has a diameter of 5 inches and a height of 20 inches. Whose cylinder has a larger surface area?	19.) A gymnasium is 45 ft long, 30 ft wide, and 14 ft high. The school is painting the four walls and the ceiling of the gym. Find the area of the painted parts of the gym.
20.) Use #18 to then find how much it costs to paint the room if the paint costs $8 a gallon and each gallon covers 300 ft of wall.	21.) Tommy and Ethan each have a cylinder. Tommy's cylinder has a diameter of 6 inches and a height of 8 inches. Ethan's cylinder has a diameter of 8 inches and a height of 6 inches. Whose cylinder has a larger surface area? Use 3.14 for π.
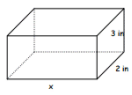
22.) Clayton wants to make a cylindrical case that will fit a bass drum with a height of 14 in. and a diameter of 24 in. What is the surface area of the case? Use 3.14 for π.	23.) **Challenge** Must try! If the lateral area of this shape is 54in², what is the value of x?

FIGURE 9.8

Vocabulary:

- Circumference
- Area
- Lateral Surface Area
- Total Surface Area
- Composite Figure
- Net
- Prism

CHAPTER **10**   # Volume of 3-D Figures

Chapter Outline

7.8a: model the relationship between the volume of a rectangular prism and a rectangular pyramid having both congruent bases and heights and connect that relationship to the formulas (not tested)

7.9a: solve problems involving the volume of rectangular prisms, triangular prisms, rectangular pyramids, and triangular pyramids

8.7a: solve problems involving the volume of cylinders, cones, and spheres

10.1 Relationship Between Volume of Prisms and Pyramids (7.8a)
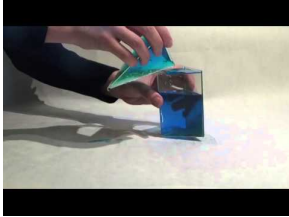
10.2 Rectangular Prisms (7.9a)

Finding the Volume of a Rectangular Prism

Volume is the measure of how much three-dimensional space an object takes up or holds.

Imagine a fish aquarium. Its length, width, and height determine how much water the tank will hold. If you fill it with water, the amount of water is the volume that the tank will hold. You measure volume in cubic units, because you are multiplying three dimensions: length, width, and height.

One way to find the volume of a prism is to consider how many unit cubes it can contain. A unit cube is simply a cube measuring one inch, one centimeter, one foot, or whatever unit of measurement you are using, on all sides. Here are some unit cubes.
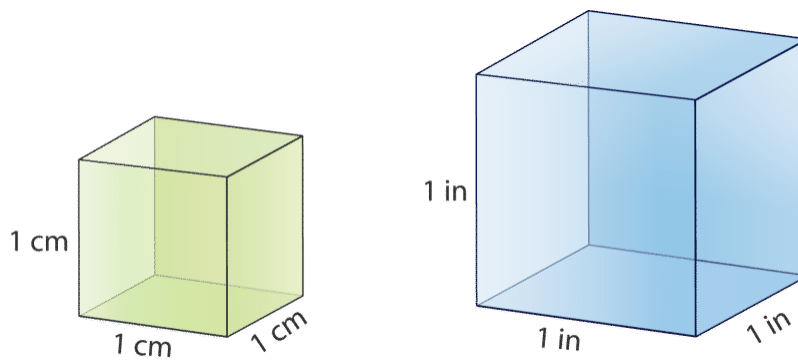
To use unit cubes to calculate volume, simply count the number of unit cubes that fit into the prism. Begin by counting the number of cubes that cover the bottom of the prism, and then count each layer. Let's see how this works.
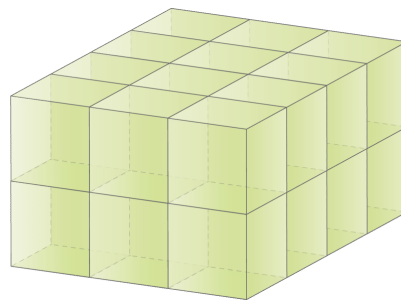
How many cubes do you see here? If you count all of the cubes, you will see that there are 24 cubes in this prism.

The volume of this prism is 24 $units^3$ or cubic units.

Find the volume of the following figure using unit cubes.
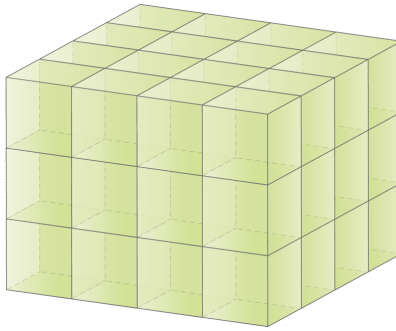
How many cubes are in this figure? If you count the cubes, you will get a total of 48 cubes. The volume of this prism is 48 cubic units or units^3.

If you look carefully, you will see that the volume of the rectangular prism is a function of multiplying the length $\times$ the width $\times$ the height. You just discovered the formula for finding the volume of a rectangular prism. Now let's refine that formula a little further. Here is the formula.

$V = Bh$

The volume is equal to the B, base area of the prism, times the height of the prism.

Let's look at an example.

Find the volume of the prism below.
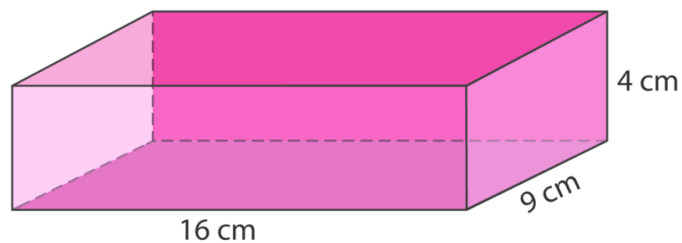
Simply put the values for the length, width, and height in for the appropriate variables in the formula, then solve for V, volume.

First find the area of the base. This is the rectangular side on the bottom. Remember, to find the area of a rectangle, multiply the length times the width.

$$B = lw$$
$$B = 16 \times 9$$
$$B = 144 \ cm^2$$

The base area is 144 square centimeters. Now multiply this by the height.

$$V = Bh$$
$$V = 144 \times 4$$
$$V = 576 \ cm^3$$

You can use the following formula for volume of a rectangular prism. This combines the two steps that you completed above:

$$V = lwh$$
$$V = (16)(9)(4)$$
$$V = 576 \ cm^3$$

The volume of this rectangular prism is 576 cubic centimeters.

You can work with the same rectangular prism, but fill it with unit cubes.
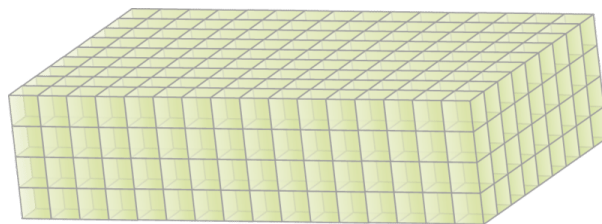
You can count the unit cubes here to find the volume of the rectangular prism. However, you save time by using the formula for volume.

Let's look at another example.

Find the volume of a container with a length of 15 ft, width of 12 ft, and height of 11 ft.

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (15)(12)(11)$$

$$V = (180)(11)$$

Next, multiply the results by the value for the height:

$$V = (180)(11)$$
$$V = 1,980$$

Then, record the answer including the appropriate unit of measurement:

$$V = 1,980 \ ft^3$$

The answer is the container has a volume of 1,980 cubic feet.

GUIDED PRACTICE

Examples

Example 1

Earlier, you were given a problem about Ben, who is searching for the box that will hold the most Legos.

Ben needs to figure out which of the following boxes has the greatest volume.

Box A: height = 5 inches, length = 18 inches, width = 10 inches

Box B: height = 8 inches, length = 10 inches, width = 12 inches

Box C: height = 6 inches, length = 14 inches, width = 10 inches

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

Box A: $V = (5)(18)(10)$
$V = (90)(10)$

Box B: $V = (8)(10)(12)$
$V = (80)(12)$

Box C: $V = (6)(14)(10)$
$V = (84)(10)$

Next, multiply the results by the value for the height:

Box A: $V = (90)(10)$
$V = 900$

Box B: $V = (80)(12)$
$V = 960$

Box C: $V = (84)(10)$
$V = 840$

Then, record the answer including the appropriate unit of measurement:

Box A: $V = 900 \ in^2$

Box B: $V = 960 \ in^2$

Box C: $V = 840 \ in^2$

The answer is Box B has the greatest volume and therefore can hold the most Legos.

Example 2

Carla is cleaning out her fish tank, so she filled the bathtub to the rim with water for her fish to swim in while she empties their tank. If the bathtub is 5.5 feet long, 3.3 feet wide, and 2.2 feet deep, how much water can it hold?

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (5.5)(3.3)(2.2)$$

$$V = (18.15)(2.2)$$

Next, multiply the results by the value for the height:

$$V = (18.15)(2.2)$$
$$V = 39.93$$

Then, record the answer including the appropriate unit of measurement:

$$V = 39.93 \ ft^3$$

The answer is Carla's bathtub can hold 39.93 cubic feet of water.

Example 3

Find the volume of a container with a length of 10 inches, width of 8 inches, and height of 6 inches.

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (10)(8)(6)$$

$$V = (80)(6)$$

Next, multiply the results by the value for the height:

$$V = (80)(6)$$
$$V = 480$$

Then, record the answer including the appropriate unit of measurement:

$$V = 480 \; in^3$$

The answer is the container has a volume of 480 cubic inches.

Example 4

Find the volume of a container with a length of 8 meters, width of 7 meters, and height of 3 meters.

First, plug the values of the dimensions into the formula for volume of a rectangular prism and multiply the values for length and width:

$$V = lwh$$
$$V = (8)(7)(3)$$

$$V = (56)(3)$$

Next, multiply the results by the value for the height:

$$V = (56)(3)$$
$$V = 168$$

Then, record the answer including the appropriate unit of measurement:

$$V = 168 \; m^3$$

The answer is the container has a volume of 168 cubic meters.

PRACTICE

Find the volume of each rectangular prism. Remember to label your answer in cubic units.

1. Length = 5 in, width = 3 in, height = 4 in
2. Length = 7 m, width = 6 m, height = 5 m
3. Length = 8 cm, width = 4 cm, height = 9 cm
4. Length = 8 cm, width = 4 cm, height = 12 cm
5. Length = 10 ft, width = 5 ft, height = 6 ft
6. Length = 9 m, width = 8 m, height = 11 m
7. Length = 5.5 in, width = 3 in, height = 5 in
8. Length = 6.6 cm, width = 5 cm, height = 7 cm

9. Length = 7 ft, width = 4 ft, height = 6 ft
10. Length = 15 m, width = 8 m, height = 10 m
11. Length = 10.5 m, width = 11 m, height = 4 m
12. Length = 12 ft, width = 12 ft, height = 8 ft
13. Length = 16 in, width = 8 in, height = 8 in
14. Length = 12 m, width = 12 m, height = 12 m
15. Length = 24 in, width = 6 in, height = 6 in

10.3 Triangular Prisms (7.9a)

In this concept, you will learn how to calculate the volume of triangular prisms.

GUIDANCE

Finding the Volume of a Triangular Prism

With a triangular prism, the two parallel faces are triangles and the other faces are rectangles.
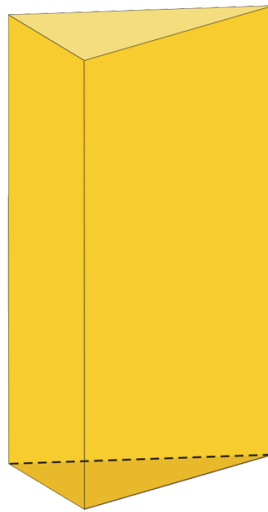
You calculate the volume of triangular prisms almost the same way that you find the volume of rectangular prisms. You still use the formula $V = Bh$. However, this time the base of the prism is a triangle, not a rectangle. Therefore, you need to use the area formula for a triangle to find the area of the base, B. Then you can multiply this amount by the height of the rectangle to find the volume of the triangular prism.
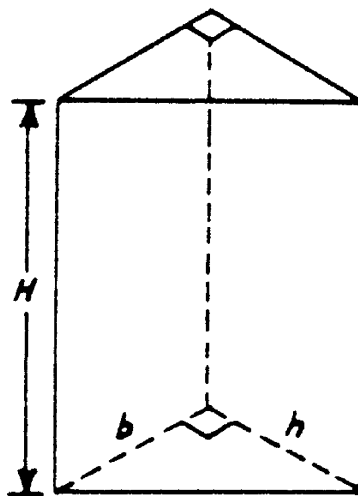
Let's look at an example.

What is the volume of this triangular prism where the height of the triangular base is 7 cm and the width of the base of the triangle is 5 cm?

First ,you need to find the area of the triangular base.   You use the formula for the area of a triangle which is $\frac{1}{2}bh$. Remember, you use the height and base measurements for the triangular face, not the height, H, measurement for the whole prism which is the length of the rectangle.

So there are two things that you need to accomplish: You need to find the area of one of the triangular bases, and then you can take that measurement and multiply it with the height of the entire prism.

$$V = BH$$
$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(5)(7)$$
$$B = 35$$
$$V = (35)H$$
$$V = (35)(12)$$
$$V = 420 \ cm^3$$

The answer is the volume of this triangular prism is 420 $centimeters^3$.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about how Maggie wants to find the volume of chocolate that will fit inside the packaging.

The triangular base is 1.4 inches; the height of the triangle is 1.25 inches, and the height of the bar when stood on its end is 8.25 inches.

First, plug in the values for the area formula of a triangle to find the base area, B.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(1.4)(1.25)$$
$$B = 0.7(1.25)$$
$$B = 0.88 \ in^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 0.88 \times 8.25$$

433

$$V = 0.88 \ \times \ 8.25$$
$$V = 7.26$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 7.26 \ in^3$$

The answer is the package has a volume of 7.26 cubic inches.

Example 2

Every year Jeanie gets a bottle of her favorite perfume for her birthday. The perfume comes in a bottle shaped like a triangular prism. How much perfume does the bottle hold when it is full?
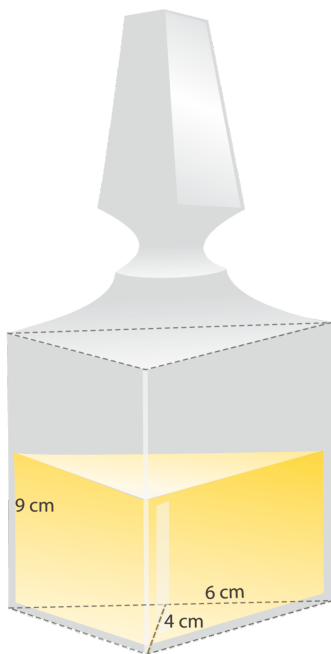
First, plug in the values for the area formula of a triangle to find the base area, B.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(6)(4)$$
$$B = 3(4)$$
$$B = 12 \ cm^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 12 \times 9$$

$$V = 12 \ \times \ 9$$
$$V = 108$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 108 \ cm^3$$

The answer is the perfume bottle holds 108 cubic centimeters of perfume when full.

Example 3

What is the volume of a triangular prism with the following dimensions: $b = 12 \ in, \ h = 10 \ in, \ H = 15 \ in$

First, plug in the values for the area formula of a triangle to find the base area, B.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(12)(10)$$
$$B = 6(10)$$
$$B = 60 \ in^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 60 \times 15$$

$$V = 60 \ \times \ 15$$
$$V = 900$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 900 \ in^3$$

The answer is the volume of this triangular prism is 900 in^3.

Example 4

What is the volume of a triangular prism with the following dimensions: $b = 7\ cm$, $h = 5\ cm$, $H = 9\ cm$

First, plug in the values for the area formula of a triangle to find the base area, B.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(7)(5)$$
$$B = 3.5(5)$$
$$B = 17.5\ cm^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 17.5 \times 9$$

$$V = 17.5 \times 9$$
$$V = 157.5$$

Then, record the answer, making sure to include the appropriate unit of measurement.

$$V = 157.5\ cm^3$$

The answer is the volume of this triangular prism is $157.5\ cm^3$.

Example 5

What is the volume of a triangular prism with the following dimensions: $b = 4\ mm$, $h = 3\ mm$, $H = 5\ mm$

First, plug in the values for the area formula of a triangle to find the base area, B.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(4)(3)$$
$$B = 2(3)$$
$$B = 6\ mm^2$$

Next, plug the triangle area value, B, and the value for the height of the prism, H, into the volume formula and multiply the values together.

$$V = BH$$
$$V = 6 \times 5$$

$$V = 6 \times 5$$
$$V = 30$$

Last, record the answer, making sure to include the appropriate unit of measurement.

$$V = 30 \ mm^3$$

The answer is the volume of this triangular prism is 30 mm^3.

PRACTICE

Find the volume of each triangular prism. Remember that h means the height of the triangular base and H means the height of the whole prism.

1. $b = 6 \ in, \ h = 4 \ in, \ H = 5 \ in$
2. $b = 7 \ in, \ h = 5 \ in, \ H = 9 \ in$
3. $b = 10 \ m, \ h = 8 \ m, \ H = 9 \ m$
4. $b = 12 \ m, \ h = 10 \ m, \ H = 13 \ m$
5. $b = 8 \ cm, \ h = 6 \ cm, \ H = 9 \ cm$
6. $b = 9 \ cm, \ h = 7 \ cm, \ H = 8 \ cm$
7. $b = 5.5 \ mm, \ h = 4 \ mm, \ H = 4 \ mm$
8. $b = 11 \ cm, \ h = 9 \ cm, \ H = 8 \ cm$
9. $b = 20 \ ft, \ h = 17 \ ft, \ H = 19 \ ft.$
10. $b = 20 \ ft, \ h = 18 \ ft, \ H = 15 \ ft.$
11. $b = 18 \ ft, \ h = 16 \ ft, \ H = 17 \ ft.$
12. $b = 24 \ ft, \ h = 21 \ ft, \ H = 19 \ ft.$
13. $b = 24.5 \ ft, \ h = 18 \ ft, \ H = 16 \ ft.$
14. $b = 99 \ ft, \ h = 80 \ ft, \ H = 75 \ ft.$
15. $b = 100 \ ft, \ h = 80 \ ft, \ H = 110 \ ft.$

10.4 Rectangular and Triangular Pyramids (7.9a)

GUIDANCE

Finding Volume of Cylinders

Volume is the amount of space contained within a solid figure. In the case of cylinders, you can think of volume as the capacity, or the amount of a liquid or substance, it can hold.

Unlike prisms, unit cubes do not fit nicely into a cylinder.
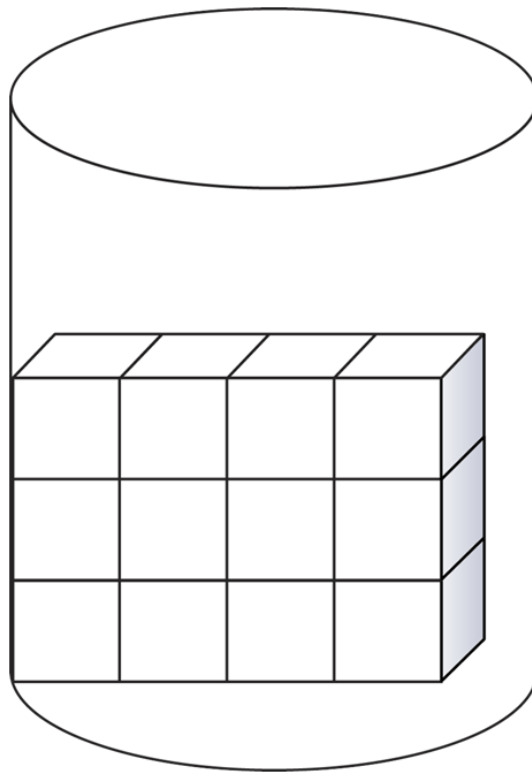
So, to calculate the volume of a cylinder accurately, you need to use a formula instead of unit cubes.

To calculate the volume of a cylinder, start by calculating the area of the circular base. That will give you a measure for the number of unit cubes that can fit across the bottom of the cylinder. The height of the cylinder will show how high cubes can be stacked inside the cylinder.

Here is the formula for finding the volume of a cylinder.

$$V = Bh$$

Let's apply this formula.
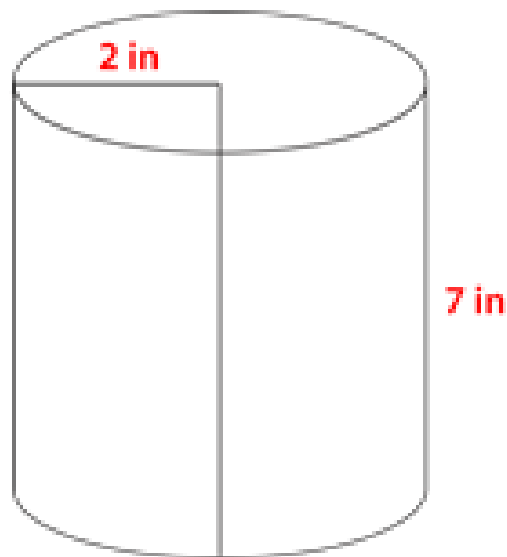
The radius of the circular base is 2 inches. The height of the cylinder is 7 inches. If you take both of these given measures and substitute them into the formula, you can solve for the volume of the cylinder.

$$
\begin{aligned}
V &= \pi r^2 h \\
V &= (3.14)(2^2)(7) \\
V &= (3.14)(4)(7) \\
V &= (3.14)(28) \\
V &= 87.92 \ in^3
\end{aligned}
$$

The volume of the cylinder is $87.92 \ in^3$.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Ramona and her extra fruit preserve.

Ramona has two food storage containers, but isn't sure which will hold the most. How can Raymond use these dimensions to figure out which will hold the most?
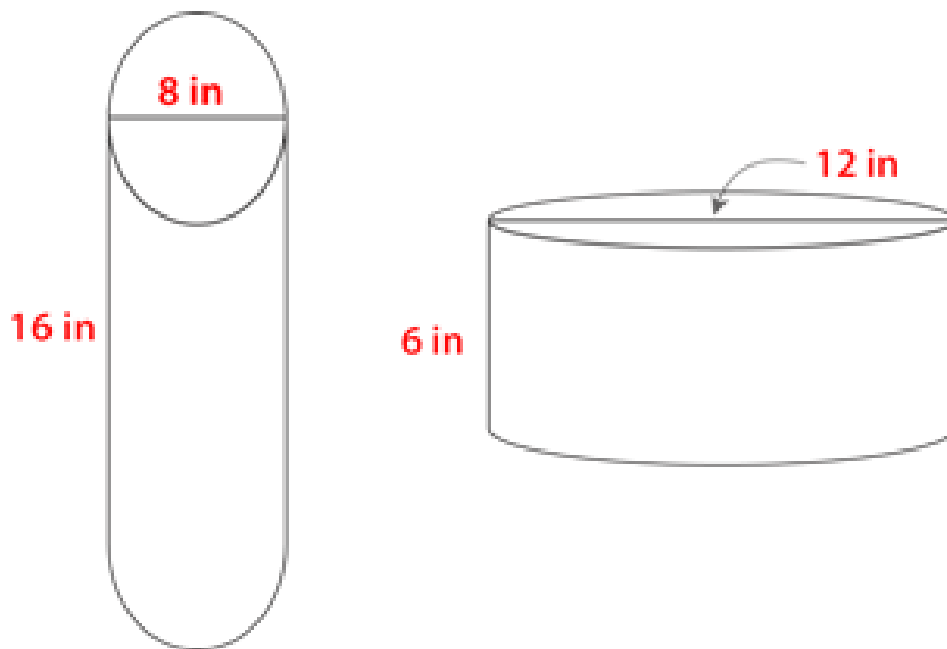
Start with the container on the left.

First, note that the diameter is twice the radius, so divide the diameter by 2 to get the radius.

$8 \div 2 = 4$

Radius is 4.

Next, substitute the given information into the formula then solve.

$$
\begin{aligned}
V &= \pi r^2 h \\
V &= \pi (4)^2 (16) \\
V &= \pi (16)(16) \\
V &= 256\pi \\
V &= 803.84 \ in.^3
\end{aligned}
$$

The first container holds 803.84 cubic inches.

Next, consider the other container.

First, note that the diameter is twice the radius, so divide the diameter by 2 to get the radius.

$12 \div 2 = 6$

Radius is 6.

Next, substitute the given information into the formula then solve.

$$
\begin{aligned}
V &= \pi r^2 h \\
V &= \pi (6)^2 (6) \\
V &= \pi (36)(6) \\
V &= 216\pi \\
V &= 678.24 \ in.^3
\end{aligned}
$$

The second container holds 678.24 cubic inches.

The container on the left has the greatest volume.

Example 2

Use the formula for finding the volume of a cylinder to answer the following question.

A water tank has a radius of 50 feet and a height of 400 feet. How many cubic feet of water will the tank hold when it is full?

First, substitute the given information into the formula and solve.

$$
\begin{aligned}
V &= \pi r^2 h \\
V &= \pi(50^2)(400) \\
V &= \pi(2,500)(400) \\
V &= 1,000,000\pi \\
V &= 3,140,000 \ in.^3
\end{aligned}
$$

The water tank will hold more than 3 *million* cubic feet of water!

Example 3

Find the volume of the cylinder.
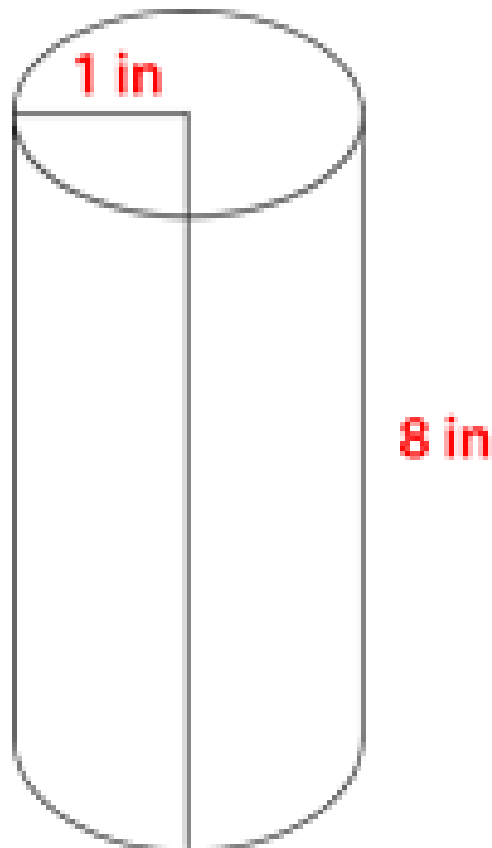
First, substitute the given information into the formula then solve.

$$\begin{aligned} V &= \pi r^2 h \\ V &= \pi(1^2)(8) \\ V &= \pi(1)(8) \\ V &= 8\pi \\ V &= 25.12 \ in.^3 \end{aligned}$$

The answer is 25.12 cubic inches.

Example 4

Find the volume of the cylinder.

$d = 10 \ ft, h = 12 \ ft$

First, note that the diameter is twice the radius, so divide the diameter by 2 to get the radius.

$10 \div 2 = 5$

Radius is 5.

Next, substitute the given information into the formula then solve.

$$\begin{aligned} V &= \pi r^2 h \\ V &= \pi(5)^2(12) \\ V &= \pi(25)(12) \\ V &= 300\pi \\ V &= 942 \ ft.^3 \end{aligned}$$

The answer is 942 cubic feet.

Example 5

Find the volume of the cylinder.

$r = 6 \ in, h = 10 \ in$

First, substitute the given information into the formula then solve.

$$\begin{aligned} V &= \pi r^2 h \\ V &= \pi(6)^2(10) \\ V &= \pi(36)(10) \\ V &= 360\pi \\ V &= 1130.4 \ in.^3 \end{aligned}$$

The answer is 1130.4 cubic inches.

PRACTICE

Find the volume of each of the following cylinders.

1. $r = 5 \ in, h = 8 \ in$

2. $r = 4 \ in, h = 7 \ in$
3. $r = 3 \ ft, h = 5 \ ft$
4. $r = 3 \ ft, h = 8 \ ft$
5. $r = 4 \ cm, h = 9 \ cm$

6.

3 in

h=8 in

7.

5 m

h=9 m

2 in

h=10 in

8.

6 cm

7 cm

9.

3 ft

h=11 ft

10.

8 in

h=8 in

11.

4 mm

h=12 mm

12.

12 in

6 in

13.

10 cm

h=11 cm

14.

20 ft

h=10 ft

15.
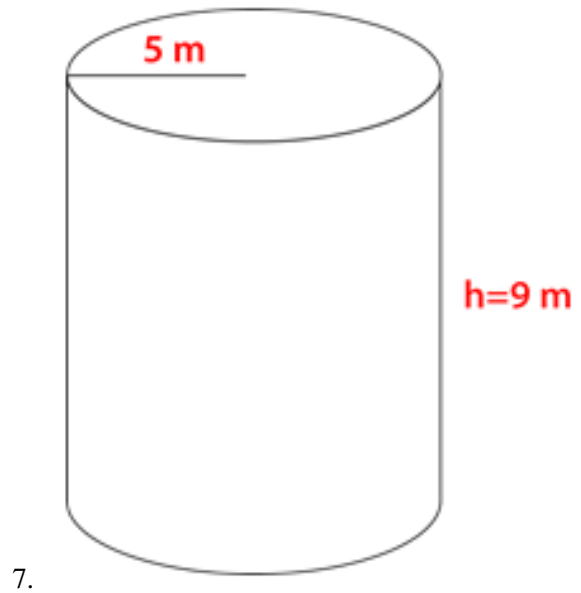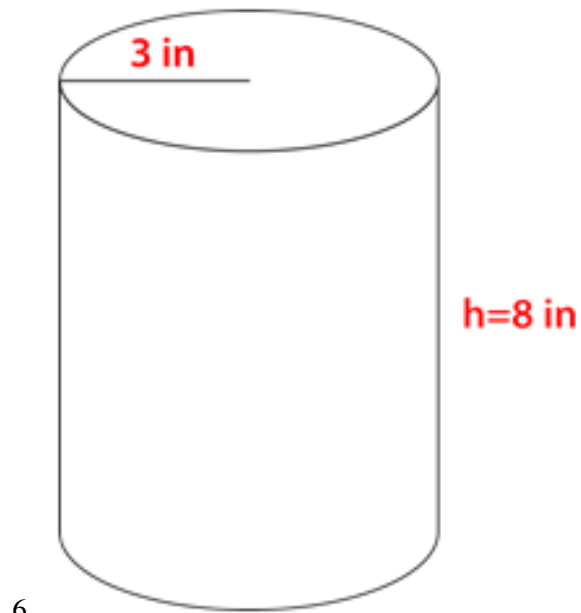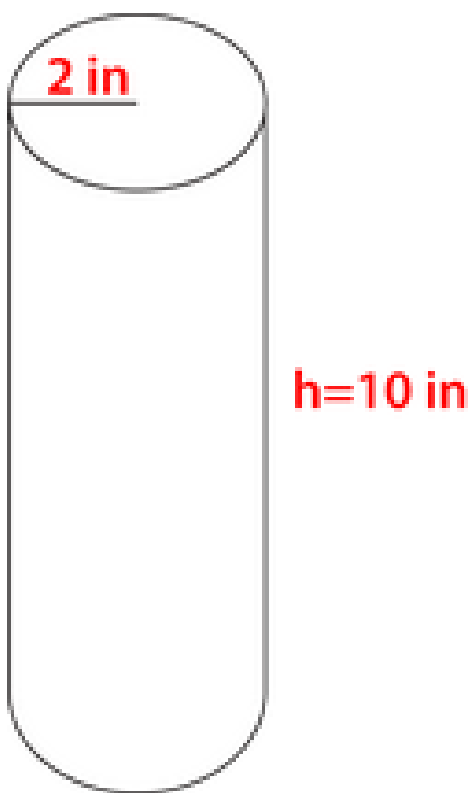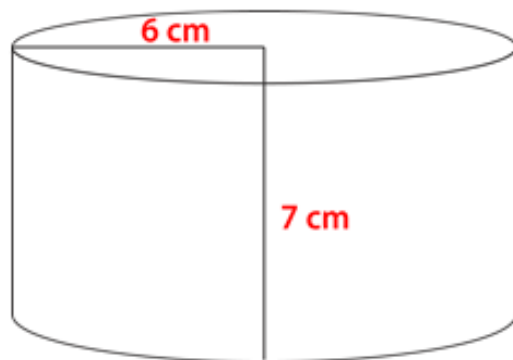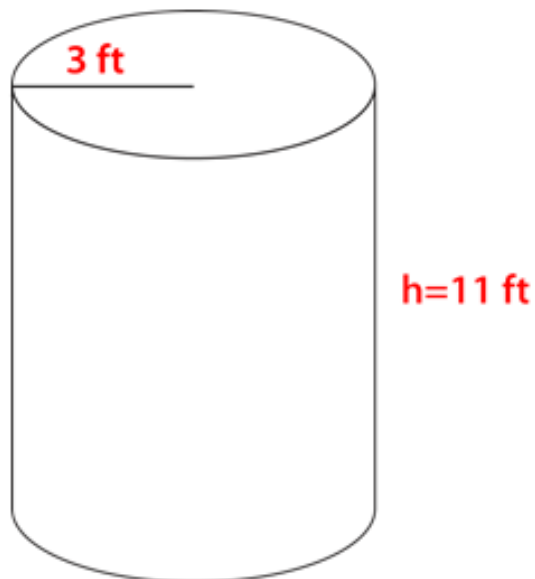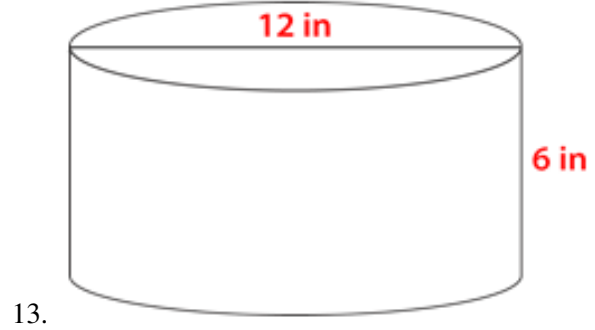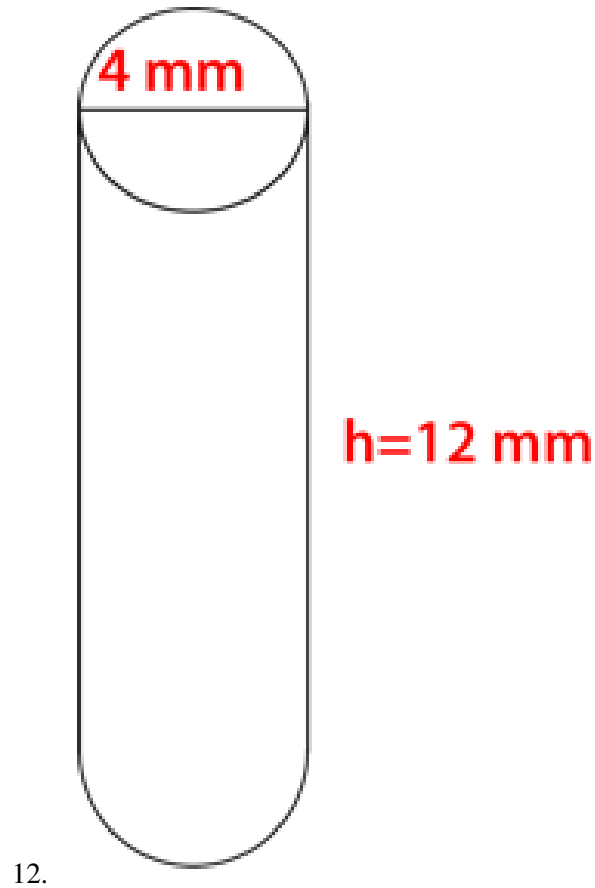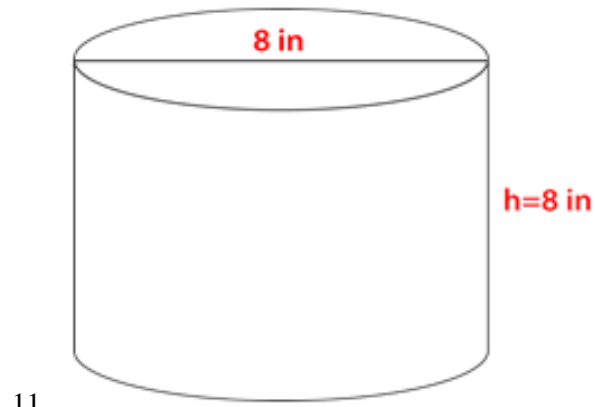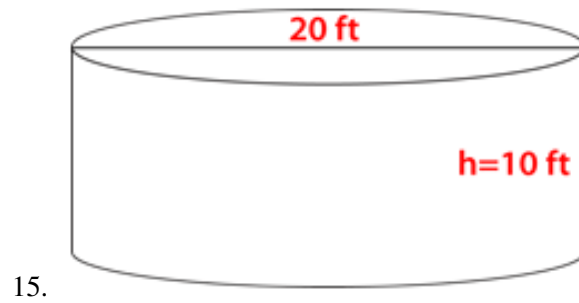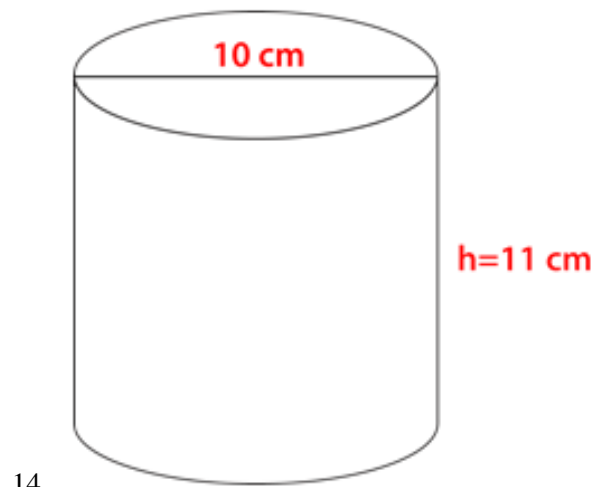
10.5 Cylinders (8.7a)

Finding the Volume of a Pyramid

Pyramids are solid shapes that exist in three-dimensional space. A pyramid has sides that are triangular faces and a base that can be any shape.
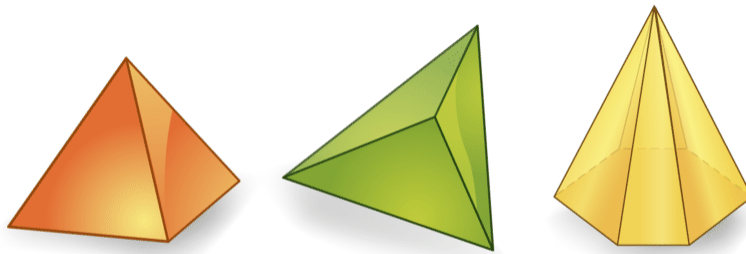
Volume is the measure of how much space a three-dimensional figure takes up or holds. Volume is measured in cubic units, because you are dealing with three dimensions: length, width, and height.

You can start to understand the volume of a pyramid by comparing a pyramid with a figure that is similar to it. You know that a prism has length, width and height. The parallel bases of a prism can be any polygon. This one has a square base; so it is called a cube.
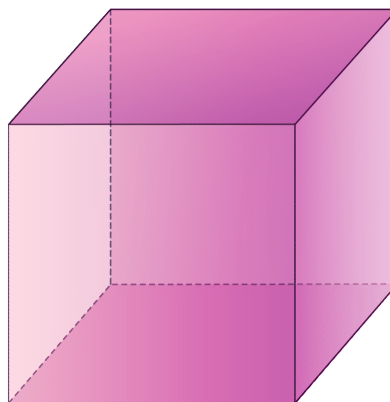
Now think about a pyramid. It has a base that can be any polygon. The base of this pyramid is a square.
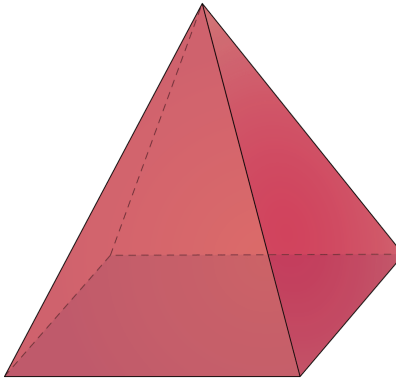
These are both similar given that they have square bases. Think about the volume of a pyramid by looking at the volume of the cube or prism. Look at this picture that demonstrates the relationship of a cube to a square pyramid:
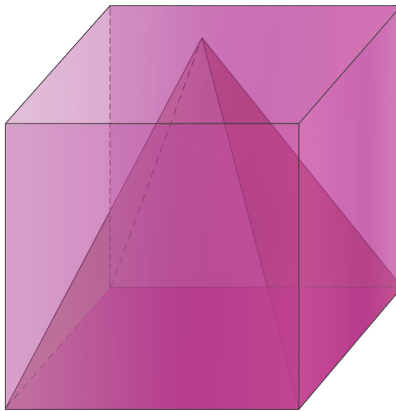
To find the volume of this cube, multiply the base times the width times the height. To find the volume of the pyramid, take the area of the base, B and multiply it times the height and then multiply it by $\frac{1}{3}$. Here is the formula for finding the volume of a pyramid.

$V = \frac{1}{3}Bh$

One thing to keep in mind is that pyramids can be tricky because they can have many different bases. B in the formula means that you need to find the area of the base. If the base is a square, you will need to use the formula for area of a square, $B = s^2$. If the base is a rectangle, the area of the base is $B = lw$. If the base is a triangle, use the formula $B = \frac{bh}{2}$. Making sure that you have the correct formula is essential in your work with pyramids.

Let's look at an example. What is the volume of the pyramid below?
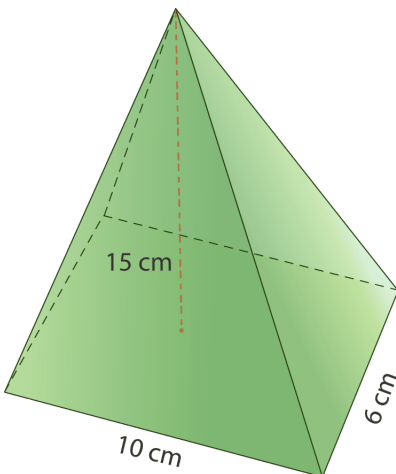
First, decide what shape the base of the pyramid is. One side is 10 centimeters and the other is 6 centimeters, so it must be a rectangle. Use the area formula for rectangles to find B, the base area.

$$B = lw$$
$$B = 10(6)$$
$$B = 60 \ cm^2$$

The area of this pyramid's base is 60 square centimeters. Now multiply this by the height and $\frac{1}{3}$ according to the formula.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(60)(15)$$
$$V = \frac{900}{3}$$
$$V = 300 \ cm^3$$

The answer is the volume of the pyramid is 300 cubic centimeters.

Remember, you measure volume in three dimensions, so write the answer in cubic units.

Let's look at another example.

Find the volume of the figure below.
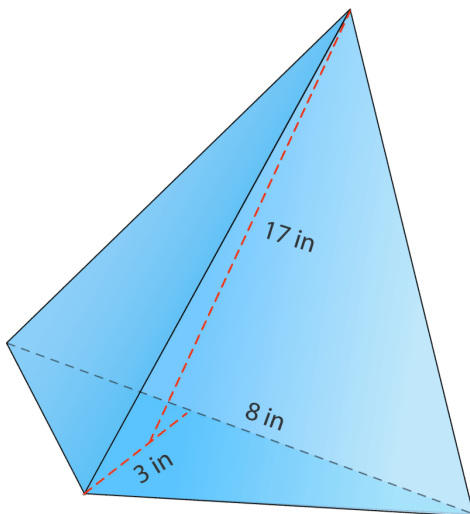
This time the base is a triangle, so you'll need to use the area formula for triangles to find the base area. Be careful not to confuse the height of the base triangle with the height of the whole pyramid.

$$B = \frac{1}{2}bh$$
$$B = \frac{1}{2}(8)(3)$$
$$B = \frac{24}{2}$$
$$B = 12 \ in.^2$$

The base area for this triangular pyramid is 12 square inches. Put this into the formula and solve for V, volume.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(12)(17)$$
$$V = \frac{204}{3}$$
$$V = 68 \ in.^3$$

The answer is the volume of this pyramid is 68 cubic inches.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Michael and Juanita's interest in the sand sculpture pyramid.

They want to figure out the volume of sand required to create a sand sculpture pyramid with base 4 feet and height 5 feet:

First, determine the shape of the base and use the correct formula to find the base area. Since the base is a square, use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (4)^2$$
$$B = 16 \ ft^2$$

The base area of the pyramid is 16 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(16)(5)$$

$$V = \frac{80}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{80}{3}$$
$$V = 26.67 \ ft^3$$

The answer is the volume of sand required to create the sand sculpture pyramid is 26.67 cubic feet.

Example 2

Felice bought the candle below for her friend's birthday. The package says that the candle burns one hour for every 20 cubic centimeters of wax. What is the volume of the candle so that Felice can figure out how long it will burn?
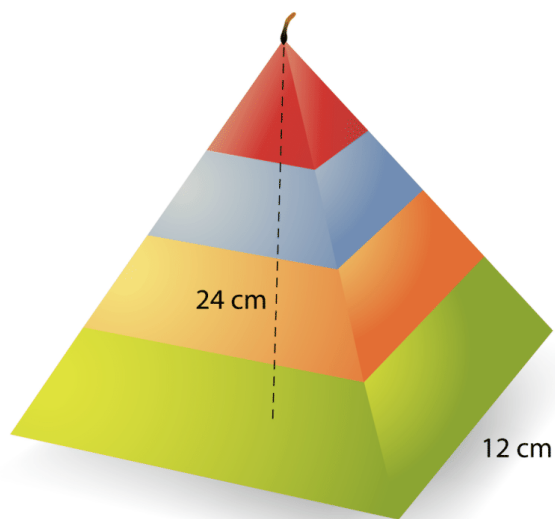
First, determine the shape of the base and use the correct formula to find the base area.   Since the base is a square, you use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (12)^2$$
$$B = 144 \ cm^2$$

The base area of the pyramid is 144 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(144)(24)$$

$$V = \frac{3456}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{3456}{3}$$
$$V = 1,152 \ cm^3$$

The answer is the volume of the candle is 1,152 cubic cm.

Example 3

Find the volume of a square pyramid with a base of 8 cm and a height of 6 cm.

First, determine the shape of the base and use the correct formula to find the base area. The base is a square, so use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (8)^2$$
$$B = 64 \ cm^2$$

The base area of the pyramid is 64 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(64)(6)$$

$$V = \frac{384}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{384}{3}$$
$$V = 128 \ cm^3$$

The answer is the volume of the pyramid is 128 cubic cm.

Example 4

Find the volume of a rectangular pyramid with a length of 10 cm, a width of 8 cm and a height of 9 cm.

First, determine the shape of the base and use the correct formula to find the base area. Since the base is a rectangle, use the area formula for squares to find the base area.

$$B = lw$$
$$B = (10)(8)$$
$$B = 80 \ cm^2$$

The base area of the pyramid is 80 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(80)(9)$$

$$V = \frac{720}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{720}{3}$$
$$V = 240 \ cm^3$$

The answer is the volume of the pyramid is 240 cubic cm.

Example 5

Find the volume of a square pyramid with a base of 5.5 in and a height of 4 in.

First, determine the shape of the base and use the correct formula to find the base area. The base is a square, so use the area formula for squares to find the base area.

$$B = s^2$$
$$B = (5.5)^2$$
$$B = 30.25 \ in^2$$

The base area of the pyramid is 30.25 square centimeters.

Next, plug the area of the base, and the height of the pyramid into the formula for volume of a pyramid and multiply the values together.

$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(30.25)(4)$$

$$V = \frac{121}{3}$$

Then, divide by 3 for the answer, making sure to include the appropriate unit of measurement.

$$V = \frac{121}{3}$$
$$V = 40.33 \ in^3$$

The answer is the volume of the pyramid is 40.33 cubic inches.

PRACTICE

Find the volume of each of the following pyramids.

1. A square pyramid with a base of 6 ft and a height of 9 ft.
2. A square pyramid with a base of 8 m and a height of 10 m.
3. A square pyramid with a base of 11 in and a height of 13 in.
4. A square pyramid with a base of 9 ft and a height of 14 ft.
5. A square pyramid with a base of 4.5 in and a height of 5 inches.
6. A rectangular pyramid with a base length of 4 in, a base width of 3 in and a height of 5 in.
7. A rectangular pyramid with a base length of 5 ft, a base width of 4 ft and a height of 6 ft.
8. A rectangular pyramid with a base length of 7 m, a base width of 4 m and a height of 9 m.
9. A triangular pyramid with a base length of 5 in and a base height of 4 inches with a pyramid height of 6 inches.
10. A triangular pyramid with a base length of 8 ft and a base height of 7 ft with a pyramid height of 9 ft.
11. A square pyramid with a base of 8 feet and height of 4 feet.
12. A rectangular pyramid with a length of 5 inches, a width of 4 inches and a height of 6 inches.
13. A square pyramid with a base of 3.5 feet and a height of 6.5 feet.
14. A square pyramid with a base of 6.5 feet and a height of 8.5 feet.
15. A rectangular pyramid with a width of 4 feet, a length of 6 feet and a height of 7.5 feet.

10.6 Cones (8.7a)

1. The base of a pyramid has an area B of 48 square millimeters and a height h of 13 millimeters. What is the volume of the pyramid?

2. The measure of each edge of a cube is represented by (2y - 3) centimeters. Find the volume of the cube when y = 7.25.

3. The height of a pyramid is 4 inches, and the base is a rectangle 6 inches long and 3.5 inches wide. What is the volume of the pyramid?

4. A right circular cylinder has a base with a radius of 24.1 centimeters and a height of 17.3 centimeters.

    1. a) Express the volume of the cylinder in terms of π.

    1. b) Find the volume to the nearest hundred cubic centimeters.

5. The base of a cone has a radius of 7 inches. The height of the cone is 5 inches.

   1. a) Find the volume of the cone in terms of π.

   1. b) Find the volume to the nearest cubic inch.

6. A sphere has a diameter of 4 feet.

   1. a) Find the volume in terms of π.

   1. b) Find the volume to the nearest cubic foot.

7. A tank in the form of a right circular cylinder is used for storing water. It has a diameter of 12 feet and a height of 14 feet. How many gallons of water will it hold (1 cubic foot contains 7.5 gallons).

8. Four pieces of cardboard that are 8.0 inches by 12 inches and two pieces that are 12 inches by 12 inches are used to form a rectangular solid.

   1. a) Find the volume in cubic inches.

1. b) Find the volume in cubic feet.

9. An official handball has a diameter of 4.8 centimeters.

1. a) Find the volume to the nearest cubic centimeter

1. b) Find the volume to the nearest cubic millimeters.

10. The largest pyramid in the world was built around 2500 B.C. by Khufu, or Cheops, a king of ancient Egypt. The pyramid had a square base 230 meters (756 feet) on each side, an da height of 147 meters (482 feet). Find the volume of Cheops' pyramid:

1. a) in cubic meters

1. b) in cubic feet

Vocabulary:

- Volume
- Prisms
- Cylinders
- Pyramids

CHAPTER **11**

Probability

Chapter Outline

7.6a: represent sample spaces for simple and compound events using lists and tree diagrams

7.6d: make predictions and determine solutions using theoretical probability for simple and compound events

7.6i: determine experimental and theoretical probabilities related to simple and compound events using data and sample spaces

7.6c: make predictions and determine solutions using experimental data for simple and compound events

7.6h: solve problems using qualitative and quantitative predictions and comparisons from simple experiments

11.1 Sample Spaces (7.6a)

GUIDANCE

Basic Terminology and Probability Rules

An **event** is something that occurs, or happens. For example, flipping a coin is an event, and so is walking in the park and passing by a bench. Anything that could possibly happen is an event.

Every event has one or more possible outcomes. While tossing a coin is an event, getting tails is the outcome of that event. Likewise, while walking in the park is an event, finding your friend sitting on the bench is an outcome of that event.

Suppose a coin is tossed once. There are two possible outcomes, either heads, H, or tails, T. Notice that if the experiment is conducted only once, you will observe only one of the two possible outcomes. An **experiment** is the process of taking a measurement or making an observation. These individual outcomes for an experiment are each called **simple events**.

A die has six possible outcomes: 1, 2, 3, 4, 5, or 6. When we toss it once, only one of the six outcomes of this experiment will occur. The one that does occur is called a simple event.

Suppose that two pennies are tossed simultaneously. We could have both pennies land heads up (which we write as HH), or the first penny could land heads up and the second one tails up (which we write as HT), etc. We will see that there are four possible outcomes for each toss, which are HH, HT, TH, and TT. The table below shows all the possible outcomes.

	H	T
H	HH	HT
T	TH	TT

Figure: The possible outcomes of flipping two coins.

What we have accomplished so far is a listing of all the possible simple events of an experiment. This collection is called the **sample space** of the experiment.

The sample space is the set of all possible outcomes of an experiment, or the collection of all the possible simple events of an experiment. We will denote a sample space by S.

As we know, there are 6 possible outcomes for throwing a die. We may get 1, 2, 3, 4, 5, or 6, so we write the sample space as the set of all possible outcomes:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Similarly, the sample space of tossing a coin is either heads, H, or tails, T, so we write $S = \{H, T\}$.
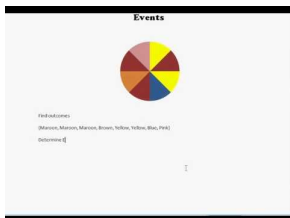
Determining the Sample Space

Suppose a box contains three balls, one red, one blue, and one white. One ball is selected, its color is observed, and then the ball is placed back in the box. The balls are scrambled, and again, a ball is selected and its color is observed. What is the sample space of the experiment?

It is probably best if we draw a **tree diagram** to illustrate all the possible selections.
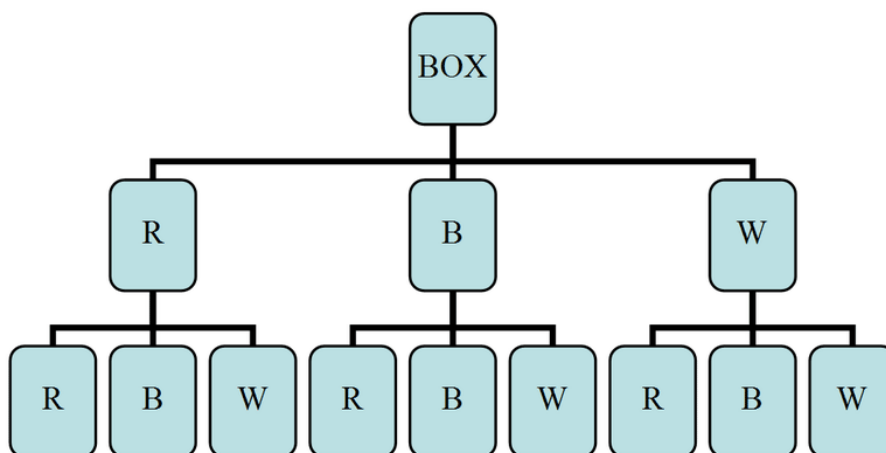
As you can see from the tree diagram, it is possible that you will get the red ball, R, on the first drawing and then another red one on the second, RR. You can also get a red one on the first and a blue on the second, and so on. From the tree diagram above, we can see that the sample space is as follows:

$$S = \{RR, RB, RW, BR, BB, BW, WR, WB, WW\}$$

Each pair in the set above gives the first and second drawings, respectively. That is, RW is different from WR.

We can also represent all the possible drawings by a table or a matrix:

	R	*B*	*W*
R	*RR*	*RB*	*RW*
B	*BR*	*BB*	*BW*
W	*WR*	*WB*	*WW*

Figure: Table representing the possible outcomes diagrammed in the previous figure. The first column represents the first drawing, and the first row represents the second drawing.

Consider the same experiment as in the last example. This time we will draw one ball and record its color, but we will not place it back into the box. We will then select another ball from the box and record its color. What is the sample space in this case?

Solution: The tree diagram below illustrates this case:
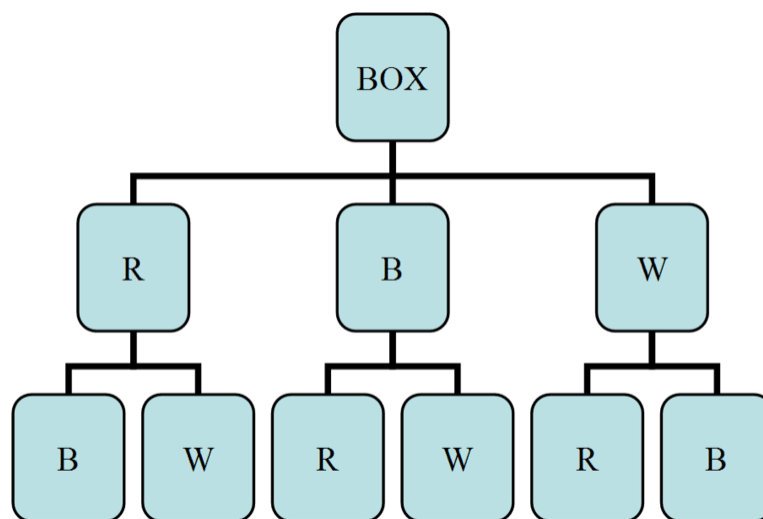
You can clearly see that when we draw, say, a red ball, the blue and white balls will remain. So on the second selection, we will either get a blue or a while ball. The sample space in this case is as shown:

$$S = \{RB, RW, BR, BW, WR, WB\}$$

Now let us return to the concept of probability and relate it to the concepts of sample space and simple events. If you toss a fair coin, the chance of getting tails, T, is the same as the chance of getting heads, H. Thus, we say that the probability of observing heads is 0.5, and the probability of observing tails is also 0.5. The probability, P, of an outcome, A, always falls somewhere between two extremes: 0, which means the outcome is an impossible event, and 1, which means the outcome is guaranteed to happen. Most outcomes have probabilities somewhere in-between.

Property 1: $0 \leq P(A) \leq 1$, for any event, A.

The probability of an event, A, ranges from 0 (impossible) to 1 (certain).

In addition, the probabilities of all possible simple outcomes of an event must add up to 1. This 1 represents certainty that one of the outcomes must happen. For example, tossing a coin will produce either heads or tails. Each of these two outcomes has a probability of 0.5. This means that the total probability of the coin landing either heads or tails is $0.5 + 0.5 = 1$. That is, we know that if we toss a coin, we are certain to get heads or tails.

Property 2: $\sum P(A) = 1$ when summed over all possible simple outcomes.

The sum of the probabilities of all possible outcomes must add up to 1.

Notice that tossing a coin or throwing a die results in outcomes that are all equally probable. That is, each outcome has the same probability as all the other outcomes in the same sample space. Getting heads or tails when tossing a coin produces an equal probability for each outcome, 0.5. Throwing a die has 6 possible outcomes, each also having the same probability, $\frac{1}{6}$. We refer to this kind of probability as classical probability. *Classical probability* is defined to be the ratio of the number of cases favorable to an event to the number of all outcomes possible, where each of the outcomes is equally likely.

Probability is usually denoted by P, and the respective elements of the sample space (the outcomes) are denoted by A, B, C, etc. The mathematical notation that indicates the probability that an outcome, A, happens is $P(A)$. We use the following formula to calculate the probability of an outcome occurring:

$$P(A) = \frac{\text{The number of outcomes for } A \text{ to occur}}{\text{The size of the sample space}}$$

Determining Probability

When tossing two coins, what is the probability of getting a head on both coins, HH? Is the probability classical?

Since there are 4 elements (outcomes) in the sample space set, $\{HH, HT, TH, TT\}$, its size is 4. Furthermore, there is only 1 HH outcome that can occur. Therefore, using the formula above, we can calculate the probability as shown:

$$P(A) = \frac{\text{The number of outcomes for } HH \text{ to occur}}{\text{The size of the sample space}} = \frac{1}{4} = 25\%$$

Notice that each of the 4 possible outcomes is equally likely. The probability of each is 0.25. Also notice that the total probability of all possible outcomes in the sample space is 1.

What is the probability of throwing a die and getting $A = 2, 3,$ or 4?

There are 6 possible outcomes when you toss a die. Thus, the total number of outcomes in the sample space is 6. The event we are interested in is getting a 2, 3, or 4, and there are three ways for this event to occur.

$$P(A) = \frac{\text{The number of outcomes for 2, 3, or 4 to occur}}{\text{The size of the sample space}} = \frac{3}{6} = \frac{1}{2} = 50\%$$

Therefore, there is a probability of 0.5 that we will get 2, 3, or 4.

Using a Probability Table

Suppose you toss two coins. Assume the coins are not balanced. The design of the coins is such that they produce the probabilities shown in the table below:

TABLE 11.1:

Outcome	Probability
HH	$\frac{4}{9}$
HT	$\frac{2}{9}$

TABLE 11.1: (continued)

Outcome	Probability
TH	$\frac{2}{9}$
TT	$\frac{1}{9}$

Figure: Probability table for flipping two weighted coins.

What is the probability of observing exactly one head, and what is the probability of observing at least one head?

Notice that the simple events HT and TH each contain only one head. Thus, we can easily calculate the probability of observing exactly one head by simply adding the probabilities of the two simple events:

$$P = P(HT) + P(TH)$$
$$= \frac{2}{9} + \frac{2}{9}$$
$$= \frac{4}{9}$$

Similarly, the probability of observing at least one head is:

$$P = P(HH) + P(HT) + P(TH)$$
$$= \frac{4}{9} + \frac{2}{9} + \frac{2}{9} = \frac{8}{9}$$

GUIDED PRACTICE

Suppose you have a jar of candies: 4 red, 5 purple and 7 green. Find the probability of the following events.

Example 1

Selecting a red candy

To find the probability of selecting a red candy, first find the total number of candies:

$total = 4 + 5 + 7 = 16$

Since there are 4 ways to get a red candy out of a total of 16 candies, the probability of selecting a red candy is:

$P(R) = \frac{4}{16} = \frac{1}{4} = 0.25$

Example 2

Selecting a purple candy.

To find the probability of selecting a purple candy, since there are 5 ways to get a purple candy out of a total of 16 candies, the probability of selecting a purple candy is:

$P(P) = \frac{5}{16} = 0.3125$

Example 3

Selecting a green candy.

To find the probability of selecting a green candy, since there are 7 ways to get a green candy out of a total of 16 candies, the probability of selecting a green candy is:

$P(G) = \frac{7}{16} = 0.4375$

Example 4

Selecting a yellow candy.

Since there are no yellow candies in the jar, the probability of selecting a yellow candy is 0.

PRACTICE

For 1-4, consider an experiment composed of throwing a die followed by throwing a coin.

a. List the simple events and assign a probability for each simple event.

b. What are the probabilities of observing the following events?

1. A 2 on the die and H on the coin.
2. A 2 on the die and H on the coin.
3. An even number on the die.
4. T on the coin.

For 5-6, the Venn diagram below shows an experiment with six simple events. Events A and B are also shown. The probabilities of the simple events

are:

$$P(1) = P(2) = P(4) = \frac{2}{9}$$
$$P(3) = P(5) = P(6) = \frac{1}{9}$$
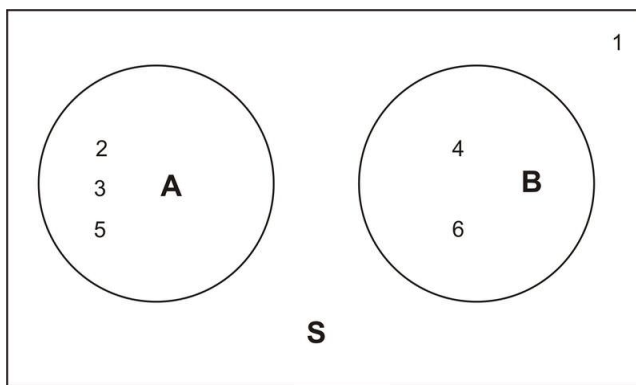
5. Find $P(A)$
6. Find $P(B)$

For 7-10, a box contains two blue marbles and three red ones. Two marbles are drawn randomly without replacement. Refer to the blue marbles as *B*1 and *B*2 and the red ones as *R*1, *R*2, and *R*3.

a. List the outcomes in the sample space.

b. Determine the probability of observing each of the event.

   7. Drawing 2 blue marbles.
   8. Drawing 1 red marble and 1 blue marble.
   9. Drawing 2 red marbles.
  10. Neither marble is blue or red.

  11. List the sample space for the following:

     a. Spinning a round spinner labeled A, B, C, D, E
     b. The sexes of a 3-child family
     c. The order in which 4 blocks A, B, C, and D can be lined up

  12. Show on a 2-dimensional grid the sample space for

     a. Rolling a die and tossing a coin at the same tie.
     b. Tossing two coins
     c. Rolling a die and spinning a spinner with sides A, B, C and D

  13. Show on a tree diagram the sample space for:

     a. Tossing a nickel and a dime at the same time.
     b. Tossing a coin three times
     c. Drawing two tickets from a hat containing a number of pink, blue and white tickets.

  14. Is each of the following values a legitimate probability value? Explain any "no" answers.

     a. .56
     b. .00
     c. 1.00
     d. 1.43
     e. -.36

  15. An appliance storeowner notices that one out of 30 customers returns the appliance within the two weeks of making the purchase.

     a. Write a sentence expressing this fact as a proportion.
     b. Write a sentence expressing this fact as a percent.
     c. Write a sentence expressing this fact as a probability.

  16. For each of the following experiments, describe the sample space.

     a. Toss a coin four times.
     b. Measure the lifetime (in hours) of a particular brand of light bulb.
     c. Record the weights of 13 day old rats.
     d. Observe the proportion of defectives in a shipment of computer components.

11.2 Theoretical Probability – Simple Events (7.6d and 7.6i)

In this concept, you will learn how to define and calculate theoretical probability.

GUIDANCE

Theoretical Probability

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/187577

Probability is defined as a mathematical way of calculating how likely an event is to occur. The probability of an event occurring is defined as the ratio of favorable outcomes to the number of possible equally likely total outcomes in a given situation.

In ratio form, the probability of an event is:

$$P(\text{event}) = \text{\# of favorable outcomes : total \# of outcomes}$$

Theoretical probability is probability that is based on an ideal situation.

For instance, since a flipped coin has two sides and each side is equally likely to land up, the theoretical probability of landing heads (or tails) is *exactly* 1 out of 2. Whether or not the coin actually lands on heads (or tails) 1 out of every 2 flips in the real world does not affect theoretical probability. The theoretical probability of an event remains the same no matter how events turn out in the real world.

Let's look at an example.

Find the probability of tossing a number cube and getting a 4.

First, find the total number of outcomes

Outcomes: 1, 2, 3, 4, 5, and 6

Total outcomes: 6

Second, find the number of favorable outcomes.

Favorable outcomes:

Getting a 4 = 1 favorable outcome

Then, find the ratio of favorable outcomes to total outcomes.

$$
\begin{aligned}
P(\text{event}) &= \text{\# of favorable outcomes : total \# of outcomes} \\
P(4) &= 1 : 6
\end{aligned}
$$

A **prediction** is a reasonable guess about what will happen in the future. Good predictions should be based on facts and probability.

Predictions based on theoretical probability. These are the most reliable types of predictions, based on physical relationships that are easy to see and measure and that do not change over time. They include such things as:

- coin flips
- spinners
- number cubes

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Jessie and the spinner.

The spinner can be used to calculate theoretical probability when it is considered without an experiment. Looking at the spinner itself, you can calculate the chances of spinning one or more colors. You haven't done an experiment with the spinner yet, you are simply calculating the probability by looking at the spinner.

The spinner has four colors so the total number of outcomes is 4. If she wanted to know the theoretical probability of spinning and getting on blue it would be 1:4 since there is only one section that is blue.

Experimental probability can be calculated once the spinner has been spun once or a number of times.

Example 2

Two number cubes are thrown. What is the theoretical probability of rolling a number greater than 8?

First, let's think about the different combinations that you can possibly roll on the number cubes. If you roll a 1 and a 2 for instance, you get a sum of 3. Look at the table below to show all of the combinations.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

The total number of outcomes is 36.

Next, the favorable outcomes are shaded in yellow in the table above.

favorable outcomes $= 10$

Then, find the ratio of favorable outcomes to total outcomes.

$$\begin{aligned} P(\text{event}) &= \text{\# of favorable outcomes : total \# of outcomes} \\ P(> 8) &= 10 : 36 \end{aligned}$$

Then, simplify the ratio.

$$\begin{aligned} P(> 8) &= 10 : 36 \\ P(> 8) &= 5 : 18 \end{aligned}$$

The answer is 5:18.

The theoretical probability of rolling a sum greater than 8 when rolling two number cubes is 5:18.

Calculate each example of theoretical probability.

Example 3

What is the probability of tossing a number cube and having it come up a two or a three?

First, find the total number of outcomes

Outcomes: 1, 2, 3, 4, 5, and 6

Total outcomes: 6

Next, find the number of favorable outcomes.

Favorable outcomes:

$$\text{Getting a 2 or a 3} = 2 \text{ favorable outcomes}$$

Then, find the ratio of favorable outcomes to total outcomes.

$$\begin{aligned} P(\text{event}) &= \text{\# of favorable outcomes : total \# of outcomes} \\ P(2 \text{ or } 3) &= 2 : 6 \\ P(2 \text{ or } 3) &= 1 : 3 \end{aligned}$$

The answer is 1:3.

The theoretical probability of rolling a 2 or a 3 on a number cube is 1:3.

Example 4

What is the probability of tossing a number cube and having it come up even?

First, find the total number of outcomes

Outcomes: 1, 2, 3, 4, 5, and 6

Total outcomes: 6

Next, find the number of favorable outcomes.

Favorable outcomes:

$$
\begin{aligned}
\text{Getting an even number} &= 2,4,6 \\
&= 3 \text{ favorable outcomes}
\end{aligned}
$$

Then, find the ratio of favorable outcomes to total outcomes.

$$
\begin{aligned}
P(\text{event}) &= \text{\# of favorable outcomes : total \# of outcomes} \\
P(\text{event \#}) &= 3:6 \\
P(\text{event \#}) &= 1:2
\end{aligned}
$$

The answer is 1:2.

The theoretical probability of rolling an even number on a number cube is 1:2.

Example 5

What is the probability of tossing a number cube and having it come up less than 6?

First, find the total number of outcomes

Outcomes: 1, 2, 3, 4, 5, and 6

Total outcomes: 6

Second, find the number of favorable outcomes.

Favorable outcomes:

$$
\begin{aligned}
\text{A number less than 6} &= 1,2,3,4,5 \\
&= 5 \text{ favorable outcomes}
\end{aligned}
$$

Then, find the ratio of favorable outcomes to total outcomes.

$$
\begin{aligned}
P(\text{event}) &= \text{\# of favorable outcomes : total \# of outcomes} \\
P(\# < 6) &= 5:6
\end{aligned}
$$

The answer is 5:6.

The theoretical probability of rolling a number less than 6 on a number cube is 5:6.

PRACTICE

Solve each problem.

A spinner has five sections: purple, yellow, green, blue and red.

1. Find the probability for the arrow landing on blue on the spinner:

   a. List each favorable outcome.
   b. Count the number of favorable outcomes.
   c. Write the total number of outcomes.
   d. Write the probability.

2. Find the probability for the arrow landing on red or green on the spinner:

   a. List each favorable outcome.
   b. Count the number of favorable outcomes.
   c. Write the total number of outcomes.
   d. Write the probability.

3. Find the probability for the arrow NOT landing on yellow on the spinner:

   a. List each favorable outcome.
   b. Count the number of favorable outcomes.
   c. Write the total number of outcomes.
   d. Write the probability.

4. Find the probability for rolling a 3 or 4 on the number cube:

   a. List each favorable outcome.
   b. Count the number of favorable outcomes.
   c. Write the total number of outcomes.
   d. Write the probability.

5. Find the probability for rolling greater than 2 on the number cube:

   a. List each favorable outcome.
   b. Count the number of favorable outcomes.
   c. Write the total number of outcomes.
   d. Write the probability.

11.3 Theoretical Probability – Compound Events (7.6i)

GUIDANCE

Independent and Dependent Events

When you work with probability, you can think about different events and how these events impact each other. Let's look at an example.
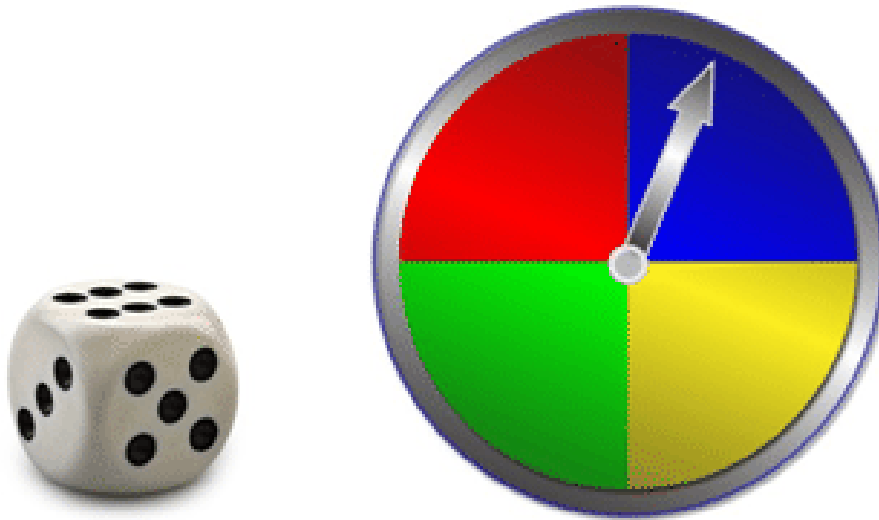
Suppose there are two events:

Event : Toss 5 on the number cube

Event : Spin blue on the spinner

The probability of each of these events by itself is easy enough to compute. In general:

If this is the case, then you can write the following ratios for rolling a 5 and spinning blue.

These two events were performed with a spinner and a number cube. Does event affect the probability of event in any way? That is, does the number cube landing on 5 affect where the arrow lands in the spinner? If not, then the two events are said to be independent events.

If the outcome of one event has no effect on the outcome of a second event, then the two events are **independent events.**

Events and above are independent events. No matter how the number cube turns up, its outcome does not affect the outcome of spinning the spinner.

Now let's think about a different kind of example, one where the outcome of one event does impact the outcome of another event.

A bag has 3 red marbles, 4 blue marbles, and 3 green marbles. Irene pulls 1 green marble out of the bag. Does this change the probability that the next marble Irene pulls out of the bag will be green?

First, find the number of favorable outcomes and the total number of outcomes for the first pick from the bag.

Next, find the probability of choosing a green marble.

Then, for the second marble, there are now only 9 marbles left in the bag and only 2 of them are green. So the probability of pulling out a green marble for the second marble is now:

Clearly, the first event affected the outcome of the second event in this situation. So the two events are NOT independent. In other words, they are dependent events.

If the outcome of one event has an effect on the outcome of a second event, then the two events are **dependent events.**

Sometimes, you have mutually exclusive events and you have events that overlap and are not mutually exclusive. Two events and that cannot occur at the same time are **mutually exclusive events**. **Mutually inclusive events** can occur at the same time.

Events and event are overlapping events or mutually exclusive events because both events share one outcome .
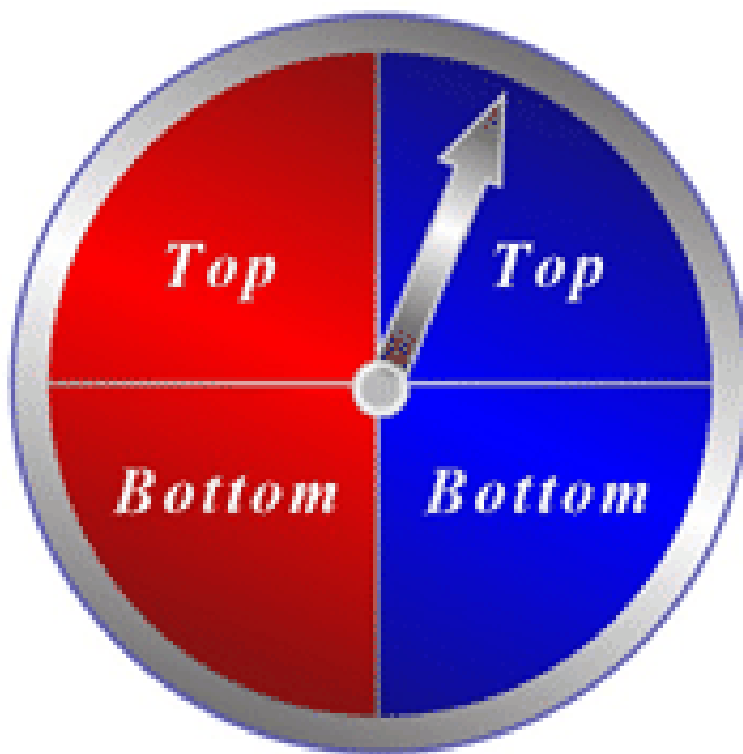
To calculate the probability of overlapping events, list the sample space and find the favorable events.

The probability of red-top is:

VIDEO REVIEW

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about the race and the two runners. Two runners are in a race. The first runner runs a 5 minute mile and the second runner runs a 10 minute mile.

The events are independent. The speed of one runner does not impact the speed of another runner.

For a single toss of a number cube, what is the probability of event and event both occurring?

First, identify the overlapping outcomes of both events.

Next, find the total number of outcomes.

Then, find the probability of the overlapping events.

Identify whether the following events are independent or dependent.

Example 2

Rolling a 1 and then rolling a 6 on a number cube.

The answer is these events are independent events because the rolling of a one does not affect the second roll and getting a six.

Example 3

A bag has a red and two blue marbles.

First, one blue marble is drawn and then replaced.

Then, a red one is drawn.

Are these two events dependent or independent?

The answer is these events are independent events because the blue marble was replaced; it does not affect the outcome of drawing of red.

Example 4

A box contains 5 white and 6 red marbles. What is the probability of successfully drawing, in order, a red marble and then a white marble?

The answer is these events are dependent events because the red marble was not replaced; the number of total outcomes will decrease from 11 to 10 when choosing the white marble.

PRACTICE

Write whether events A and B are dependent or independent.

1. A : Doug flips a coin. B : Marlene chooses a card out of a deck.

2. A : In a bag with 5 white marbles and 5 black marbles, Sanjay pulls out a white marble. B : Without returning the marble to the bag, Sanjay pulls out a second marble.

3. A : Eddie chooses the color blue for his new bike. B : Eddie chooses lasagne from the dinner menu.

4. A : The probability that it will rain tomorrow. B : The probability that the Red Wings hockey team will win their game tomorrow.

5. A : The probability that it will rain tomorrow. B: The probability that the baseball team will have a rain delay.

6. A : From a deck of cards, the probability of one player drawing a heart from the deck. B : On the next player's turn, the probability of drawing another heart.

7. A : The probability of a spinner landing on blue 6 times in a row. B : The probability of the spinner landing on blue on the next spin.

8. A : The probability of flipping a coin and having it come up heads. B : The probability of flipping it again and having it come up heads.

9. A : The probability that it will snow tomorrow. B : The probability of having a snow day from school.

10. A : The probability that it will be 90 degrees. B : The probability of enjoying a hot day at the beach.

11. A : The probability that it will rain tomorrow. B : The probability of getting an A on a math test.

12. A: The probability that the Rockies will be in the playoffs. B : The probability that the Rockies will win the World Series.

13. A : The probability that tomorrow will be sunny. B : The probability that tomorrow will be a full moon.

14. A : The probability that tomorrow will be sunny. B : The probability that tomorrow will be cloudy.

15. A : The probability that it will cold today. B : The probability that it will be a full moon tomorrow.

11.4 Experimental Probability (7.6c and 7.6i)

GUIDANCE

Jackson has just been told by his football coach that he will be the starting kicker in this week's football game, meaning the team will be depending on him to kick the field goals. Jackson wants a little reassurance before the game that he will have a successful first starting game. He decides to count the number of successful field goals he kicks. Out of 20 tries, he kicks 16 field goals and misses 4 times. What is the experimental probability that Jackson will kick a field goal during the game?

In this concept, you will learn about experimental probability.

Experimental Probability

Probability is based on simple facts. For example, since there are two sides of a coin, heads and tails, and each side has an equal chance of turning up, it makes sense to say the probability of heads turning up is 50 percent, or 1 out of 2.

However, making sense and being true are often two different things. It may make sense to say the probability of heads is 50 percent, but does a real coin in the real world actually turn up heads 50 percent of the time? To answer

that question, you need to learn about **empirical probability**, also called **experimental probability**. **Experimental probability** is probability based on doing actual experiments - flipping coins, spinning spinners, picking ping pong balls out of a jar, and so on.

To compute the experimental probability of the spinner landing on the red section you would need to conduct an experiment. Suppose you were to spin the spinner 60 times.
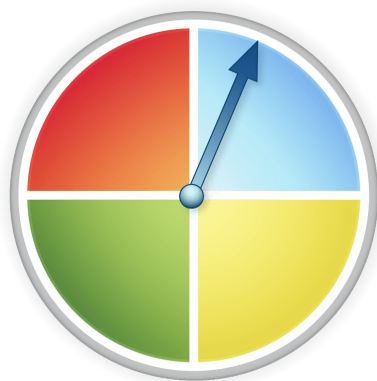
Favorable outcomes: red

Total outcomes: 60 spins

$$\underline{\text{Experimental probability}} : \ P(\text{red}) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{\text{number of red}}{\text{total number of spins}}$$

Now you can use this data to conduct an experiment.

What is the experimental probability of having the arrow of the spinner land in the red section?

TABLE 11.2:

Trial	1	2	3	4	5	6	Total
red favorable outcomes	2	3	1	5	2	4	**17**
total spins total outcomes	10	10	10	10	10	10	**60**
experimental probability: ratio of favorable outcomes to total outcomes	x	x	x	x	x	x	**17:60**

Solve the problem by spinning the spinner 60 times in 6 trials of 10 spins each as shown in the table above. Then follow the steps below.

First, total up the number of favorable outcomes - the number of times the spinner landed on red. This is shown in red in the final column of the table as 17.

Next, add up the number of total outcomes - the total number of spins. This is shown in red in the final column of the table as 60.

Then, compute the experimental probability - the ratio of favorable outcomes to total outcomes in percent form. The experimental probability, in ratio form, is 17:60. Convert to a percentage:

$$60\overline{)17.00}^{.28} = 28\%$$

The answer is the experimental probability of favorable outcomes to total outcomes is 28%.

A number cube was rolled in a probability experiment 40 times. The results are shown in the table. Compute the experimental probability of rolling a 5.

TABLE 11.3:

Dots on number cube	.	..	. . .	.. ..	. . . ..	. . . . . .	Total
	1	2	3	4	5	6	
number of times cube landed	|||||	||||	|||||	|||||	|||||	|||||	x
total from tally	6	4	9	8	6	7	**40**
favorable outcomes	x	x	x	x	6	x	**6**
experimental probability: ratio of favorable outcomes to total outcomes	x	x	x	x	x	x	6:40 = 3:20

First, add up the tallies to get the number of total number of outcomes. This is recorded as 40 in the fifth row of the table.

Next, find the number of favorable outcomes - the number of times the number cube landed on 5. This is recorded as 6 in the sixth row of the table.

Then compute the experimental probability by finding the ratio of favorable outcomes to total outcomes, making sure to simplify if possible: 6:40 = 3:20

Then answer is the experimental probability of rolling a 5 a 3:20 ratio.

Most probability experiments are conducted as a process of testing a hypothesis. A **hypothesis** is a statement that you want to test to see if it's true.

Here are some examples of hypotheses.

- A flipped coin comes up heads 50 percent of the time. True or false?
- A number cube will land on *three* $\frac{1}{6}$ th of the time. True or false?
- It rains more on the weekend than it does on weekdays. True or false?
- You perform better on math tests when you get a good night's sleep. True or false?

• A flipped coin will land on heads $\frac{1}{2}$, or 50 percent of the time. True or false?

To compute the experimental probability of the hypothesis you would need to conduct an experiment. Suppose you were to flip the coin 50 times. To find out how often it lands on heads, follow the steps below.

First, state your hypothesis including the **theoretical probability** and make a prediction.

Hypothesis: The coin will land on heads $\frac{1}{2}$, or 50 percent of the time.

Use the theoretical probability to make a prediction. Since there are 2 different outcomes, and 1 of those outcomes is heads, it make sense to predict that heads will come up 1 out of 2, or 50 percent of the time.

$$P(\text{heads}) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{1}{2} \Longrightarrow \text{prediction:heads} = 50\%$$

Next, conduct an experiment and collect data. You can use a tally table like the one shown below. Fill in your predicted values and tally as you conduct the experiment.

TABLE 11.4:

Trial	1	2	3	4	5	Prediction	Total
tally	||||	|||||	|||	|||||	|||||	x	x
heads	4	5	3	6	5	25	23
total flips	10	10	10	10	10	50	50

Then, analyze your data and see how well it agrees with your prediction. You can see that 23 out of 50 flips came up heads. The data does not perfectly agree with the prediction, but 23 is close to 25. So the results are fairly close to the prediction. In general, the more total outcomes included in the experiment, the more likely the experimental probability will agree with the predicted probability.

The answer is the prediction is 50% and the experimental probability is

$\frac{23}{50} = 46\%$.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Jackson's experiment to predict the probability of him kicking a field goal during his football game.

He kicked the ball 20 times. He kicked 16 field goals and missed 4 times. What is the experimental probability that Jackson will kick a field goal during the game?

First, determine the number of successful outcomes:

Jackson succeeded in kicking 16 field goals. There are 16 successful outcomes.

Next, detemine the total number of attempts (trials attempted):

Jackson attempted to kick a field goal 20 times. There are 20 trials attempted.

Then, create a ratio from the data and convert to a percent:

$$\text{Probability} = \frac{successful\ outcomes}{trials\ attempted}$$
$$16 : 20 = \frac{16}{20}$$
$$\frac{16}{20} = 0.8 = 80\ percent$$

The answer is Jackson's experimental probability of kicking a field goal during the football game is 80%.

Example 2
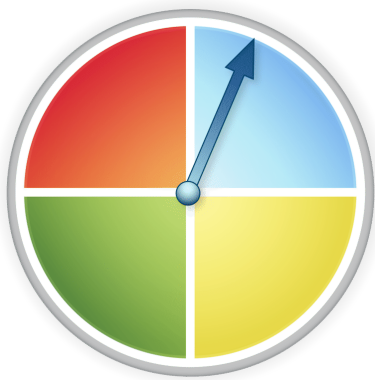
Use the table to compute the experimental probability of the arrow in the spinner landing on yellow and record your answer as a percentage.

TABLE 11.5:

Trial	1	2	3	4	5	6	Total
number of times arrow landed on yellow	4	2	3	1	2	5	
total number of spins	10	10	10	10	10	10	
experimental probability	x	x	x	x	x	x	

First, calculate the total attempts:

There were 6 x 10 total attempts to have the spinner land on yellow.

Next, calculate the number of attempts that landed on yellow:

If you add up each success out of 10 we have: $4 + 2 + 3 + 1 + 2 + 5 = 17$

Then, record the results as a ratio and convert to a percent.

$17 : 60 = \frac{17}{60} = .2833 = 28\%$

The answer is the experimental probability of the spinner landing on yellow is 28% of the time.

Use the below information the answer the questions that follow:

Jennifer attempted to juggle three pins for five minutes. She made five attempts and dropped a club twice.

Example 3

Write a probability that shows successful attempts.

First, determine the number of successful attempts (favorable outcomes):

The number of successful attempts is 3.

Next, determine the total number of attempts (total outomes):

The total number of attempts is 5.

Then, create a ratio out of the data: 3:5

The answer is the probability of successful attempts is 3:5.

Example 4

Write a probability that shows drops.

First, determine the number of successful attempts (favorable outcomes):

If the number of successful attempts is 3 out of a total of 5 total attempts, then there were 2 drops.

Next, determine the total number of attempts (total outomes):

The total number of attempts is 5.

Then, create a ratio out of the data: 2:5

The answer is the probability of drops is 2:5.

Example 5

What is the percent of success?

First, convert the ratio into a fraction: 3:5 = 3/5

Next, divide the numerator by the denominator:

$$\overset{.60}{5\overline{)3.00}}$$

Then, convert the decimal value into a percentage: $0.60 = 60\%$

The answer is the probability of successful attempts is 60%.

PRACTICE

Use what you have learned about experimental probability to complete each problem.
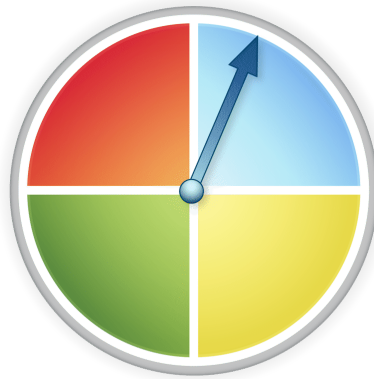
Use the table to compute the experimental probability of the arrow in the spinner landing on yellow.

TABLE 11.6:

Trial	1	2	3	4	5	6	Total
number of times arrow landed on yellow	4	2	3	1	2	5	
total number of spins	10	10	10	10	10	10	
experimental probability	x	x	x	x	x	x	

1. How many favorable outcomes were there in the experiment?
2. How many total outcomes were there in the experiment?
3. What is the experimental probability of the arrow landing on yellow?

Use the table to compute the experimental probability of the arrow in the spinner above landing on blue or green.

TABLE 11.7:

Trial	1	2	3	4	5	Total
number of times arrow landed on blue or green	4	5	6	5	4	
total spins	10	10	10	10	10	
experimental probability	x	x	x	x	x	

4. How many favorable outcomes were there in the experiment?
5. How many total outcomes were there in the experiment?
6. What is the experimental probability of the arrow landing on yellow?

Use the table to compute the experimental probability of the arrow landing on any color but blue.

TABLE 11.8:

Trial	1	2	3	4	5	Total
number of times arrow landed on any color but blue	15	17	14	16	16	
total spins	20	20	20	20	20	
experimental probability						

7. How many favorable outcomes were there in the experiment?
8. How many total outcomes were there in the experiment?
9. What is the experimental probability of the arrow landing on yellow?

A number cube was rolled in a probability experiment 40 times. The results are shown in the table. Compute the experimental probability of rolling a 2.

TABLE 11.9:

Dots on number cube	.	..	. . .	.. ..	. . . ..	. . . . . .	Total
	1	2	3	4	5	6	
number of times cube landed	⫽⫽	⫽⫽	⫽⫽	⫽⫽	⫽⫽	‖‖	x
	‖‖	‖‖	‖	‖	‖		
total from tally	9	8	6	7	6	4	
favorable outcomes	x		x	x	x	x	
experimental probability	x	x	x	x	x	x	

10. How many favorable outcomes were there in the experiment?
11. How many total outcomes were there in the experiment?
12. What is the experimental probability of the rolling a 2?

A number cube was rolled in a probability experiment 50 times. The results are shown in the table. Compute the experimental probability of rolling a 3 or a 4.

TABLE 11.10:

Dots on number cube	·	..	· · ·	.. ..	· · · ..	· · · · · ·	Total
	1	2	3	4	5	6	
number of times cube landed	卌	卌	卌	卌	卌	卌	x
	卌	|	||||	||	卌	||	
					|		
total from tally	10	6	9	7	11	7	
favorable outcomes	x	x	9	7	x	x	
experimental probability	x	x	x	x	x	x	

13. How many favorable outcomes were there in the experiment?
14. How many total outcomes were there in the experiment?
15. What is the experimental probability of the rolling a 2?

Resources

According to theoretical probability, how many times can we expect to land on each color if we take 16 spins?

Green	Yellow	Red	White	Purple
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{4}{16}$	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{2}{16}$	$\frac{2}{16}$

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/183925
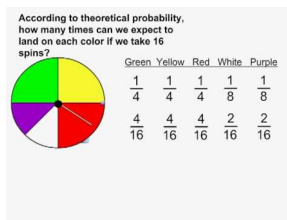
11.5 Probability Applications (7.6h)

Multiple Choice

Identify the choice that best completes the statement or answers the question.

1. A group of volleyball players consists of four grade-11 students and six grade-12 students. If six players are chosen at random to start a match, what is the probability that three will be from each grade?
2. If a bowl contains ten hazelnuts and eight almonds, what is the probability that four nuts randomly selected from the bowl will all be hazelnuts?
3. Without looking, Jenny randomly selects two socks from a drawer containing four blue, three white, and five black socks, none of which are paired up. What is the probability that she chooses two socks of the same col- our?
4. A euchre deck has 24 cards: the 9, 10, jack, queen, king, and ace of each suit. If you were to deal out five cards from this deck, what is the probability that they will be a 10, jack, queen, king, and ace all from the same suit?.
5. A bag contains 26 tiles, each marked with a different letter of the alphabet. What is the probability of being able to spell the word *math* with four randomly selected tiles that are taken from the bag all at the same time?

Short Answer

6. Participants in marathons are often given numbers to wear, so that race officials can identify individual run- ners more If the numbers are assigned randomly, what is the probability that the eight fastest runners will finish in the order of their assigned numbers, assuming that there are no ties?
7. A club with eight members from grade 11 and five members from grade 12 is to elect a president, vice-presid- ent, and What is the probability (as a percentage to one decimal place) that grade 12 students will be elected for all three positions, assuming that all club members have an equal chance of being elected?
8. A four-member curling team is randomly chosen from six grade-11 students and nine grade-12 What is the probability that the team has at least one grade-11 student?
9. If a CD player is programmed to play the CD tracks in random order, what is the probability that it will play six songs from a CD in order from your favourite to your least favourite?
10. What is the probability that at least two people in a class of 30 students have the same birthday? Assume that no one in the class was born on February

Problem

11. Suppose you randomly draw two marbles, without replacement, from a bag containing six green, four red, and three black

   1. Draw a tree diagram to illustrate all possible outcomes of this
   2. Determine the probability that both marbles are
   3. Determine the probability that you pick at least one green
   4. A six-member working group to plan a student common room is to be selected from five teachers and nine If the working group is randomly selected, what is the probability that it will include at least two teachers?
   5. Len just wrote a multiple-choice test with 15 questions, each having four Len is sure that he got ex- actly 9 of the first 12 questions correct, but he guessed randomly on the last 3 questions. What is the probabil- ity that he will get at least 80% on the test?
   6. Leela has five white and six grey huskies in her If a wilderness expedition chooses a team of six sled dogs at random from Leela's kennel, what is the probability the team will consist of

      1. all white huskies?
      2. all grey huskies?
      3. three of each colour?

15. Six friends go to their favourite restaurant, which has ten entrees on the If the friends are equally likely to pick any of the entrees, what is the probability that at least two of them will order the same one?

Vocabulary:

- Sample Spaces
- Probability
- Theoretical Probability
- Experimental Probability
- Favorable Outcomes
- Simple Probability
- Compound Probability
- Independent Events
- Dependent Events

CHAPTER **12**

Graphs

Chapter Outline

7.6g: solve problems using data represented in bar graphs, dot plots, and circle graphs, including part-to-whole and part-to- part comparisons and equivalents

7.12a: compare two groups of numeric data using comparative dot plots or box plots by comparing their shapes, centers, and spreads

7.12c: compare two populations based on data in random samples from these populations, including informal comparative inferences about differences between the two populations

7.12b: use data from a random sample to make inferences about a population

12.1 Bar Graphs (7.6g)

Mr. Davison has a number of students who do really well in his English class and some who don't seem very interested at all. He is coming up with a plan for an amazing field trip to reignite his class's interest. To get a good idea of what the current level of engagement is, Mr. Davison wants to visualize the grade distribution and he thinks a bar graph is the best way to do this.

TABLE 12.1: Mr. Davison's English Class Grades

Grade	Number of Students
A	15
B	24
C	33
D	9
F	6

How can Mr. Davison create a bar graph?

In this concept, you will learn how to create and read bar graphs.

Creating and Reading Bar Graphs

Data is a set of numerical or non-numerical information. Data can be analyzed in many different ways. In this concept you will analyze numerical data using a bar graph.

One way to display data is in a **bar graph.** A bar graph shows the relationship between different values of data. Each data value is represented by a column in the graph. The categories of different kinds of data are listed along the horizontal, or *x*, axis. The quantity or amount of data is listed along the vertical, or *y*, axis.

Let's look at an example.

A seventh grade class recorded their favorite after school activity in a table. Create a bar graph to display the data from the table. Then, list two conclusions that can be made about the data.

TABLE 12.2: Seventh Graders' Favorite Activities

Favorite Activity:	Number of Students:
Watching T.V.	9
Playing sports/exercising	6
Reading	7
Hanging out with friends	5
Babysitting	3

First, draw the horizontal (*x*) and vertical (*y*) axes.

y

x

Next, label the horizontal axis. The horizontal axis lists the different categories of data. In this case, the category will be "favorite activity."

Next, label the vertical axis. The vertical axis lists the quantity or amount of the data. In this case, the category will be "Number of Students."

Next, title the graph. The title of the graph should be short and clear. It should explain what data is presented in the graph. In this case, the title will be "Seventh Graders' Favorite Activities."

Then, determine the units on the vertical axis. To do this, start by reviewing the smallest and largest values in the table. The smallest value is 3 and the largest is 9. Based on these values label the vertical axis from 0-10. Further, since the values are whole numbers and between 3 and 9, a unit of 1 should be used. Therefore, the vertical axis will start at 0 and go to 10 by increments of 1.

Next, draw the vertical columns. To do this, write each activity along the horizontal axis. Be sure to leave space between each one. Then draw the column vertically until it reaches the quantity for that activity. For example, draw a vertical column to the number "9" for the activity "Watching T.V."

Lastly, state two conclusions from the graph. To do this, analyze the data presented in the table by comparing the heights of the bars. The first conclusion is that watching T.V. is the most common favorite activity among seventh graders because the column is the highest. A second conclusion is that babysitting is the least most common favorite activity because it is the shortest column in the graph.

The first answer is the graph should look like the one below. The second answer is two conclusions that can be made from the data are: watching T.V. is the most common favorite activity among seventh graders and babysitting is the least most common favorite activity among seventh graders.
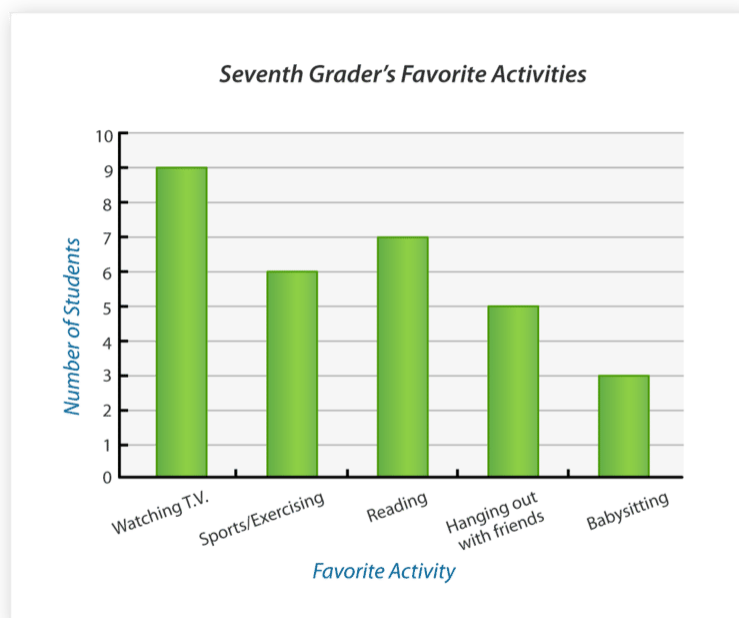
GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Mr. Davison, who wants to illustrate the grade distribution of all seventh grade students in English using a bar graph.

He organized the data into the following table.

TABLE 12.3: Mr. Davison's English Class Grades

Grade	Number of Students
A	15
B	24
C	33
D	9
F	6

To make the bar graph, first Mr. Davison will draw the horizontal (x) and vertical (y) axes.

Next, he will label the horizontal axis. The horizontal axis lists the different categories of data. In this case, the category will be "grade."

Next, he will label the vertical axis. The vertical axis lists the quantity or amount of the data. In this case, the category will be "Number of Students."

Next, he will title the graph. The title should explain what data is presented in the graph. In this case, the title will be "Seventh Grader's English Grade."

Then, he will determine the units on the vertical axis. To do this, determine the smallest and largest values in the table. The smallest value is 6 and the largest is 33. Based on these values label the vertical axis from 0-35. Since the values are whole numbers and have some distance between them a unit of 5 should be used. Therefore, the vertical axis will start at 0 and go to 35 by increments of 5.

Next, draw the vertical columns. To do this, write each grade along the horizontal axis, leaving space between each one. Then draw each column vertically until it reaches the quantity for that grade. For example, draw a vertical column to the number "15" for the grade "A."

The graph should look like the one below.

Example 2

The graph below shows the six countries per capita GDP for 2013. List three conclusions that can be made about the data.
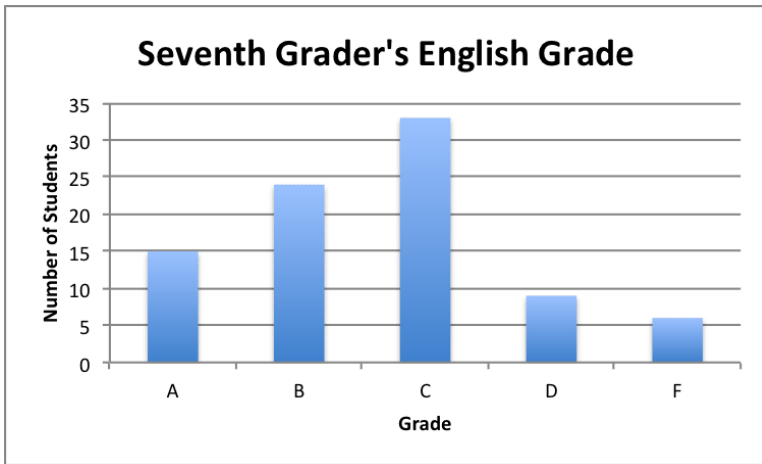
FIGURE 12.1
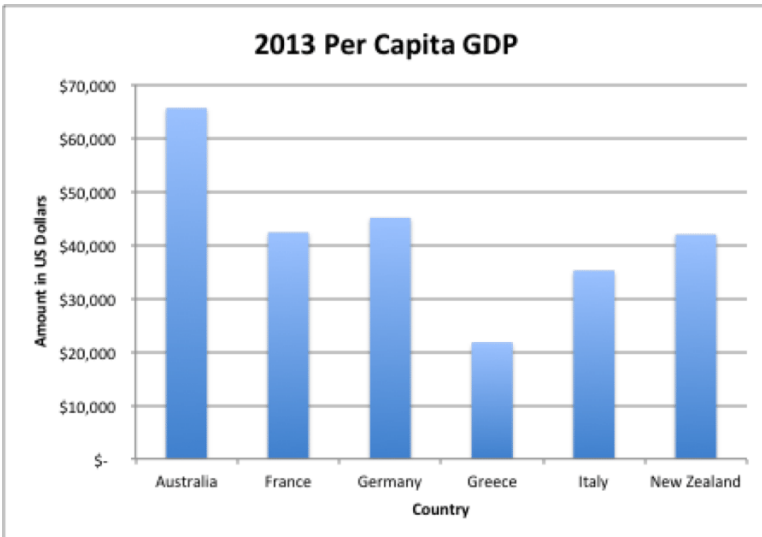
FIGURE 12.2

The answer is three conclusions that can be made from the graph are: Australia has the highest per capita GDP of the countries listed; Greece has the lowest per capita GDP of the countries listed; and the range of the per capita GDP is approximately $44,000.

Example 3

The table below shows the recommended number of hours of sleep people need each day by age. Create a bar graph to display the data from the table. Then, state one conclusion that can be made about the data.

TABLE 12.4: Recommended Hours of Sleep Per Day

Age Group:	Recommended Hours of Sleep:
Infants (0-1 years old)	15 hours
Children (2-5 years old)	13 hours
Children (6-11 years old)	11 hours

<div align="center">**TABLE 12.4:** (continued)</div>

Age Group:	Recommended Hours of Sleep:
Teens	9 hours
Adults	8 hours

First, create the bar graph. To do this, draw the horizontal (*x*) and vertical (*y*) axes.

y

x

Next, label the horizontal axis. The horizontal axis lists the different categories of data. In this case, the category will be "age group."

Next, label the vertical axis. The vertical axis lists the quantity or amount of the data. In this case, the category will be "hours."

Next, title the graph. The title of the graph should be short and clear. It should explain what data is presented in the graph. In this case, the title will be "Recommended Hours of Sleep Per Day."

Then, determine the units on the vertical axis. To do this, start by reviewing the smallest and largest values in the table. The smallest value is 8 and the largest is 15. Based on these values label the vertical axis from 0-16. Since the values are whole numbers and are less than 16, a unit of 2 can be used. Therefore, the vertical axis will start at 0 and go to 16 by increments of 2.

Next, draw the vertical columns. To do this, write each age group along the horizontal axis, leaving space between each one. Then draw the column vertically until it reaches the quantity for that age group. For example, draw a vertical column to the number "15" for the age group "Infants (0-1 years old)."

Finally, state one conclusion from the graph. To do this, analyze the data presented in the table by comparing the heights of the bars.

The first answer is the graph should look like the one below. The second answer is: one conclusion that can be made from the graph is that as people age the number of recommended hours of sleep per day decreases.

Example 4

The graph below shows the fifteen most populous countries in the world. List three conclusions that can be made about the data.

The answer is three conclusions that can be made from the graph are: China is the most populated country in the world, India is the second most populated country in the world, and Egypt is ranked last in the top fifteen most populous countries in the world.

Example 5

The graph below shows the estimated number of diabetes cases in 2000. List three conclusions that can be made from the graph.

First, analyze the graph by comparing the heights of the columns.
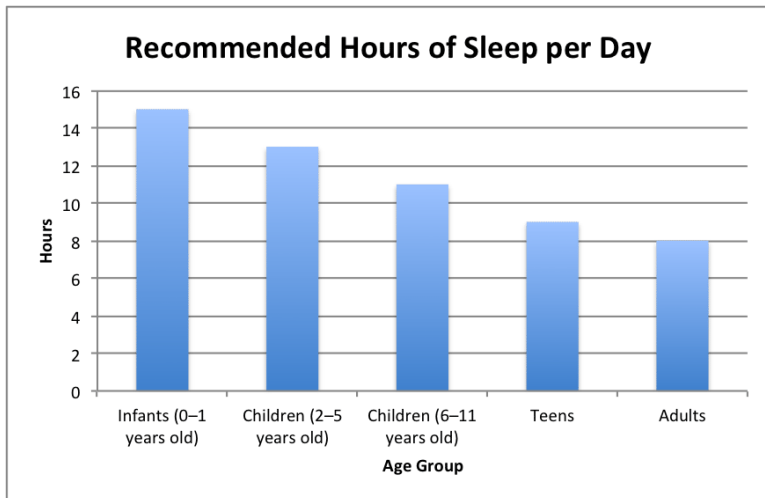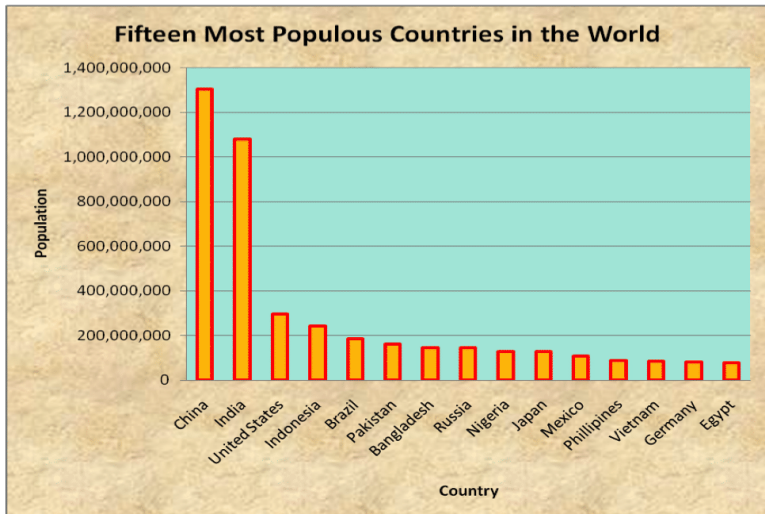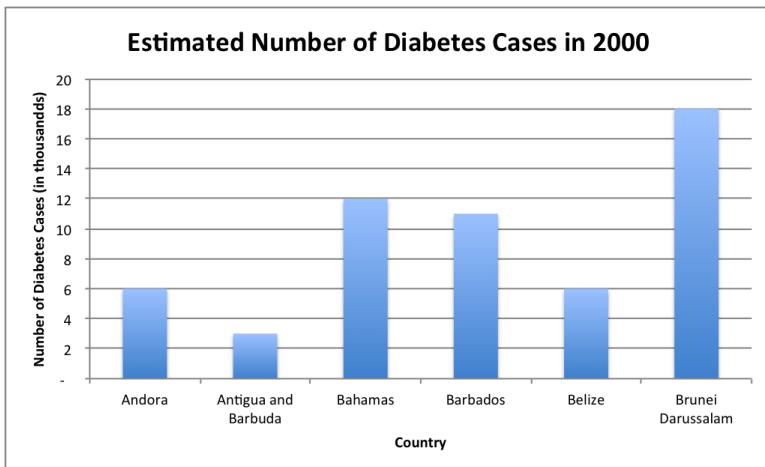
The answer is three conclusions that can be made from the graph are: Brunei Darussalam had the greatest number of estimated diabetes cases in 2000 at 18,000; Andora and Belize had the same number of estimated diabetes cases in 2000 at 6,000; and Antigua and Barbuda had the smallest number of estimated diabetes cases in 2000.

PRACTICE

Use the bar graph to answer the following questions.
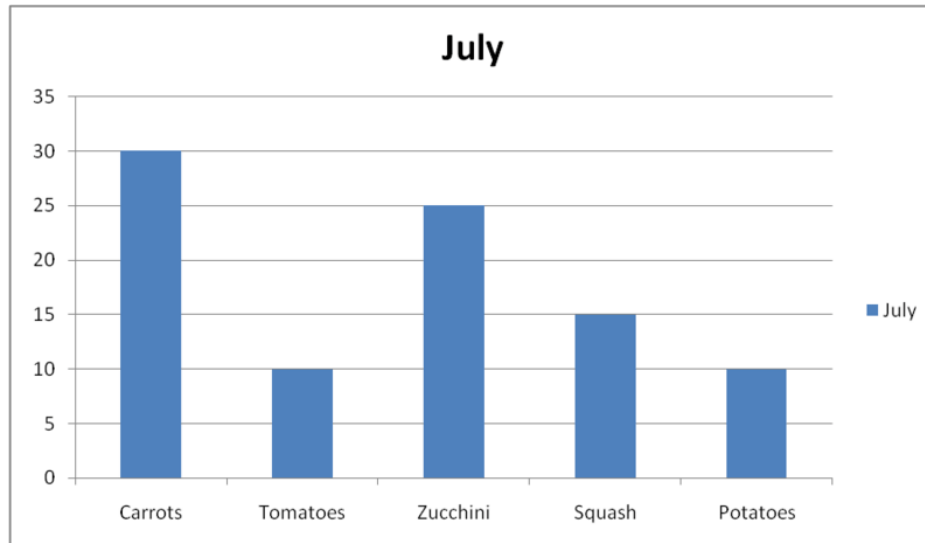
1. How many different types of vegetables are on the graph?
2. What is the range of the data?
3. What vegetable was the most popular?
4. Which one was the least popular?
5. How many carrots were picked?
6. How many potatoes?
7. How many squash?
8. If ten more zucchini was picked, how would this change the data?
9. If each category doubled, what would the new values be?
10. If each category was divided in half, what would the new values be?
11. What was the total number of vegetables picked?
12. What is the difference between carrots and zucchini picked?
13. What scale was used for this graph?
14. What interval was used in the scale?
15. What is the difference between carrots and potatoes?

12.2 Circle Graphs (7.6g)

GUIDANCE

Circle Graphs

A **circle graph**, sometimes called a pie graph, is a visual way to display data using circles and parts of a circle. In fact, circle graphs are one of the most popular means of displaying real-world data.
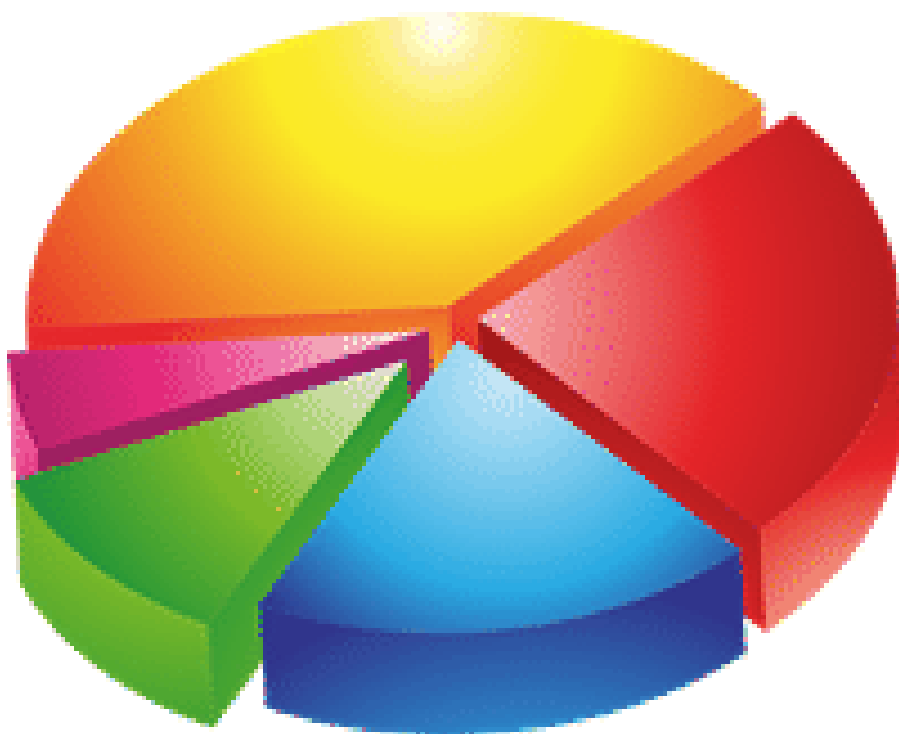
In a circle graph, the entire circle represents 100% and each section of the circle represents some part out of 100.

The circle graph below shows how a graph is divided into sections according to the data. Data Set 1 is 25% of the graph. This is displayed numerically as well as visually. Notice that the green section of the circle is one-fourth, or 25%, of the circle.

Understanding Circle Graphs
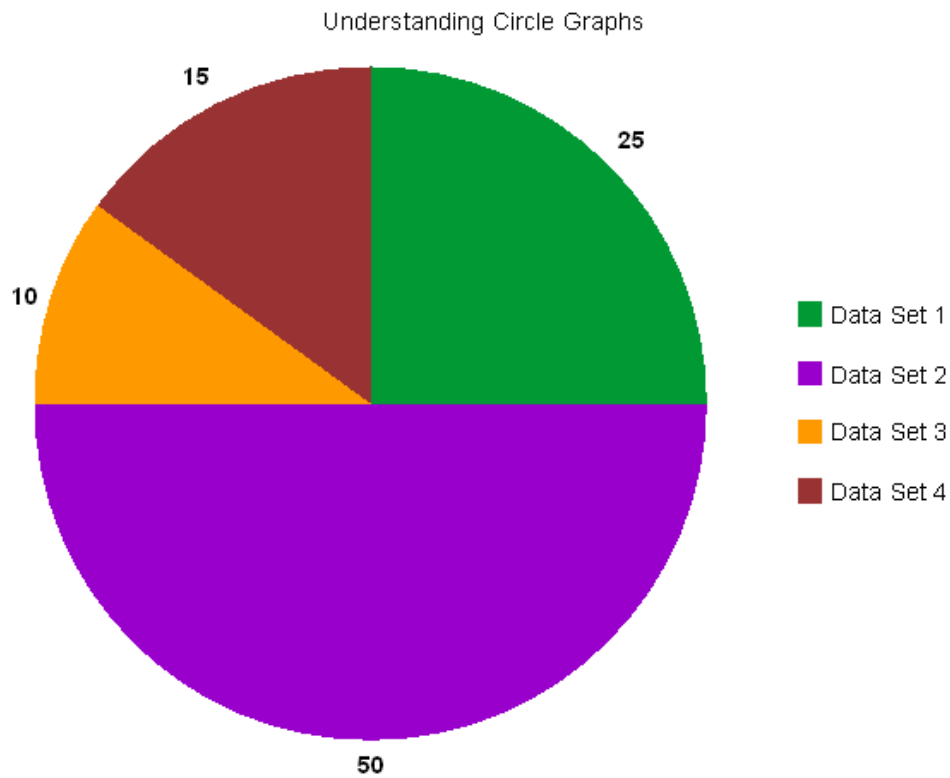
Data Set 2 is 50% or one-half of the entire circle. Again, this is represented numerically and by the proportion of the circle it occupies.

Data Set 3, with 10%, and Data Set 4, with 15%, account for a total of one-fourth, or 25%, of the circle.

You can use circle graphs to easily answer questions about data and to compare and contrast sections of data with other sections and against the whole data set.

The circle graph below displays data about the spending habits of a teenager.

Spending Habits
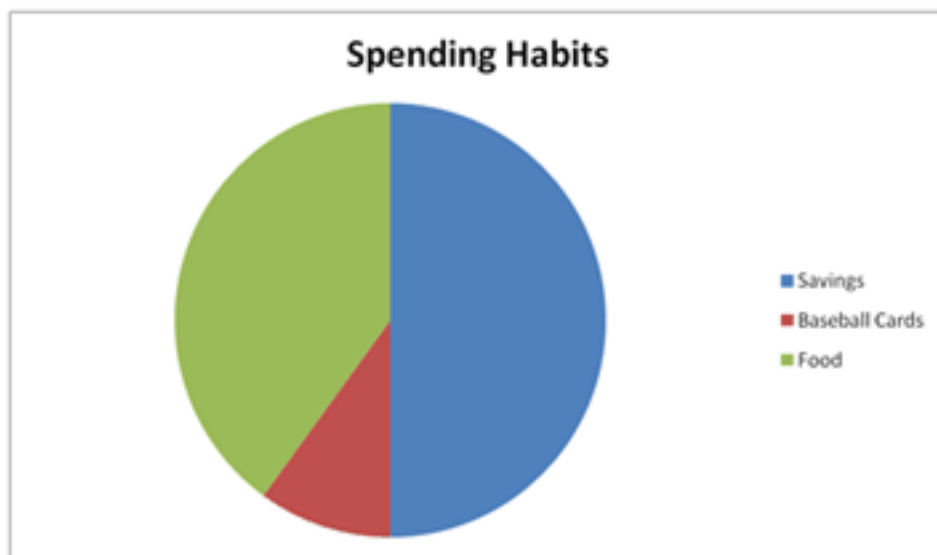
Even though this graph does not contain any numerical data, the way it is sectioned, you can estimate that 50% of this teen's money went into savings, while 40% was spent on food and 10% of his money was spent on baseball cards.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about the election results from the mayor's race.

Glenda has decided to display the results of the mayor's race in a circle graph to accompany her story. She uses a program on her computer to design the circle graph. Now, she has to display the results on the graph. The results of the election are below.

John Williams - 43%

Rachel Clark - 26%

Roger Parks - 19%

Paul Taylor - 12%

How should the sections of the graph be proportioned and labeled so that viewers will be able to quickly compare the data?

First, take another look at Glenda's data. The data is arranged and ordered.

John Williams - 43%

Rachel Clark - 26%

Roger Parks - 19%

Paul Taylor - 12%

Next, Glenda needs to create a circle graph with sections corresponding to the numerical data.
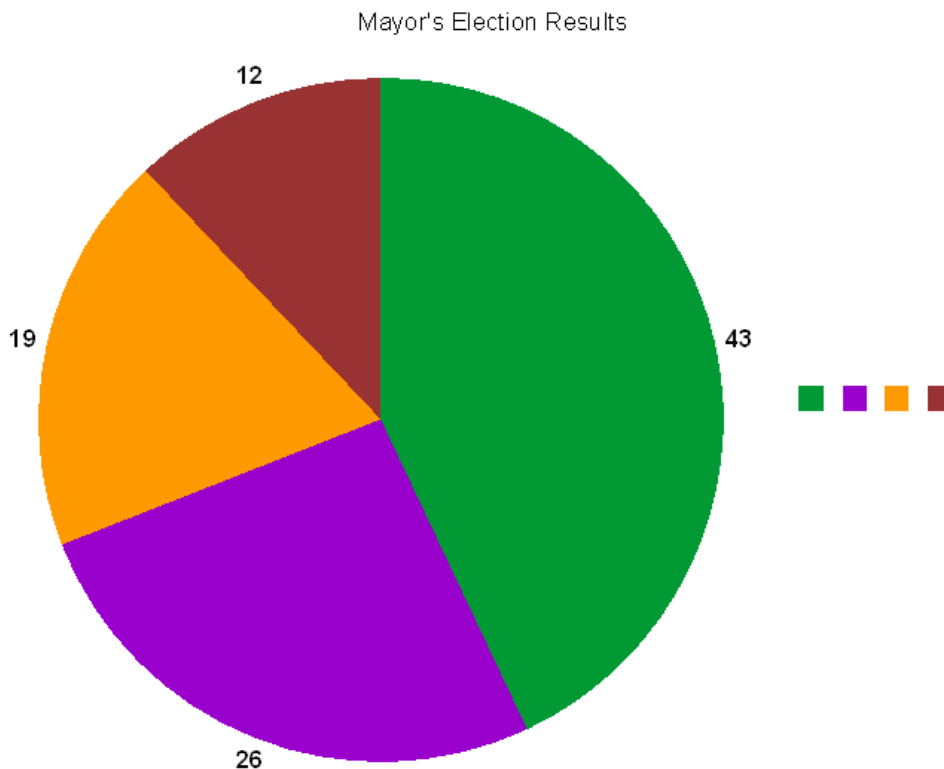
Then, she needs to add labels to help readers readily identify the data.

Mayor's Election Results
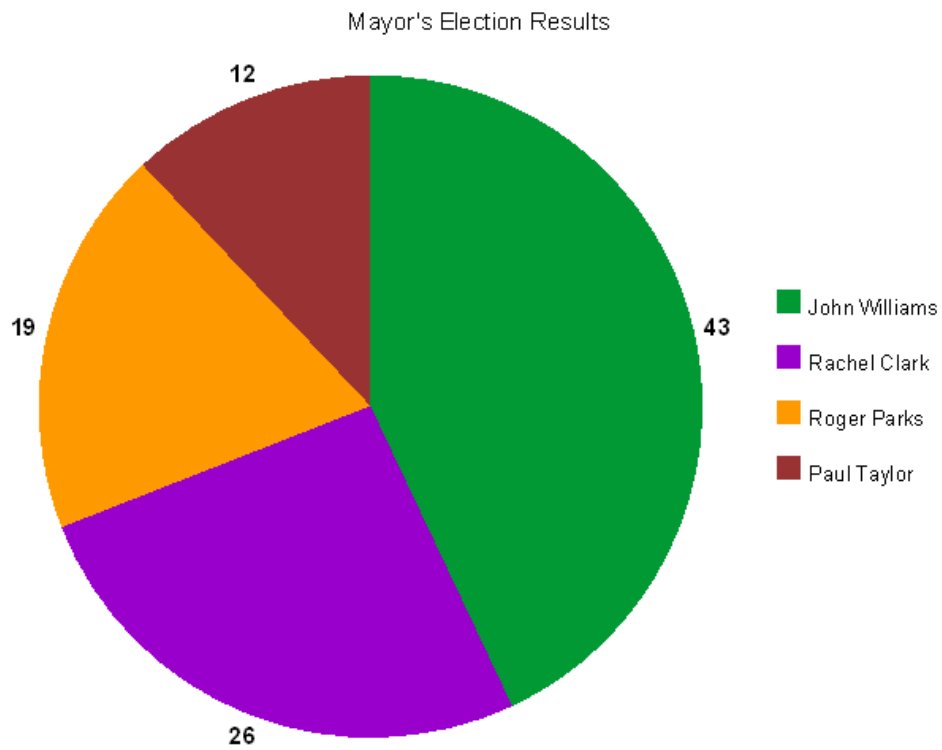
This is what Glenda's finished circle graph should look like.

Example 2

Answer the following question about the data displayed in this circle graph.

Students' Future Occupation Goals
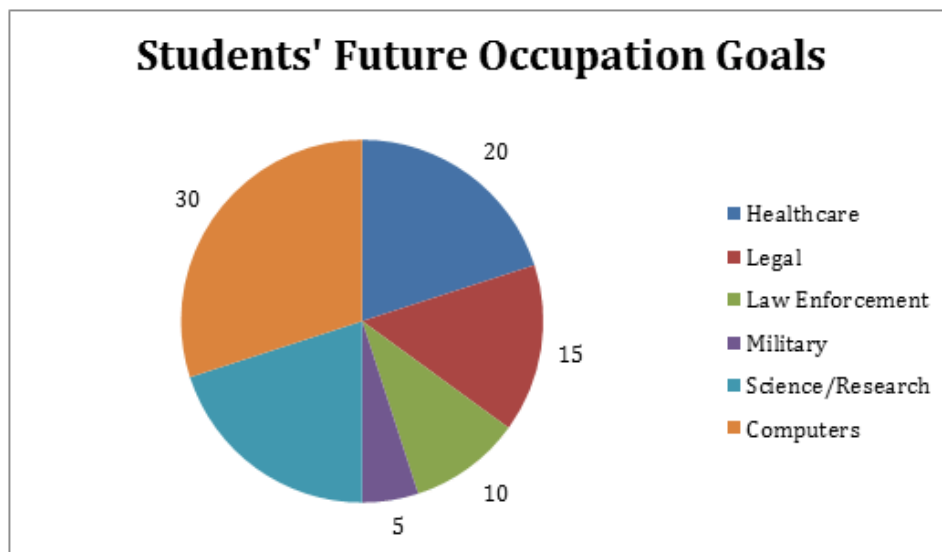
What percent of the students aspire to work in science and research?

To figure this out, look at the whole of the circle and remember that it is equal to 100%.

First, note the known percentages.

20% want a career in healthcare.

498

15% want a career in the legal field.

10% want a career in law enforcement.

5% want a career in the military.

30% want a career in computers.

Then, there is an unknown percent of the students that want to work in science / research.

Next, add the known percentages and subtract the total from 100 percent.

$$\begin{aligned} 20+15+10+5+30 &= 80 \\ 100-80 &= 20 \end{aligned}$$

The answer is 20% of the students want a career in science and research.

Take a look at the circle graph below and answer the questions in the following examples.

Favorite Type of Music
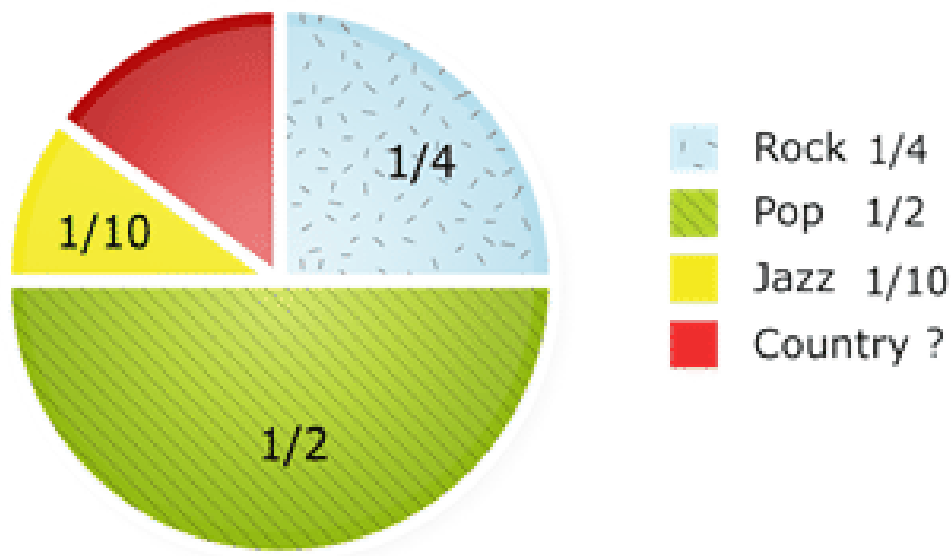
Example 3

What is the most popular music selection?

First, look at the circle and choose the largest section.

Pop music covers half of the circle.

The answer is pop music.

Example 4

What percent of the students chose jazz music?

First, find the section on the graph corresponding to jazz music.

The yellow-shaded area represents jazz music.

The answer is one tenth, or 10 percent.

Example 5

What percent of the students chose country music?

To figure this out, look at the whole of the circle and remember that it is equal to 100%.

First, note the known percentages.

One quarter is the same as 25%, so 25% represents Rock music.

One tenth is the same as 10%, so 10% represents Jazz music.

One half is the same as 50%, so 50% represents pop music.

There is an unknown percent of the students that enjoy country music.

Next, add the known percentages and subtract the total from 100 percent.

$$\begin{aligned} 25 + 10 + 50 &= 85 \\ 100 - 85 &= 15 \end{aligned}$$

The answer is 15% of the students favor country music.

PRACTICE

Use the results of the survey, which is displayed in the circle graph, to answer each question.

A survey of 300 people asked them to name their favorite spectator sport. The results are shown in the circle graph below.

Favorite Spectator Sport

■	Baseball	30%
▨	Basketball	20%
▦	Tennis	10%
▧	Soccer	17%
■	Ice Hockey	8%
■	Football	? %
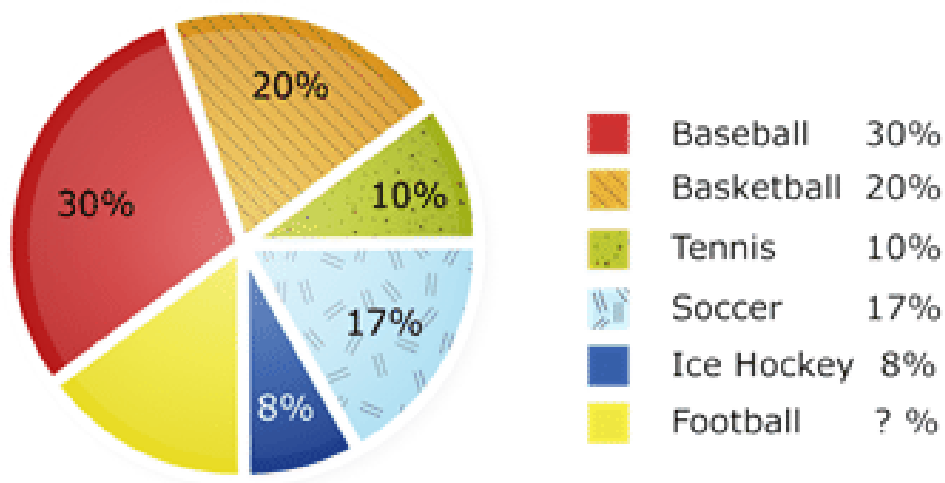
1. What was the most favorite spectator sport of the people surveyed?
2. What was the least favorite spectator sport of the people surveyed?
3. What percent of the people surveyed said that football was their favorite spectator sport
4. How many people said that basketball was their favorite spectator sport?
5. How many more people said that soccer was their favorite sport than ice hockey?
6. What percent of the people chose baseball and soccer as their favorite sports?
7. What percent of the people did not choose baseball?
8. What percent of the people did not choose hockey?
9. What percent of the people chose tennis or soccer as their favorite sports?
10. What percent of the people did not choose basketball or tennis?
11. What percent of the people did choose tennis as a favorite sport?
12. What percent of the people did not choose football?

The table shows the how much money the students in the seventh grade have raised so far for a class trip. Make a circle graph that shows the data.

TABLE 12.5:

Fundraiser	Amount
Car Wash	$150
Book sale	$175
Bake sale	$100
Plant sale	$75

13. Make a list of 5 popular ice cream flavors. Then survey your classmates asking them which of the 5 flavors is their favorite ice cream flavor. Use the data to make a circle graph.
14. Use a newspaper to locate a circle graph of some data. Then write five questions about the data.
15. Now create your own circle graph. Share it with a partner.

12.3 Dot Plots (7.6g)

GUIDANCE

As you might guess, a dotplot is made up of dots plotted on a graph. Here is how to interpret a dotplot.

- Each dot represents a specific number of observations from a set of data. (Unless otherwise indicated, assume that each dot represents one observation. If a dot represents more than one observation, that should be explicitly noted on the plot.)
- The dots are stacked in a column over a category, so that the height of the column represents the relative or absolute frequency of observations in the category.
- The pattern of data in a dotplot can be described in terms of symmetry and skewness only if the categories are quantitative. If the categories are qualitative (as they often are), a dotplot cannot be described in those terms.

GUIDED PRACTICE

Here is an example to show what a dotplot looks like and how to interpret it. Suppose 30 first graders are asked to pick their favorite color. Their choices can be summarized in a dotplot, as shown below.

TABLE 12.6:

* * * * * * * * * * *	* * *	* * * * *	* * * * * * *	*		* * *
Red	Orange	Yellow	Green	Blue	Indigo	Violet

Each dot represents one student, and the number of dots in a column represents the number of first graders who selected the color associated with that column. For example, Red was the most popular color (selected by 9 students), followed by Blue (selected by 7 students). Selected by only 1 student, Indigo was the least popular color.

In this example, note that the category (color) is a qualitative variable; so it is not appropriate to talk about the symmetry or skewness of this dotplot. The dotplot in the next section uses a quantitative variable, so we will illustrate skewness and symmetry of dotplots in the next section.

Test Your Understanding

Problem 1

The dotplot below shows the number of televisions owned by each family on a city block.

TABLE 12.7:

*	* * *	* * * * *	* * * *	* * *	* *	*	*	*
0	1	2	3	4	5	6	7	8

Which of the following statements are true?

(A) The distribution is right-skewed with no outliers. (B) The distribution is right-skewed with one outlier. (C) The

distribution is left-skewed with no outliers. (D) The distribution is left-skewed with one outlier. (E) The distribution is symmetric.

Solution

The correct answer is (A). Most of the observations are on the left side of the distribution, so the distribution is right-skewed . And none of the observations is extreme, so there are no outliers .

Note: Because the categories are quantitative (i.e., numbers), it is appropriate to describe the skewness of the data in this dotplot.

PRACTICE

1. The students in one social studies class were asked how many brothers and sisters (siblings) they each have. The dot plot here shows the results.
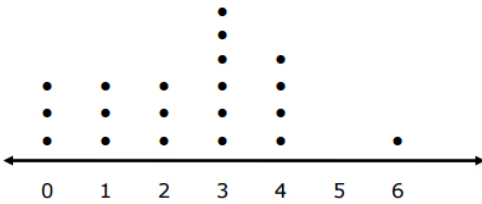
FIGURE 12.6

a. How many of the students have six siblings?

b. How many of the students have no siblings?

c. How many of the students have three or more siblings?

3. The height's of 20 basketball players, in inches, are given below.

68, 70, 70, 71, 75, 80, 81, 82, 84, 75

75, 80, 75, 77, 75, 80, 83, 80, 71, 70

a) Make a **dot plot** using the number line below.

FIGURE 12.7

68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84
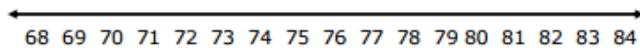
b) What is the spread (range) of the data?

c) What is the mode of the data?

d) How many players are greater than 70 inches tall?

4. The ages of 22 students in a karate class are given below.

11, 5, 9, 13, 8, 9, 9, 11, 10, 8, 6, 7,

12, 11, 13, 12, 7, 6, 11, 12, 10, 8

a) Make a **line plot** using the number line below.

FIGURE 12.8

5   6   7   8   9   10   11   12   13
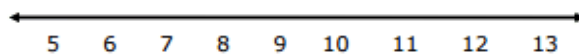
b) What is the spread (range) of the data?

c) What is the mode of the data?

d) How many students are *at least* 10 years old?

12.4 Box Plots (7.12a)

GUIDANCE

First watch this video to learn about box-and-whisker plots.
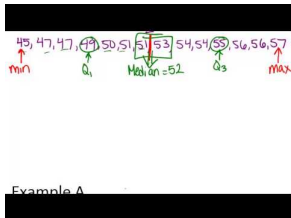
MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/137070
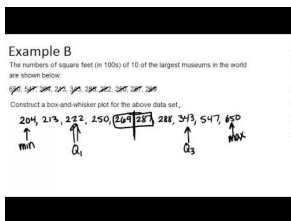
MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/137071

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/40

and plots
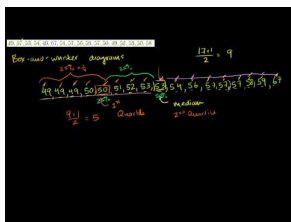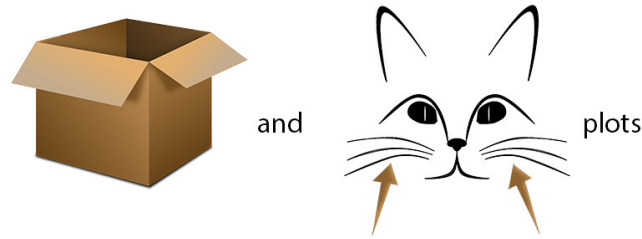
In traditional statistics, data is organized by using a frequency distribution. The results of the frequency distribution can then be used to create various graphs, such as a histogram or a frequency polygon, which indicate the shape or nature of the distribution. The shape of the distribution will allow you to confirm various conjectures about the nature of the data.

To examine data in order to identify patterns, trends, or relationships, exploratory data analysis is used. In exploratory data analysis, organized data is displayed in order to make decisions or suggestions regarding further actions. A **box-and-whisker plot** (often called a box plot) can be used to graphically represent the data set, and the graph involves plotting 5 specific values. The 5 specific values are often referred to as a **five-number summary** of the organized data set. The five-number summary consists of the following:

1. The lowest number in the data set (minimum value)
2. The median of the lower quartile: Q_1 (median of the first half of the data set)
3. The median of the entire data set (median)
4. The median of the upper quartile: Q_3 (median of the second half of the data set)
5. The highest number in the data set (maximum value)

The display of the five-number summary produces a box-and-whisker plot as shown below:
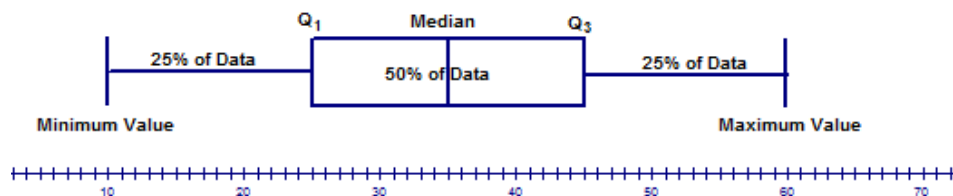
The above model of a box-and-whisker plot shows 2 horizontal lines (the whiskers) that each contain 25% of the data and are of the same length. In addition, it shows that the median of the data set is in the middle of the box, which contains 50% of the data. The lengths of the whiskers and the location of the median with respect to the center of the box are used to describe the distribution of the data. It's important to note that this is just an example. Not all box-and-whisker plots have the median in the middle of the box and whiskers of the same size.

Information about the data set that can be determined from the box-and-whisker plot with respect to the location of the median includes the following:

a. If the median is located in the center or near the center of the box, the distribution is approximately symmetric.

b. If the median is located to the left of the center of the box, the distribution is positively skewed.

c. If the median is located to the right of the center of the box, the distribution is negatively skewed.

Information about the data set that can be determined from the box-and-whisker plot with respect to the length of the whiskers includes the following:

a. If the whiskers are the same or almost the same length, the distribution is approximately symmetric.

b. If the right whisker is longer than the left whisker, the distribution is positively skewed.

c. If the left whisker is longer than the right whisker, the distribution is negatively skewed.

The length of the whiskers also gives you information about how spread out the data is.

A box-and-whisker plot is often used when the number of data values is large. The center of the distribution, the nature of the distribution, and the range of the data are very obvious from the graph. The five-number summary divides the data into quarters by use of the medians of the upper and lower halves of the data. Many data sets contain values that are either extremely high values or extremely low values compared to the rest of the data values. These values are called **outliers**. There are several reasons why a data set may contain an outlier. Some of these are listed below:

1. The value may be the result of an error made in measurement or in observation. The researcher may have measured the variable incorrectly.
2. The value may simply be an error made by the researcher in recording the value. The value may have been written or typed incorrectly.
3. The value could be a result obtained from a subject not within the defined population. A researcher recording marks from a math 12 examination may have recorded a mark by a student in grade 11 who was taking math 12.
4. The value could be one that is legitimate but is extreme compared to the other values in the data set. (This rarely occurs, but it is a possibility.)

If an outlier is present because of an error in measurement, observation, or recording, then either the error should be corrected, or the outlier should be omitted from the data set. If the outlier is a legitimate value, then the statistician must make a decision as to whether or not to include it in the set of data values. There is no rule that tells you what to do with an outlier in this case.

One method for checking a data set for the presence of an outlier is to follow the procedure below:

1. Organize the given data set and determine the values of Q_1 and Q_3.
2. Calculate the difference between Q_1 and Q_3. This difference is called the **interquartile range (IQR)**: $IQR = Q_3 - Q_1$.
3. Multiply the difference by 1.5, subtract this result from Q_1, and add it to Q_3.
4. The results from Step 3 will be the range into which all values of the data set should fit. Any values that are below or above this range are considered outliers.

Example A

For each box-and-whisker plot, list the five-number summary and describe the distribution based on the location of the median.
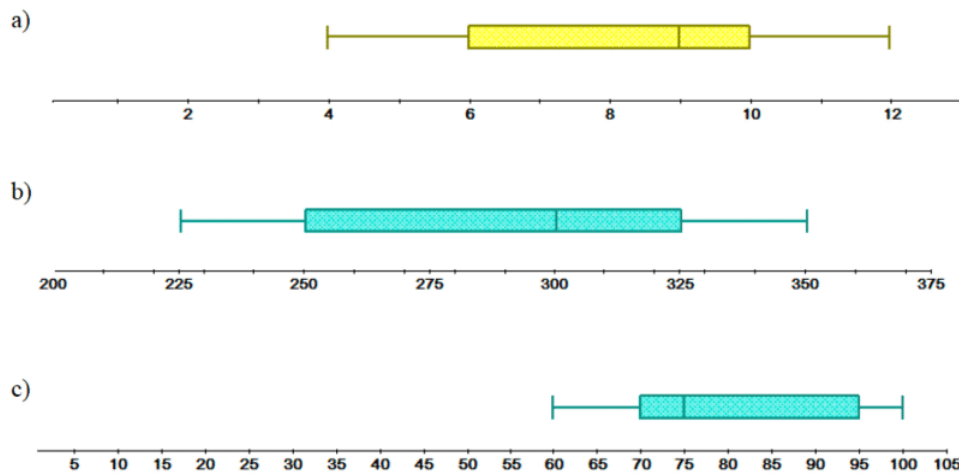
a. Minimum value $\rightarrow 4$

$Q_1 \rightarrow 6$

Median $\rightarrow 9$

$Q_3 \rightarrow 10$

Maximum value $\rightarrow 12$

The median of the data set is located to the right of the center of the box, which indicates that the distribution is negatively skewed.

b. Minimum value $\rightarrow 225$

$Q_1 \rightarrow 250$

Median $\rightarrow 300$

$Q_3 \rightarrow 325$

Maximum value $\rightarrow 350$

The median of the data set is located to the right of the center of the box, which indicates that the distribution is negatively skewed.

c. Minimum value $\rightarrow 60$

$Q_1 \rightarrow 70$

Median $\rightarrow 75$

$Q_3 \rightarrow 95$

Maximum value $\rightarrow 100$

The median of the data set is located to the left of the center of the box, which indicates that the distribution is positively skewed.

Example B

The numbers of square feet (in 100s) of 10 of the largest museums in the world are shown below:

650, 547, 204, 213, 343, 288, 222, 250, 287, 269

Construct a box-and-whisker plot for the above data set and describe the distribution.

The first step is to organize the data values as follows:

$$20{,}400 \quad 21{,}300 \quad 22{,}200 \quad 25{,}000 \quad 26{,}900 \quad 28{,}700 \quad 28{,}800 \quad 34{,}300 \quad 54{,}700 \quad 65{,}000$$

Now calculate the median, Q_1, and Q_3.

$$20{,}400 \quad 21{,}300 \quad 22{,}200 \quad 25{,}000 \quad \boxed{26{,}900 \quad 28{,}700} \quad 28{,}800 \quad 34{,}300 \quad 54{,}700 \quad 65{,}000$$

$$\text{Median} \rightarrow \frac{26{,}900 + 28{,}700}{2} = \frac{55{,}600}{2} = 27{,}800$$

20,400   21,300   22,200   25,000   26,900 | 28,700   28,800   34,300

54,700   65,000
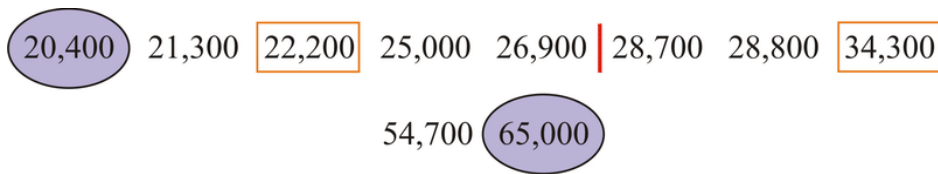
$Q_1 = 22{,}200$

$Q_3 = 34{,}300$

Next, complete the following list:

Minimum value $\rightarrow 20{,}400$

$Q_1 \rightarrow 22{,}200$

Median $\rightarrow 27{,}800$

$Q_3 \rightarrow 34{,}300$

Maximum value $\rightarrow 65{,}000$
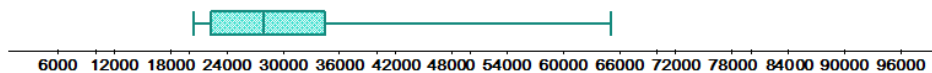
The right whisker is longer than the left whisker, which indicates that the distribution is positively skewed.

Example C

Using the procedure outlined above, check the following data sets for outliers:

a. 18, 20, 24, 21, 5, 23, 19, 22

b. 13, 15, 19, 14, 26, 17, 12, 42, 18

a. Organize the given data set as follows:

18, 20, 24, 21, 5, 23, 19, 22

5, 18, 19, 20, 21, 22, 23, 24

Determine the values for Q_1 and Q_3.

$$5, \boxed{18, \ 19}, \ 20, \ 21, \ \boxed{22, \ 23}, \ 24$$

$$Q_1 = \frac{18+19}{2} = \frac{37}{2} = 18.5 \qquad Q_3 = \frac{22+23}{2} = \frac{45}{2} = 22.5$$

Calculate the difference between Q_1 and Q_3: $Q_3 - Q_1 = 22.5 - 18.5 = 4.0$.

Multiply this difference by 1.5: $(4.0)(1.5) = 6.0$.

Finally, compute the range.

$$Q_1 - 6.0 = 18.5 - 6.0 = 12.5$$

$$Q_3 + 6.0 = 22.5 + 6.0 = 28.5.$$

Are there any data values below 12.5? Yes, the value of 5 is below 12.5 and is, therefore, an outlier.

Are there any values above 28.5? No, there are no values above 28.5.

b. Organize the given data set as follows:

$$13, \ 15, \ 19, \ 14, \ 26, \ 17, \ 12, \ 42, \ 18$$
$$12, \ 13, \ 14, \ 15, \ 17, \ 18, \ 19, \ 26, \ 42$$

Determine the values for Q_1 and Q_3.

$$12, \boxed{13, \ 14}, \ 15, \ \boxed{17}, \ 18, \ \boxed{19, \ 26}, \ 42$$

$$Q_1 = \frac{13+14}{2} = \frac{27}{2} = 13.5 \qquad Q_3 = \frac{19+26}{2} = \frac{45}{2} = 22.5$$

Calculate the difference between Q_1 and Q_3: $Q_3 - Q_1 = 22.5 - 13.5 = 9.0$.

Multiply this difference by 1.5: $(9.0)(1.5) = 13.5$.

Finally, compute the range.

$$Q_1 - 13.5 = 13.5 - 13.5 = 0$$

$$Q_3 + 13.5 = 22.5 + 13.5 = 36.0$$

Are there any data values below 0? No, there are no values below 0.

Are there any values above 36.0? Yes, the value of 42 is above 36.0 and is, therefore, an outlier.

Points to Consider

- Are there still other ways to represent data graphically?
- Are there other uses for a box-and-whisker plot?
- Can box-and-whisker plots be used for comparing data sets?

–>

GUIDED PRACTICE

a. For the following data sets, determine the five-number summaries:

i. 12, 16, 36, 10, 31, 23, 58

ii. 144, 240, 153, 629, 540, 300

b. Use the data set for part i of the previous question and the five-number summary to construct a box-and-whisker plot to model the data set.

Answer:

a. i. The first step is to organize the values in the data set as shown below:

$$12, \ 16, \ 36, \ 10, \ 31, \ 23, \ 58$$
$$10, \ 12, \ 16, \ 23, \ 31, \ 36, \ 58$$

$$\boxed{10}, \boxed{12}, 16, \boxed{23}, 31, \boxed{36}, \boxed{58}$$

Now complete the following list:

Minimum value $\rightarrow 10$

$Q_1 \rightarrow 12$

Median $\rightarrow 23$

$Q_3 \rightarrow 36$

Maximum value $\rightarrow 58$

ii. The first step is to organize the values in the data set as shown below:

$$144, \ 240, \ 153, \ 629, \ 540, \ 300$$
$$144, \ 153, \ 240, \ 300, \ 540, \ 629$$

$$\boxed{144}, \ 153, \boxed{240, \ 300}, \ 540, \boxed{629}$$

$$\frac{240 + 300}{2} = \frac{540}{2} = 270$$

$$144, \boxed{153}, \ 240 \mid 300, \boxed{540}, \ 629$$

Now complete the following list:

Minimum value → 144

Q_1 → 153

Median → 270

Q_3 → 540

Maximum value → 629

b. The five-number summary can now be used to construct a box-and-whisker plot for part i. Be sure to provide a scale on the number line that includes the range from the minimum value to the maximum value.

Minimum value → 10

Q_1 → 12

Median → 23

Q_3 → 36

Maximum value → 58
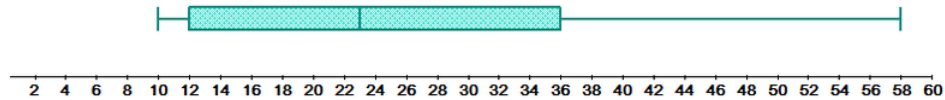
It is very visible that the right whisker is much longer than the left whisker. This indicates that the distribution is positively skewed.

PRACTICE

1. Which of the following is not a part of the five-number summary?

   a. Q_1 and Q_3
   b. the mean
   c. the median
   d. minimum and maximum values

2. What percent of the data is contained in the box of a box-and-whisker plot?

   a. 25%
   b. 100%
   c. 50%
   d. 75%

3. What name is given to the horizontal lines to the left and right of the box of a box-and-whisker plot?

   a. axis
   b. whisker
   c. range
   d. plane

4. What term describes the distribution of a data set if the median of the data set is located to the left of the center of the box in a box-and-whisker plot?

   a. positively skewed
   b. negatively skewed
   c. approximately symmetric

> d. not skewed

5. What 2 values of the five-number summary are connected with 2 horizontal lines on a box-and-whisker plot?

> a. Minimum value and the median
> b. Maximum value and the median
> c. Minimum and maximum values
> d. Q_1 and Q_3

6. For the following data sets, determine the five-number summaries:

> a. 74, 69, 83, 79, 60, 75, 67, 71
> b. 6, 9, 3, 12, 11, 9, 15, 5, 7

7. For each of the following box-and-whisker plots, list the five-number summary and comment on the distribution of the data:

a.

b.
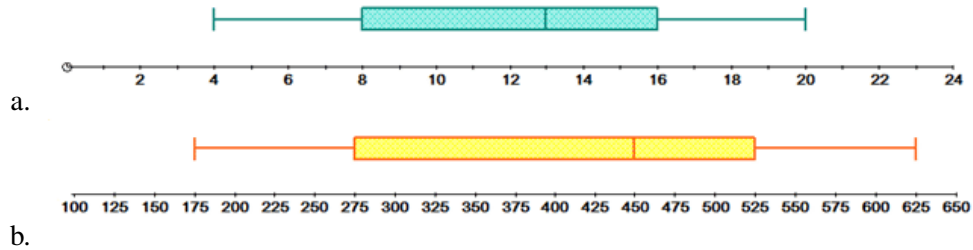
8. The following data represents the number of coins that 12 randomly selected people had in their piggy banks:

$$35 \quad 58 \quad 29 \quad 44 \quad 104 \quad 39 \quad 72 \quad 34 \quad 50 \quad 41 \quad 64 \quad 54$$

Construct a box-and-whisker plot for the above data.

9. The following data represent the time (in minutes) that each of 20 people waited in line at a local book store to purchase the latest Harry Potter book:

$$15 \quad 8 \quad 5 \quad 10 \quad 14 \quad 17 \quad 21 \quad 23 \quad 6 \quad 19 \quad 31 \quad 34 \quad 30 \quad 31$$
$$3 \quad 22 \quad 17 \quad 25 \quad 5 \quad 16$$

Construct a box-and-whisker plot for the above data. Are the data skewed in any direction?

10. Firman's Fitness Factory is a new gym that offers reasonably-priced family packages. The following table represents the number of family packages sold during the opening month:

24	21	31	28	29
27	22	27	30	32
26	35	24	22	34
30	28	24	32	27
32	28	27	32	23
20	32	28	32	34

Construct a box-and-whisker plot for the data. Are the data symmetric or skewed?

11. Shown below is the number of new stage shows that appeared in Las Vegas for each of the past several years. Construct a box-and-whisker plot for the data and comment of the shape of the distribution.

$$31 \quad 29 \quad 34 \quad 30 \quad 38 \quad 40 \quad 36 \quad 38 \quad 32 \quad 39 \quad 35$$

12. The following data represent the average snowfall (in centimeters) for 18 Canadian cities for the month of January. Construct a box-and-whisker plot to model the data. Is the data skewed? Justify your answer.

TABLE 12.8:

Name of City	Amount of Snow(cm)
Calgary	123.4
Charlottetown	74.5
Edmonton	80.6
Fredericton	73.8
Halifax	64.0
Labrador City	110.4
Moncton	82.4
Montreal	63.6
Ottawa	48.9
Quebec City	53.8
Regina	35.9
Saskatoon	25.4
St. John's	97.5
Sydney	44.2
Toronto	21.8
Vancouver	12.8
Victoria	8.3
Winnipeg	76.2

13. Using the procedure outlined in this concept, check the following data sets for outliers:

     a. 25, 33, 55, 32, 17, 19, 15, 18, 21
     b. 149, 123, 126, 122, 129, 120

12.5 Comparing Groups of Data (7.12a)

GUIDANCE

Choices for Data Display

Data is a collection of numbers brought together in order for analysis. Temperature, rainfall, salinity, money spent, percentage of minority students, etc. are all different types of data.

Often, raw data will be displayed in a **table** which is organized in some sort of clear order but without any analysis having been performed.

A **graph** is a way of representing data that includes analysis. The right graph for the data set will make important trends in the data clear. There are many different kinds of graphs, and each is particularly effective with different kinds of data.

A **bar graph** displays the frequency of data, or how often data occurs. Here is an example.
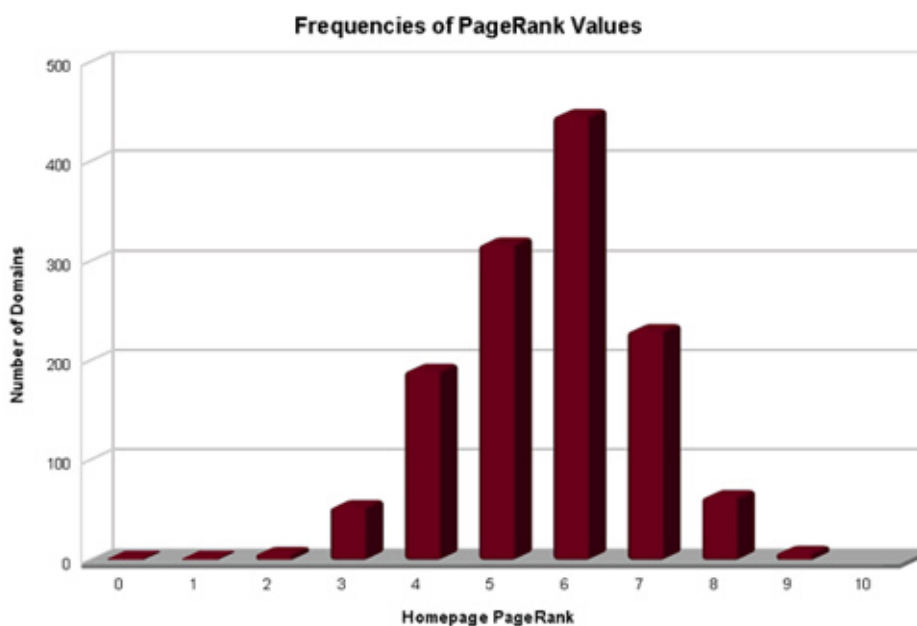
A **double bar graph** compares the frequency of two sets of data. Here is an example.
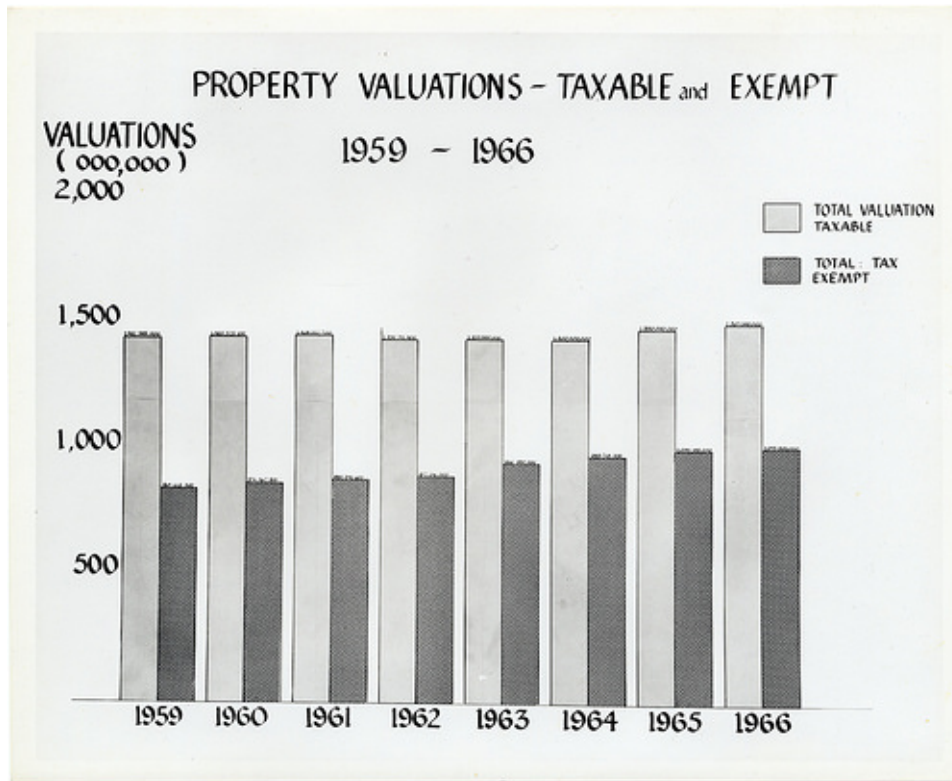
A **line graph** shows how data changes over time. Here is an example.

A **double line graph** compares how two sets of data change over time. This example compares six sets of data.
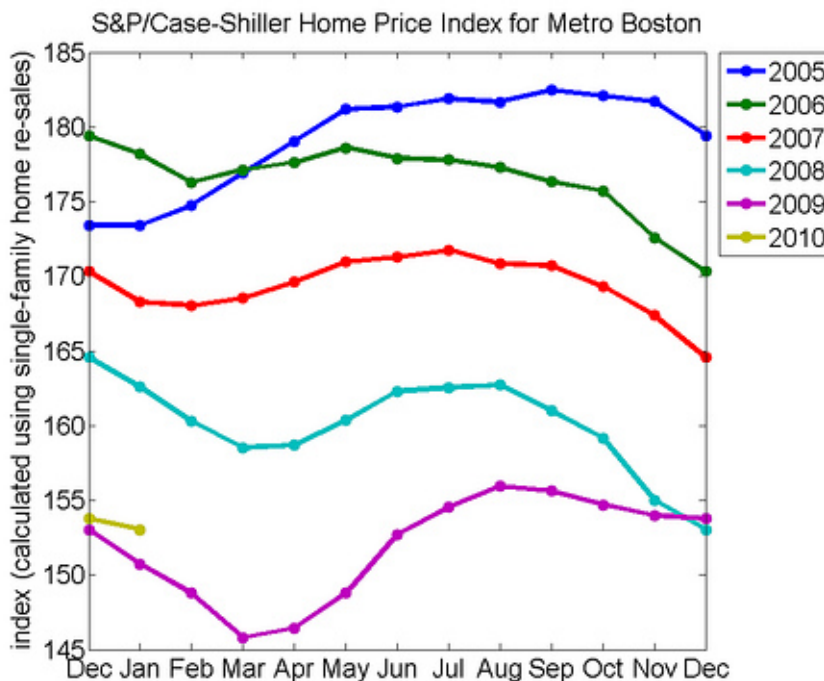
A **pie chart** shows a percentage out of a whole. Here is an example.
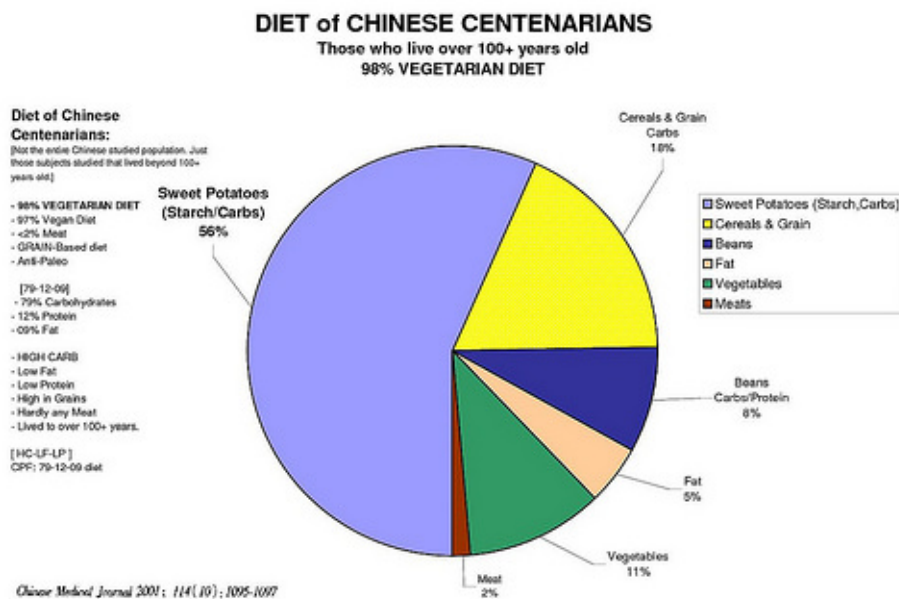
So, if a girl scout sells 20 boxes of thin mints, 13 boxes of samoas, and 5 boxes of tagalongs a table showing this data would look like:

TABLE 12.9:

	Thin Mints	Samoas	Tagalongs
Girl Scout 1	20	13	5

But if she wanted to represent her data in a visual way so that it was immediately clear which cookie sold the best, she would choose to display her data with a bar graph because number of cookie boxes sold is a type of frequency.

Now, if she wanted to compare her cookie sales to those of another girl scout, she would choose a double bar graph because that would immediately show the relative frequencies of the two.

GUIDED PRACTICE

Example 1

Earlier, you were given a problem about Rahul and his question about California rainfall.

He is trying to decide how best to display his data from the table below.

TABLE 12.10:

Season (July 1-June 30) (Year given represents end of season)	Inches Above/Below (+/-) 135 Year Average
2012	-6.29
2011	+5.22
2010	+1.38
2009	-5.90
2008	-1.45
2007	-11.77
2006	-1.79
2005	+22.98
2004	-5.73
2003	+1.44
2002	-10.56
2001	+2.96
2000	-3.41
1999	-5.89
1998	+16.03
1997	-2.58
1996	-2.54
1995	+9.37
1994	-6.87
1993	+12.38
1992	+6.02
1991	-2.99
1990	-7.63
1989	-6.90
1988	-2.50
1987	-7.32
1986	+2.88
1985	-2.16
1984	-4.55
1983	+16.3
1982	-4.27
1981	-6.02
1980	+12.00

1979	+4.69
1978	+18.46
1977	-2.68
1976	-7.77
1975	-0.63
1974	-0.06
1973	+6.28
1972	-7.81
1971	-2.66
1970	-7.24
1969	+12.49
1968	+1.60
1967	+7.02
1966	+5.46
1965	-1.30
1964	-7.05
1963	-6.60

First, he determines what type of data is given: frequency, change over time, or percentage of a whole.

In this case, it is change in rainfall over time. So it would best be described using a line graph.

Next, he determines how many lines of data there are.

He only has one rainfall value/year and his chart does not have any other type of data, so he concludes that a single line graph would be best.

Example 2

Determine which type of graph will best display the data below.

Keith gathered the following data about how bus fare changed in his town over a period of five years. He wants to bring it to the PTA to show that free bus passes ought to be given to students.

2007 .35

2008 .35

2009 .40

2010 .40

2011 .45

First, determine what type of data is given. Is the data a matter of frequency, change over time, or percentage of a whole?

In this case, it is change over time. So it would best be described using a line graph.

Next, determine whether there is just one set of data that needs to be shown or if there are multiple lines of data to compare.

In this case, there is just one bus fare, so a single line graph will suffice.

The answer is a single line graph.

Example 3

What graph would best show how often someone eats ice cream?

First, determine what type of data is given. Is the data a matter of frequency, change over time, or percentage of a whole?

In this case, the data is about frequency. So it would best be described using a bar graph.

Next, determine whether there is just one set of data that needs to be shown or if there are multiple lines of data to compare.

In this case, there is just one person eating ice cream, so a single bar graph will suffice.

The answer is a single bar graph.

Example 4

What graph would best describe how much of someone's monthly budget is allocated to rent, food, and other bills?

First, determine what type of data is given. Is the data a matter of frequency, change over time, or percentage of a whole?

In this case, it is the percentage of someone's entire budget. So it would best be described using circle graph.

The answer is a pie chart.

Example 5

What would be the best way to show that climate is changing over time?

First, determine what type of data is given. Is the data a matter of frequency, change over time, or percentage of a whole?

In this case, it is change over time. So it would best be described using a line graph.

Next, determine whether there is just one set of data that needs to be shown or if there are multiple lines of data to compare.

In this case, it would depend. You could just show that the temperature was raising over time. Or you could add on top of that other indicators like CO2 levels or levels of water melted in polar icecaps that have also changed over time.

So this data could be presented either with a single or multiple line graph.

PRACTICE

Select the best display for each description of data. Choose from circle graph, line graph, double line graph, bar graph or double bar graph.

1. The percentages of people who enjoy ice cream
2. How stamp prices have changed over time
3. How stamp prices changed in 1996 and in 1998.
4. The number of students who attended college in 1990, 1991, and 1992
5. The percentages of people who prefer chocolate, vanilla or strawberry ice cream.
6. The changes in prices at one movie theater over a period of three years.
7. The changes in prices at two different movie theaters over a period of three years.
8. A graph showing how sales had declined during the past month
9. A graph showing the number of students with perfect attendance during the past three months.

10. A graph showing the number of students with perfect attendance at two different schools during the past three months.
11. The percentages of students who complete homework.
12. The percentages of students who enjoy playing particular sports.
13. How the percentages of students who attend college has changed over time.
14. How the price of a hamburger has changed over time.
15. The percentages of people who enjoy watching particular sports events.

12.6 Population Inferences (7.12c)

GUIDANCE

In this concept you will learn how to accurately gather data about a large population without needing to get responses from each and every member.

Concept

Suppose you were chosen to help pick out a theme for your school prom. Out of all of the initial suggestions offered by your team, you have narrowed the options down to 3: Famous Couples through the Ages, Romance Under the Sea, and Stairway to Heaven.

Since this is the Senior Prom, you feel that the Senior Class should make the final call. Unfortunately, there are over three hundred seniors in your school, and your deadline for a decision is in one hour! How could you get a good idea of the preference of the class as a whole in such a limited time?

By the end of this lesson, you should have no problem suggesting a good solution!

Watch This

MEDIA
Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/62198
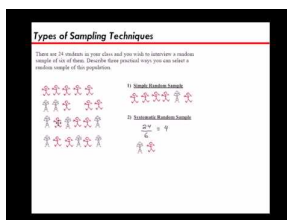
Guidance continued

There are many situations in life where we need to gather data on a very large or difficult to study ***population.*** Certainly it is ideal in most cases to be able to individually ***poll*** each and every member, but sometimes that just isn't feasible.

In such cases, the solution is to use a ***sample*** or ***subset*** that is carefully picked to accurately represent the full population. An experiment conducted on a well-chosen sample should provide an accurate representation of the results you would get by performing the same experiment on the population from which the sample was created.

There are many different ways to choose a sample, and all have applications for which they are more or less appropriate.

A few examples of sampling methods:

- Random Sampling (choosing representatives by rolling a die, for instance)
- Stratified Sampling (choosing a proportional number of representatives from each of a number of subgroups of the initial population) These divisions are chosen based on the belief that the subgroups differ significantly with respect to the variable that you are measuring. For example you might stratify by age or by income.
- Cluster Sampling (choosing representatives which are close to other representatives based on a particular factor such as location, age, color, size, etc.)
- Multi-Stage Sampling (narrowing down a field of representatives by successively applying multiple different sampling methods) For example you might stratify and then take a simple random sample from each stratum.

Example A

Would it be necessary to use a sample group to evaluate the effects of too much sugar on a group of 15 elementary-school children? What about a playground full of 300 children?

Solution: 15 children certainly seems like a manageable size group for study, so choosing a sample to represent the whole group is probably not necessary from that standpoint. However, this is the type of study where a ***control group*** would be an important consideration. If you just gave an extra handful of candy to every child, you would not know how much of the later energy actually came from the sugar, and how much was just a result of age. By pulling aside a control group of perhaps 6 students who would *not* get the extra sugar, you could better evaluate the *difference* in energy actually due to diet rather than age.

With 300 children all running around a playground, collecting them all together and attempting to organize a study might prove a daunting task. If you just chose a sample of perhaps 30 of them, some a little older, some younger, some boys, some girls, you could get an estimate of what would happen if you applied the study to the entire group.

Example B

Suppose you wanted to study the effect of rubbing marbles with candle wax before playing a classic game of marbles. After setting aside a control group, you are ready to choose a sample set of marbles to rub with the wax. Would a stratified sampling of the remaining marbles be a good choice in this situation?

Solution: Probably not. Marbles are generally created to be as alike as possible in every way other than appearance, and since appearance is unlikely to have an effect on the result of the wax experiment, it would not make sense to carefully attempt to represent each color or type of decoration. A random sample would be simpler and would very likely yield the same results.

Example C

The student council at your school has been given an assignment to find a good use for a grant that the school received to make school more enjoyable for the students. After a week or two of deliberation, the council announces that the studies they have conducted suggest that providing the cheerleading squad with new pom-poms is the #1 priority of 90% of the students in the school. Of course, the chess club members disagree and conduct their own study. If the chess team chooses a sample the same way the student council did, and their results suggest that 90% of respondents think that the money should go toward new chess clocks, what error do you think both groups committed in the choice of sample groups for study?

Solution: It would certainly appear that both groups were guilty of a process called 'cherry-picking', which means that they deliberately chose to question people who shared the same interests in order to get favorable results from their polls. Obviously neither group's results are likely to be representative of the entire student body, but rather only represent the views of the chess team and the cheerleading squad!

Concept Problem Revisited

Suppose you were chosen to help pick out a theme for your school prom. Out of all of the initial suggestions offered by your team, you have narrowed the options down to 3: Famous Couples through the Ages, Romance Under the Sea, and Stairway to Heaven.

Since this is the Senior Prom, you feel that the Senior Class should make the final call. Unfortunately there are over three hundred seniors in your school, and your deadline for a decision is in one hour! How could you get a good idea of the preference of the class as a whole in such a limited time?

This is an excellent case of the need for a representative sample of a population. Without having the time to poll all of the members of the senior class, you could get an idea of what the most popular theme would be by choosing a smaller number of seniors to represent the entire class. Just be careful to minimize the chance that your chosen representatives have any sort of *bias* that might keep them from properly representing the class as a whole.

Vocabulary

A *population* is the complete set (every single member) of a group of possible items to be studied.

To *poll* the members of a group means to question them regarding a specific topic.

A *sample* or *subset* is a smaller group of members chosen to represent a larger group. Properly chosen, a sample should provide the same results (on a smaller scale) as the population from which it was created.

A *control* group is a set of members deliberately kept as separate as possible from a particular study so as to provide an example of how the members should appear if unchanged.

Bias refers to a desire to achieve a specific result from a particular study, regardless of the data.

Random Sampling (choosing representatives by rolling a die, for instance)

Stratified Sampling (choosing a proportional number of representatives from each of a number of subgroups of the initial population) These divisions are chosen based on the belief that the subgroups differ significantly with respect to the variable that you are measuring. For example you might stratify by age or by income.

Cluster Sampling (choosing representatives which are close to other representatives based on a particular factor such as location, age, color, size, etc.)

Multi-Stage Sampling (narrowing down a field of representatives by successively applying multiple different sam-

pling methods) For example you might stratify and then take a simple random sample from each stratum.

GUIDED PRACTICE

1. What kind of sampling would you expect was used if the sample group was composed of 5 yellow, 3 green, 4 red, and 6 blue members, and the population included 48 blue, 32 red, 24 green, and 40 yellow members?
2. What type(s) of sampling method(s) might be most appropriate for approximating the number of cutthroat trout in a 25-mile section of river?
3. Would you reasonably expect bias to have affected a sample composed of 75% Toyota vehicles in a study of the most common cars in large U.S. cities?
4. Would a random sampling of students be the most appropriate method of sampling for a study of the most enjoyable after-school club in a large public school?
5. What might you conjecture about a study that claims 100% of respondents preferred "Super Sweet and Crunchy" cereal over "Super Duper Sweet" cereal?

Solutions:

1. Since the sample group contains exactly $\frac{1}{8}$ as many members of each color as the entire population, it is reasonable to suspect that a ***stratified sampling*** was used.
2. A 25-mile-long section of river is likely to include a number of different types of ecosystems that each would harbor a different density of fish. In order to get a good sample, a ***multi-stage*** sampling method comprised of a stratified sample of different ecosystems followed by a random sampling of fish in each ecosystem would probably be a good choice.
3. Although Toyota is a very popular vehicle manufacturer, 75% is an extremely high percentage of vehicles in a large city (reasonable estimates put Toyota somewhere between 25 and 30 percent). Such a huge number would definitely suggest sample bias.
4. Probably not, since a random sampling would likely include a large number of students who either have no opinion or have no experience with any after school clubs. More accurate results would be obtained by a multi-stage sample that first identified club members, and then randomly selected representatives from them.
5. There are a number of reasonable specific conjectures we might make, most related to inaccurate sampling methodology. Perhaps the sample was chosen from employees of the "Super Sweet and Crunchy" cereal company, perhaps respondents were offered a reward for choosing one option over the other, perhaps there was only a single member of the sample group or the "study" didn't include milk for the other cereal, or didn't offer samples of "Super Duper Sweet" to respondents at all

PRACTICE

1. Margo collected 12 carrots in a bag. She drew 5 carrots out of the bag. Is this a random sample of the carrots in the bag?

2. Chris put some assorted colored kerchiefs into a box. He looks into the box and pulls out the blue kerchiefs. Is this is a random sample of the kerchiefs in the bag?

3. Sue had red and white beans in a jar. She reached in and pulled out 10 beans, without looking in the jar. Is this a random sample of beans from the jar?

For questions 4-6, identify the population and the sample from each:

For example: In a class of 20 students, where each student is asked if they have gone to the movies in the past month, you would identify the population as 20 Students, and the sample as 20 students.

4. People aboard a plane who have aisle seats are asked if they travel more than 5000 miles per year.

a. Population:

b. Sample:

5. A team of marketing specialists survey every sixth child entering a park to find out how many rides they plan to go on while playing in the park.

a. Population:

b. Sample:

6. Every 15th adult at the exit door of the grocery store is questioned to find out if the store should increase its hours of operation.

a. Population:

b. Sample:

7. Luke wants to find out where most high school students buy their food for lunch. He surveys every fourth student he sees in the high school parking lot and asks them where they get food for lunch. Which would have been an improvement in Luke's experiment?

a. Survey all of the students in the school.

b. Survey all people in the parking lot.

c. Survey students in the lunch hall.

8. Sue is trying to determine the best location to sell snow cones. There are 4 locations in the city (on a side street, downtown, near a park and at a school. Sue observed that many people visit the downtown area and the park. Sue decided to sell snow cones in the downtown area where she saw the most people gather. What changes to Sue's sample would have given her a better understanding of where to sell snow cones?

9. Kerry collected shells from a visit to the ocean in a shoebox. She takes out a handful of shells from the box. Is this a random sample of shells in a box?

10. There are four dentists in a city. Their offices are located in four different parts of the city. Jake wants to attempt to figure out which dentist has the most patients. He observed that the Downtown and West Street areas have larger populations. He concurred that the dentists in those areas must have more patients. After comparing those two areas, he decided that the West Street dentist had the most patients because the area had more traffic. What changes to Jakes technique would have given him a better understanding of which doctor had the most patients?

11. Caroline wants to predict which restaurant will have less business during the Christmas season. There are three restaurants in the city. Two are on the outskirts of a city and one is in the city. She knows that two hotels situated on the outskirts are fully booked because one has Christmas show and one has a huge indoor pool. From this information she inferred that the restaurant in the city will have less business during the Christmas season. What could Caroline due to improve her experiment?

a. Ask people at the hotels if they like fast food.

b. Survey all people to see which December holiday they celebrate.

c. Look at the past holiday performance of the restaurants.

The table gives information about the number of girls in each of four schools.

TABLE 12.11:

School	A	B	C	D	Total
Number of Girls	126	82	201	52	461

12. Jenny did a survey of these girls. She used a stratified sample of exactly 80 girls according to school. Cal-

culate the number of girls from each school that were in her sample of 80. Complete the table.

TABLE 12.12:

School	A	B	C	D	Total
Number of Girls					80

12.7 Interpreting Data (7.6g and 7.12b)

Four Ways to Describe Data Sets

When you compare two or more data sets, focus on four features:

- **Center.** Graphically, the center of a distribution is the point where about half of the observations are on either side.
- **Spread.** The spread of a distribution refers to the variability of the data. If the observations cover a wide range, the spread is larger. If the observations are clustered around a single value, the spread is smaller.
- **Shape.** The shape of a distribution is described by symmetry , skewness , number of peaks, etc.
- **Unusual features.** Unusual features refer to gaps (areas of the distribution where there are no observations) and outliers .

The remainder of this lesson shows how to use various graphs to compare data sets in terms of center, spread, shape, and unusual features. (This is a skill that students are expected to master for the Advanced Placement Statistics Exam.)

Dotplots
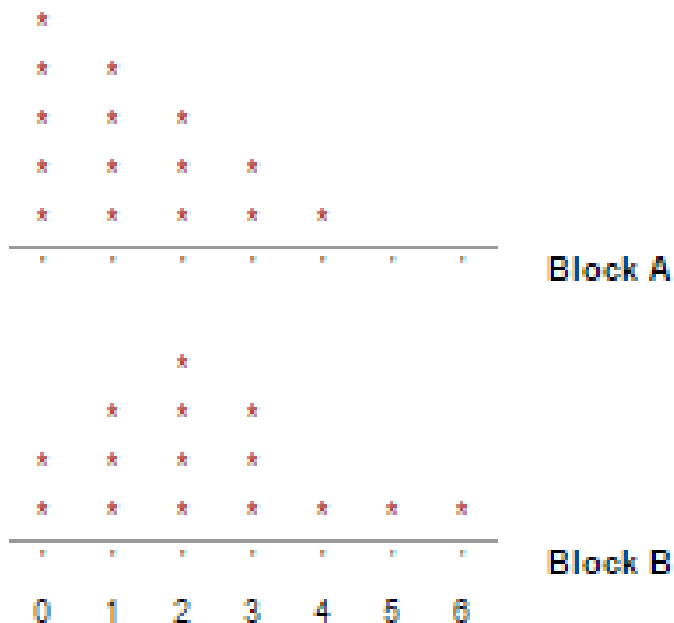
FIGURE 12.9

When dotplots are used to compare data sets, they are positioned one above the other, using the same scale of measurement, as shown on the right.

The dotplot on the right shows pet ownership in homes on two city blocks. Pet ownership is a little lower in block A. In block A, most households have zero or one pet; in block B, most households have two or more pets. In block A, pet ownership is skewed right; in block B, it is roughly bell-shaped. In block B, pet ownership ranges from 0 to 6 pets per household versus 0 to 4 pets in block A; so there is more variability in the block B distribution. There are no outliers or gaps in either data set.

Back-to-Back Stemplots

```
    Boys       Girls

       7 │ 0 │
       1 │ 1 │ 1
   1 4 6 │ 2 │ 2 6 8
   4 5 8 │ 3 │ 3 4 4 6 6 8 9
1 2 2 2 8 9 │ 4 │ 4 3 6
 3 4 7 9 │ 5 │ 4
   2 5 8 │ 6 │
     1 3 │ 7 │
```
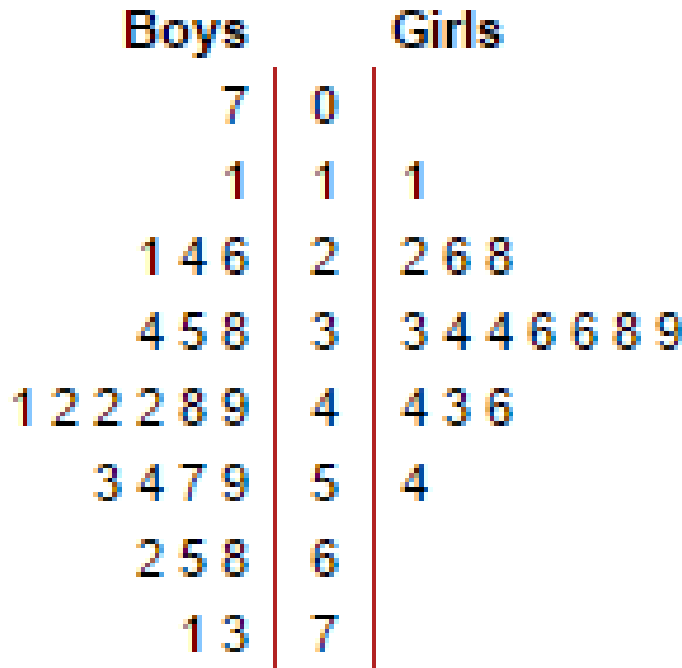
FIGURE 12.10

The back-to-back stemplots are another graphic option for comparing data from two groups. The center of a back-to-back stemplot consists of a column of stems, with a vertical line on each side. Leaves representing one data set extend from the right, and leaves representing the other data set extend from the left.

The back-to-back stemplot on the right shows the amount of cash (in dollars) carried by a random sample of teenage boys and girls. The boys carried more cash than the girls - a median of $42 for the boys versus $36 for the girls. Both distributions were roughly bell-shaped, although there was more variation among the boys. And finally, there were neither gaps nor outliers in either group.

Parallel Boxplots

With parallel boxplots (aka, side-by-side boxplots), data from two groups are displayed on the same chart, using the same measurement scale.

The boxplot to the right summarizes results from a medical study. The treatment group received an experimental
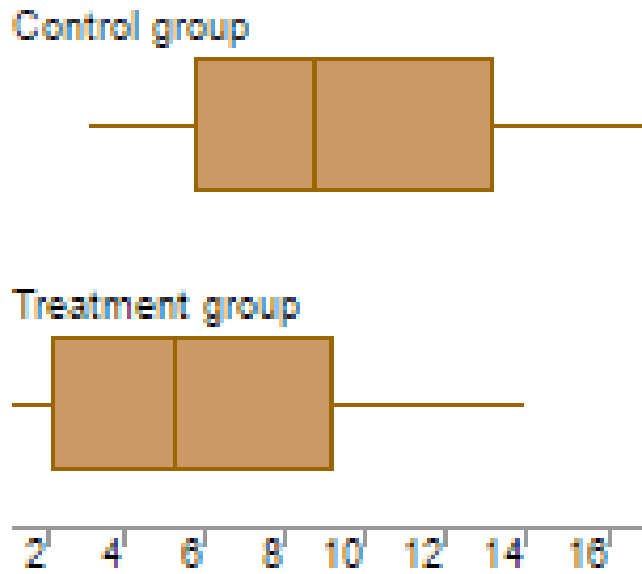
FIGURE 12.11

drug to relieve cold symptoms, and the control group received a placebo. The boxplot shows the number of days each group continued to report symptoms.

Neither boxplot reveals unusual features, such as gaps or outliers. Both plots are skewed to the right, although the skew is more prominent in the treatment group. Patient response was slightly less variable in the treatment group than in the control group. In the treatment group, cold symptoms lasted 1 to 14 days (range = 13) versus 3 to 17 days (range = 14) for the control group. The median recovery time is more telling - about 5 days for the treatment group versus about 9 days for the control group. It appears that the drug had a positive effect on patient recovery.

Double Bar Charts
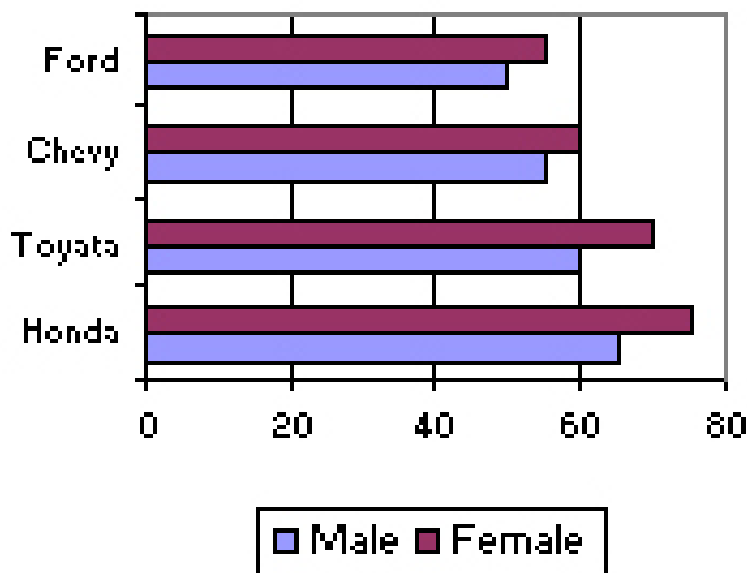
FIGURE 12.12

A double bar chart is similar to a regular bar chart , except that it provides two pieces of information for each category rather than just one. Often, the charts are color-coded with a different colored bar representing each piece of information.

To the right, a double bar chart shows customer satisfaction ratings for different cars, broken out by gender. The blue rows represent males; the red rows, females.

Both groups prefer the Japanese cars to the American cars, with Honda receiving the highest ratings and Ford receiving the lowest ratings. Moreover, both genders agree on the rank order in which the cars are rated. As a group, the men seem to be tougher raters; they gave lower ratings to each car than the women gave.

Test Your Understanding

Problem

The back-to-back stemplot on the right shows the number of books read in a year by a random sample of college and high school students. Which of the following statements are true?

I. Seven college students did not read any books. II. The college median is equal to the high school median. III. The mean is greater than the median in both groups.

(A) I only (B) II only (C) III only (D) I and II (E) II and III

Solution

The correct answer is (E). None of the college students failed to read a book during the year; the fewest read was seven. In both groups, the median is equal to 24. And the mean number of books read per year is 25.3 for high school students versus 30.4 for college students; so the mean is greater than the median in both groups.

Vocabulary:

- Dot plot
- box plot

```
   College          High school
         7 | 0 |
       3 6 6 | 1 | 0 0 3 5
     1 2 3 4 | 2 | 1 2 4 4 6
     6 8 8 9 | 3 | 1 8 9
         2 8 | 4 | 0 1
             | 5 |
             | 6 |
           3 | 7 |
```

FIGURE 12.13

- Sample
- Population
- Inference

12.8 References

1. Joe Shlabotnik. https://www.flickr.com/photos/joeshlabotnik/2207112346 .
2. alexkerhead. https://www.flickr.com/photos/alexkerhead/3127788202/ .

CHAPTER **13** # Financial Literacy II

Chapter Outline

7.13a: calculate the sales tax for a given purchase and calculate income tax for earned wages

7.13b: identify the components of a personal budget, including income; planned savings for college, retirement, and emergencies; taxes; and fixed and variable expenses, and calculate what percentage each category comprises of the total budget

7.13d: use a family budget estimator to determine the minimum household budget and average hourly wage needed for a family to meet its basic needs in the student's city or another large city nearby

7.13c: create and organize a financial assets and liabilities record and construct a net worth statement

13.1 Income Tax (7.13a)

GUIDANCE

Understanding Income tax

Most wage earners pay a federal income tax based on the amount of their earnings. If they are employed, their employer must withhold money from each pay check and send it to the federal government for their wages. This is called **withholding.** If you are self-employed, you must estimate your tax amount each quarter and send that amount to the federal government yourself. Each year on or before April 15, unless April 15 is on a weekend, workers must file a federal income tax return. When the return is filed, you may owe more taxes, or you may get a refund from the federal government depending on several things, but mainly on your amount you have already paid through withholding or quarterly payments.

A worker's pay before any amounts are deducted is called the **gross pay.** **Net pay** is the gross pay less any deductions, including withholding. Other deductions from your pay are FICA (social security) at 6.2% and Medicare at 1.45%. These are both federally mandated.

The amount of income tax a worker owes is determined by using a tax table. On a tax return Form 1040, line 43(taxable income) is the amount that determines your tax amount.

The tax table for a single person for taxable income between $36,000 and $36,250 is shown below

If Form 1040 line 43 (taxable income)-		And you are single
At least	But less than	Your tax is-
$36,000	$36,050	$4,958
$36,050	$36,100	$4,965
$36,100	$36,150	$4,973
$36,150	$36,200	$4,980
$36,200	$36,250	$4,988

FIGURE 13.1

GUIDED PRACTICE

PRACTICE

To determine your tax using a tax table, you must locate the line that includes your taxable income. The amount of tax you owe will be in the right column. There are tax tables for single persons, married filing jointly, and married filing separately.

Using the table above, if you were a single person whose taxable income is $36,025, your tax amount would be $4,958.

After totaling your withholding you have already paid to the federal government, you will know if you owe more or if you will get a refund. Just compare you withheld amount to the tax amount on the appropriate line of the table.

FIGURE 13.2

Example: Mr. James, a single person, has a gross monthly pay of $3468. His withholding each month is $329.90.
In April when he filed his taxes, his taxable income was $32,217.

- Using the tax table below, what is his tax amount?

If Form 1040 line 43 (taxable income)-		You are single	Married filing jointly	Married filing separately
At least	But less than	Your tax is-	Your tax is-	Your tax is-
$32,000	$32,050	$4,358	$3,911	$4,358
$32,050	$32,100	$4,365	$3,919	$4,365
$32,100	$32,150	$4,373	$3,926	$4,373
$32,150	$32,200	$4,380	$3,934	$4,380
$32,200	$32,250	$4,388	$3,941	$4,388

He is single, so use the third column. His taxable income was $32,217. Use the last line.

His tax is $4,388.

- Does Mr. James owe additional tax or does he get a refund?

  He has had $329.90 withheld each month. He has paid a total withholding of 12 × $329.90. His total withholding is $3,958.80.

  He has paid less withholding than his tax amount. $3,958.80 > $4,388. He will owe the difference between the amounts.

  $4,388 − $3,958.80 = $429.20. He will owe the federal government an additional $429.20..

FIGURE 13.3

Problem 1: Lauren's gross monthly pay is $2,818. Her federal withholding is 12% of her gross pay. How much will her employer deduct for federal withholding each month?

Her Medicare tax is 1.45% of her gross monthly income. How much will her employer deduct for Medicare each month?

Her FICA (social security) tax is 6.2% of her gross monthly income. How much will her employer deduct for FICA each month?

What is her net pay each month?

How much federal withholding will she have paid at the end of the year?

FIGURE 13.4

Video

MEDIA

Click image to the left or use the URL below.
URL: https://www.ck12.org/flx/render/embeddedobject/187722

Problem 2: After deductions, Jeremiah has a taxable income of $33,143. He has paid a total of $4,328 in federal withholding. Decide if Jeremiah owes more taxes at filing time, or will he get a refund.

Analyze: Taxable income of _____. He has already paid _____. Does he owe or will he get a refund?

Plan: Use a tax table to find the amount of tax he owes.

Solve:

If Form 1040 line 43 (taxable income)-		You are single	Married filing jointly	Married filing separately
At least	But less than	Your tax is-	Your tax is-	Your tax is-
$33,000	$33,050	$4,508	$4,061	$4,508
$33,050	$33,100	$4,515	$4,069	$4,515
$33,100	$33,150	$4,523	$4,076	$4,523
$33,150	$33,200	$4,530	$4,084	$4,530
$33,200	$33,250	$4,538	$4,091	$4,538

His tax will be _____ (Circle the amount on the table to show which column and row you used)

He has already paid _____.

Compare the tax he owes and the amount he has already paid. _____ < _____

Does he get a refund or still owe additional taxes? Explain

FIGURE 13.5

Problem 3: After deductions, LeAnn and her husband have a taxable income of $37,683. They have paid a total of $4,942 in federal withholding. They file married jointly. Decide if LeAnn and her husband owe additional income tax, or will they get a refund.

Analyze: Taxable income of _____.   They has already paid _____.   Do they owe or will they get a refund?

Plan: Use a tax table to find the amount of tax they owe.

Solve:

If Form 1040 line 43 (taxable income)-		You are single	Married filing jointly	Married filing separately
At least	But less than	Your tax is-	Your tax is-	Your tax is-
$37,500	$37,550	$5,310	$4,736	$5,310
$37,550	$37,600	$5,323	$4,744	$5,323
$37,600	$37,650	$5,335	$4,751	$5,335
$37,650	$37,700	$5,348	$4,759	$5,348
$37,700	$37,750	$5,360	$4,766	$5,360

Their tax will be _____ (Circle the amount on the table to show which column and row you used)

They have already paid _____.

Compare the tax they owe and the amount they have already paid.   _____ < _____

Do they get a refund or still owe additional taxes? Explain

FIGURE 13.6

13.2 **Personal Budget (7.13b)**

GUIDANCE

A personal **budget** is a plan for the amount of money you earn and the amounts of money you spend. Every family should have their own family budget. A budget is a plan to help you reach your financial goals.

Income and **expenses** are the two components of a budget. **Income** is the amount of money earned. **Expenses** are the amounts of money you spend. Expenses can include savings for college or retirement, living expenses, entertainment, and taxes. The goal of a budget is help you spend no more than you earn, and actually save some money if possible.

FIGURE 13.7

GUIDED PRACTICE

PRACTICE

MEDIA

Click image to the left or use the URL below.
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Understanding Family Budgets and the Percent Allocated for Each Expense

One way to represent a family budget is to use a circle graph. The circle represents the amount of income. The circle is divided into sectors representing each expense category. These sectors can be labeled with the amount of money allocated for that category or they can be labeled with a percent of the total income that is allocated for that category.

For most families, the largest sector will be their home expense. This can be a lease, rent, or mortgage payment. The other sectors can vary but will usually include food, medical, entertainment, transportation, utilities, savings, clothing, college fund, life insurance, emergency fund, and retirement savings.

Example: The circle graph below shows the Wang's family budget.
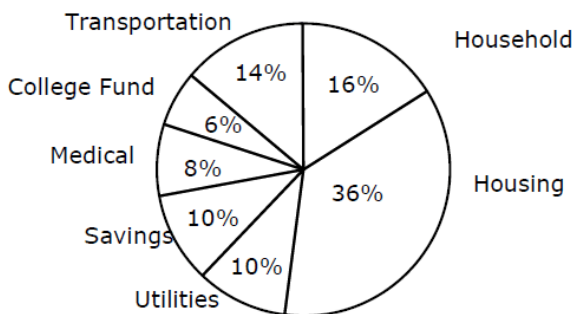
FIGURE 13.8

- If the Wang's monthly income is $5,000, how much money have they budgeted for medical expenses?

    The budget circle graph shows they budget 8% of their income for medical expenses.
    To find 8% of $5000, we will multiply 5000 by 0.08. $5000 \times 0.08 = 400$

FIGURE 13.9

Example: Lorraine is saving for clothes for college. She is making a 4-week budget. The table below shows her total income and the various expenses she anticipates she will incur during these 4 weeks, including saving for her clothes.

Item	Income (+)	Expense (-)	Amount Available
Earnings ($85 per week for part time job)			
College account			
Saving for Clothes			
Entertainment			
Car expense			

FIGURE 13.10

Video

Her planned expenses for the 4 weeks:
  Entertainment : $40   Car expense: $60
  College Savings Account: $75   Savings for Clothes: $75

Enter the expenses in the appropriate column and calculate the amount available at the end of the 4 weeks.

Item	Income (+)	Expense (-)	Amount Available
Earnings ($85 per week for part time job)	$340		$340
College Savings Account		$75	$265
Saving for Clothes		$75	$190
Entertainment		$40	$150
Car expense		$60	$90

She has an available amount of $90. She should put the additional amount into the College Savings Account or the Saving for Clothes Account.

If Lorraine was ill and missed half a week's work, her income would decrease by $42.50. She would still be able to cover her expenses.

Income is called an **asset** and expenses are called **debits**. A person must make sure that the assets are more than the debits.

There are two types of expenses or debits. One is called fixed expense. **Fixed expenses** are expenses that occur regularly and do not change from month to month. Car loan payments and mortgage payments are two examples of fixed expenses. The other type of expense is variable expense. **Variable expenses** occur regularly but can change from week to week or month to month. Electricity bills and grocery costs are examples of two variable expenses. Variable expenses are necessary expenses but you can have some control over their amounts. You can adjust your living style to decrease them if necessary. Entertainment is one expense that you can adjust if money is short.

FIGURE 13.11

Problem 1: The Thompson family has a net monthly income of $5,600. The table below shows the categories of expenses and the amount budgeted for each category.

The Thompson Family Budget

Expense Item	Amount	Percent
Mortgage and Insurance	$1,800	
Entertainment	$200	3.6%
Clothes	$300	
Utilities	$600	
Car Payment and Insurance	$800	
Household Expenses (food and supplies)	$825	14.7%
Savings	$300	
Cell Phone	$125	2.2%
Medical/Dental Expenses	$300	
Miscellaneous	$350	6.3%
Total		

Determine the percent (to the nearest tenth of a percent) budgeted for each expense that is missing in the table. Show your work.

Do the percents total to 100%? If not, can you explain why?

Problem 2: The Smith family has a net monthly income of $4,200. They budget $1,680 for their home mortgage, insurance, and taxes. What percent of their monthly income do they budget for this category? Show your work.

Problem 3: The Garza family has planned a monthly budget. The table below shows the expense categories and the percent of their monthly income they have budgeted for each item.

The Garza Family Budget

Expense Item	Percent
Mortgage and Insurance	32%
Entertainment	4%
Clothes	5%
Utilities	12%
Car Payment and Insurance	18%
Household Expenses (food and supplies)	15%
Savings	8%
Cell Phone/Internet Service	6%

FIGURE 13.13

The circle graph below represents the Garza family budget. Label each sector with the appropriate category based on the information in the table.

Garza Family Budget
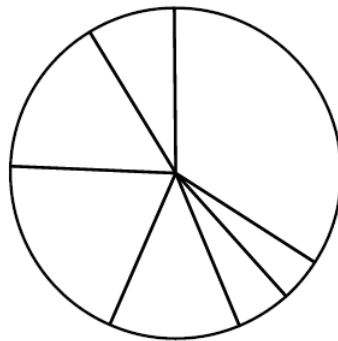
- If the Garza family's net monthly income is $5,800, what amount did they budget for mortgage and insurance expenses? Show your work.

- Are mortgage and insurance expenses a fixed expense or a variable expense? Explain.

- How much would they spend on this category in a year?

FIGURE 13.14

13.3 Family Budget (7.13d)

GUIDANCE

An expense is money being spent, and income is money being earned. To be financially stable, your income must be more than your expenses. Some expenses are fixed expenses. This type of expense remains the same every month. A fixed expense is a car payment, house payment, or rent. Other expenses are variable expenses. Food, transportation, and entertainment are examples of variable expenses.

A family budget is a financial plan based on the amount of money the family needs to live. The amount of expenses must not be more than the amount of income for the family. Things that affect a family budget are the following: where the family lives, number of children in the family, number of wage earners in the family, family health, employment benefits, etc.

FIGURE 13.15

GUIDED PRACTICE

Example: Mr. Smith estimates his monthly expenses to be the following. Determine the total expenses.

Expense	Amount		Expense	Amount
Food	$425		Entertainment	$200
House	$1055		Utilities	$500
Transportation	$650		Clothing	$300
Medical	$400		Savings	$400

The total expenses will be the sum of all the expenses. This total is $3930

FIGURE 13.16

PRACTICE

What does Mr. Smith need to earn per hour if he works 40 hours per week to meet his expenses?

If he works 40 hours a week for 4 weeks, that is 160 hours. Divide 3930 by 160. The quotient is $24.56. This will just meet his expenses. This does not include deductions from his pay for income tax, social security, or Medicare. These could easily total about $500 per month. He would probably need to search for a job that pays about $30 an hour.

The website www.familybudgets.org will give you a budget estimator that will help you decide what type of earning you need to meet your expenses. It allows you to input the area of Texas where you live. It will ask you if you pay for your own health insurance, how many adults live in the household, and how many children live in the household. It will also ask you if you are saving money, what you are saving money for. It will then give you a budget breakdown into categories and how much money a month you will need to meet expenses.

FIGURE 13.17

Problem 1: Determine the hourly wage needed to meet a budget of the following weekly or monthly amounts. Assume you work 40 hours per week.

- $890 per week

- $4200 per month

- $1200 per week

- $3850 per month

FIGURE 13.18

Problem 2:

- If you can't find a job that will pay the amount you need to meet expenses, what can you do to lower the amount of expenses?

- If your job doesn't pay enough to meet your expenses, is it a good idea to charge your groceries or car payment to a credit card each month? Explain

FIGURE 13.19

3. Mr. Willis and his 2 children live in Cleburne, Texas. He estimates his monthly living expenses to be $3,100. His payroll and income taxes are $318. What is a necessary monthly wage to meet his expenses each month including the payroll and income taxes? Show your work.

4. Ms. Bailey has a gross monthly wage of $4,002. Her social security deduction is 6.2% from her wages. How much will be deducted each month for social security? Show your work.

Her Medicare deduction is 1.45%. How much is deducted for Medicare?

Her income tax deduction is $750 per month.

If she estimates her expenses at $3,200 a month, does her net pay meet her expenses? Explain.

FIGURE 13.20

13.4 Net Worth-Assets and Liabilities (7.13c)

GUIDANCE

A person's net worth is a picture of a person's financial position at a given moment. A person's net worth is the difference between the person's assets and the person's liabilities. This can be written as an equation as $N = A - L$.

An asset is something of value the person owns. It can be items such as a house, a car, investments, or bank accounts. Liabilities are things that the person owes for. Liabilities are things such as a mortgage, a car, loans, and credit card bills.

A car can be both an asset and a liability. If the car is paid for it is an asset. The amount that is owed on the car is a liability. The more you pay on the car the more the value of the car moves to the asset side and the liability side decreases. The same is true for a home. If you own a home valued at $80,000 on the market, but your mortgage on the home is $30,000, then the asset part of the home is $80,000 and the liability part of the home is $30,000. The net worth of the home is $80,000 - $30,000 = $50,000.

In the world today, the value of an item such as a home, boat, or car, can vary as the market for these items vary. If you buy and pay for a new $25,000 car, it immediately decreases in value. You would need a "book value" of the car, which shows the value of the car at the present time, to know the value of it as an asset. It will NOT be $25,000.

 A person needs to know the value of his assets and his liabilities to be able to create a net worth statement.

FIGURE 13.21

GUIDED PRACTICE

Example: Sheila pays cash for a new computer with a value of $900. She buys new furniture with her credit card for a total of $1500. Identify these items as an asset or a liability.

The computer is an asset because it is paid for.
The furniture is a liability because it was paid for with a credit card which has to be paid for.

FIGURE 13.22

Did Sheila paying cash for her computer affect her net worth?
No, she gained the computer as an asset, but she decreased her cash amount by the value of the computer. Her net worth was not affected by paying cash for her computer.

Did Sheila using her credit card to pay for her furniture affect her net worth?
Yes, the cost of the furniture became a liability and thus reduced her net worth.

FIGURE 13.23

Example: Stanley has stocks worth $35,000. His home is valued at $250,000. His car is valued at $28,000. He has $11,000 in a savings account. He has a mortgage for $220,000 on his home, and he owes $18,000 on his car. He also has student loans for a total of $15,000. What is Stanley's net worth?

Stanley's assets: $35,000; $250,000; $28,000; $11,000   Total assets: $324,000

Stanley's liabilities: $220,000; $18,000; $15,000   Total liabilities: $253,000

Stanley's net worth is total assets minus total liabilities which is $324,000 − $253,000 = $71,000

PRACTICE

Problem 1: Identify the following as an asset (A) or a liability (L).

_____Savings account
_____Certificate of Deposit (CD)
_____Car payment
_____Home mortgage
_____Apartment rent
_____School loan
_____Stocks
_____Retirement account
_____Credit card balance
_____Rare coin collection

FIGURE 13.24

Problem 2:

- List 5 things that can be considered an asset.

- List 5 things that can be considered a liability.

Problem 3: Bradley owns a home worth $210,000. His home mortgage is $185,000. He owes $6,000 on a car valued at $28,000. He has a bank loan with a balance of $12,000. He owns technology equipment worth $6,000. He has a credit card balance of $3,270. He has $5,400 in a checking account and $12,000 in a savings account. Place the items in the correct column of the Net Worth Table below. Then calculate Bradley's net worth.

Bradley's Net Worth

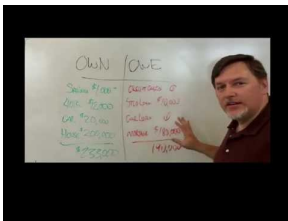| Item	Asset	Liability
Home		
Home Mortgage		
Car		
Car Debt		
Bank Loan		
Technology Equipment		
Credit Card Balance		
Checking Account		
Savings Account		
Total		
Net Worth		

Problem 4:
- Explain the difference between an asset and a liability.

- Explain net worth.

- If a person owes 8 monthly payments of $250 on a new roof, how would that be listed on a net worth statement?

- Bobby wants to use her credit card to buy some new appliances for a total of $3000. She knows the value of the appliances will decrease 15% after she purchases them. How will this purchase affect her net worth?

Problem 5: Give an example of a person's assets and liabilities if he has 4 assets and 6 liabilities and a net worth of $4,500.

FIGURE 13.27

MEDIA

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Vocabulary:

- Income Tax
- Assets
- Liability
- Financial Statements
- Budget
- Net Worth Statements