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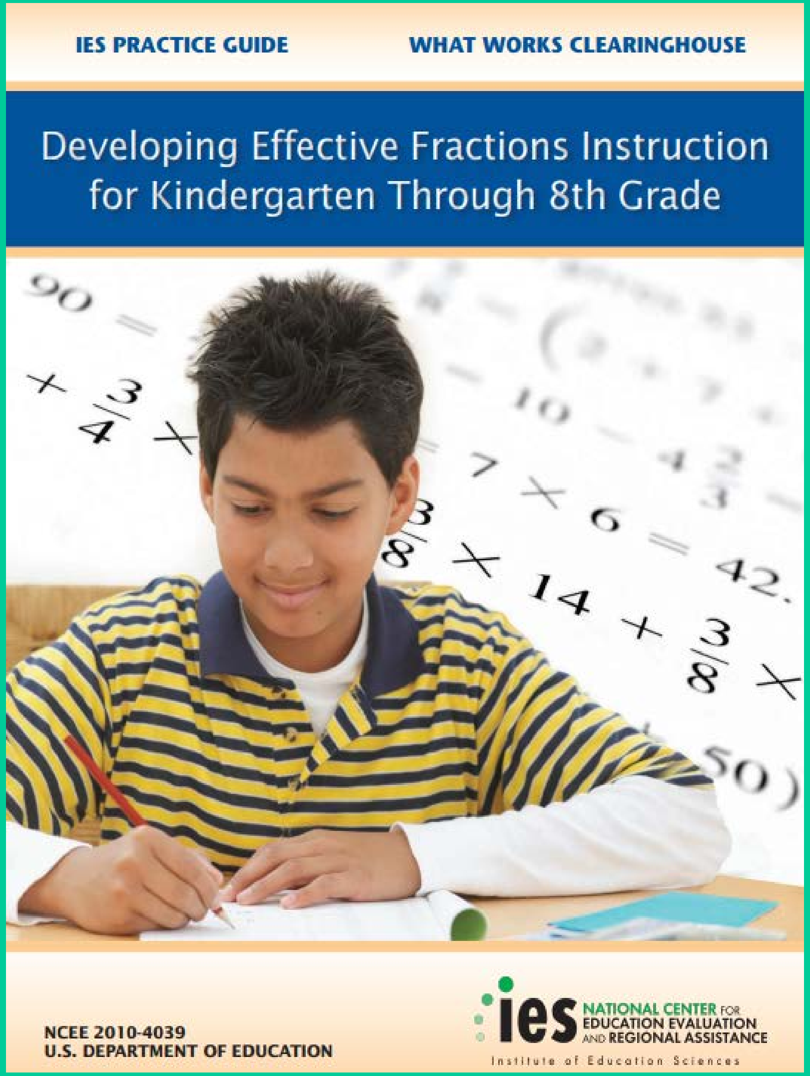


Developing Effective Fractions Instruction for Kindergarten through 5th Grade

February 24, 2021

Recommendation 1

Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.

- Use equal-sharing activities to introduce the concept of fractions. These sharing activities should involve dividing sets of objects as well as single whole objects.

- Extend equal-sharing activities to develop students' understanding of ordering and equivalence of fractions.

- Build on students' informal understanding to develop more advanced understanding of proportional reasoning concepts.

- Include activities that involve similar proportions, and transfer into activities that involve ordering different proportions.

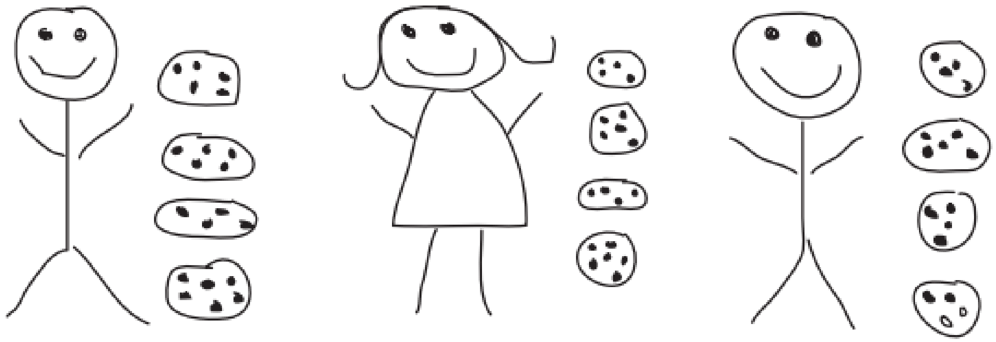
We can begin with activities that involve dividing or separating groups of objects equally.

Problem

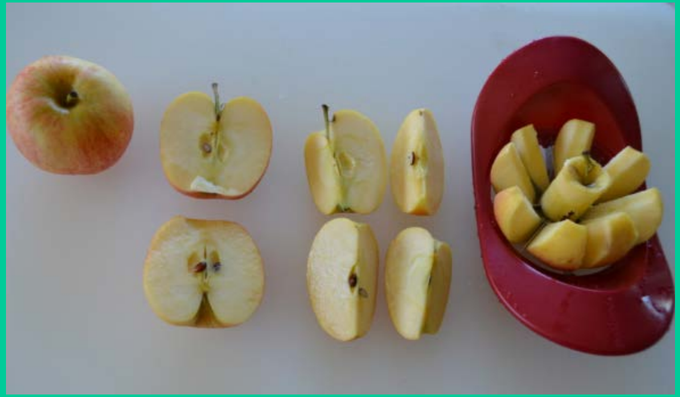
Three children want to share 12 cookies so that each child receives the same number of cookies. How many cookies should each child get?

Examples of Solution Strategies

Students can solve this problem by drawing three figures to represent the children and then drawing cookies by each figure, giving one cookie to the first child, one to the second, and one to the third, continuing until they have distributed 12 cookies to the three children, and then counting the number of cookies distributed to each child. Other students may solve the problem by simply dealing the cookies into three piles, as if they were dealing cards.



Then we can introduce the new challenge of unit fractions, involving partitioning of a single object among sharers so that each receives equal shares.

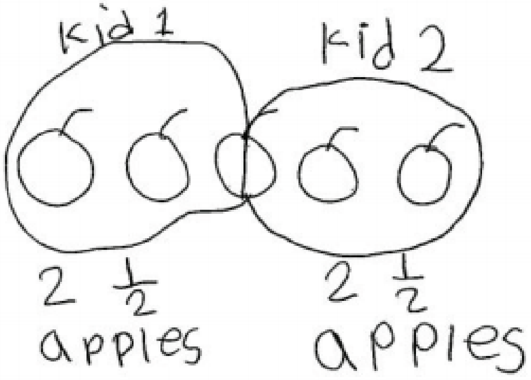


Problem

Two children want to share five apples that are the same size so that both have the same amount to eat. Draw a picture to show what each child should receive.

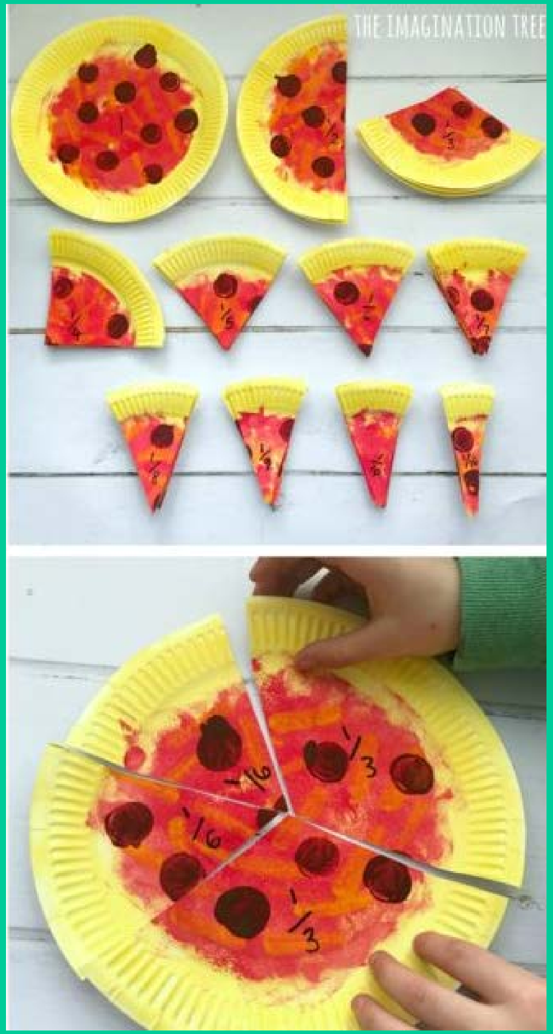
Examples of Solution Strategies

Students might solve this problem by drawing five circles to represent the five apples and two figures to represent the two children. Students then might draw lines connecting each child to two apples. Finally, they might draw a line partitioning the final apple into two approximately equal parts and draw a line from each part to the two children. Alternatively, as in the picture to the right, children might draw a large circle representing each child, two apples within each circle, and a fifth apple straddling the circles representing the two children. In yet another possibility, children might divide each apple into two parts and then connect five half apples to the representation of each figure.



These sharing activities can be used to illustrate concepts such as halves, thirds, and fourths, as well as more general concepts relevant to fractions.

Children may begin to understand that increasing the number of people among whom an object is divided results in a smaller fraction of the object for each person.



Recommendation 2

Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.

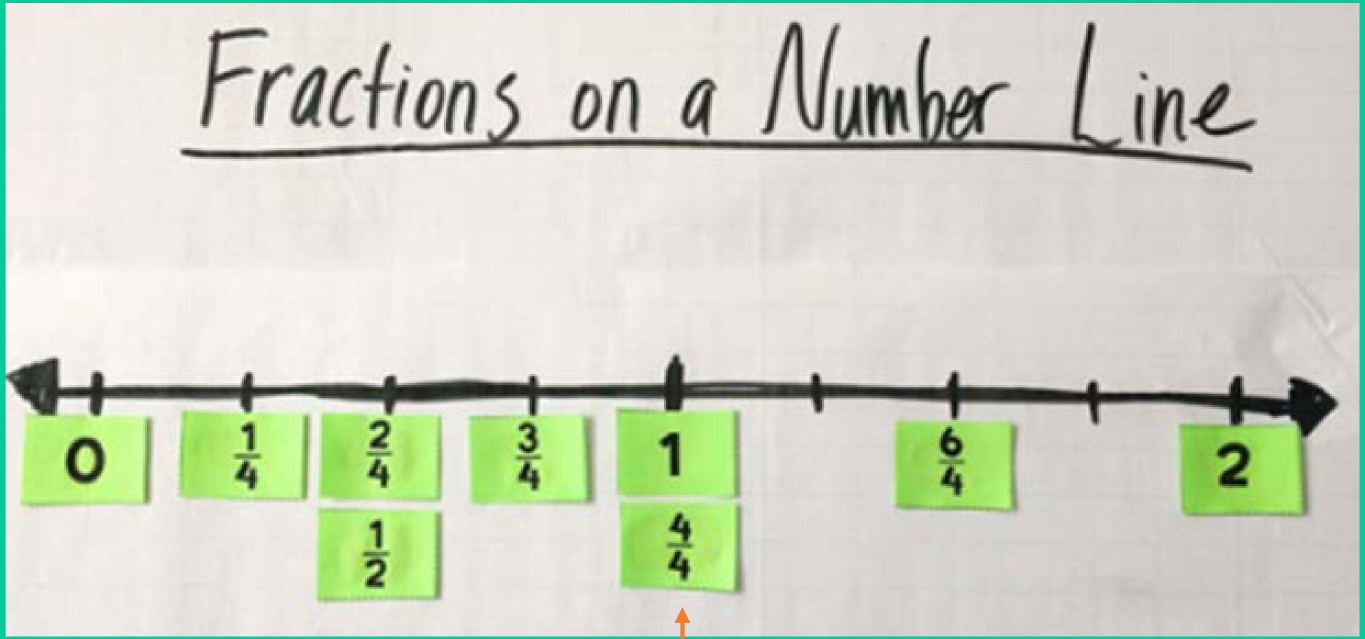
- Use measurement activities and number lines to help students understand that fractions are numbers, with all the properties that numbers share.

- Provide opportunities for students to locate and compare fractions on number lines.

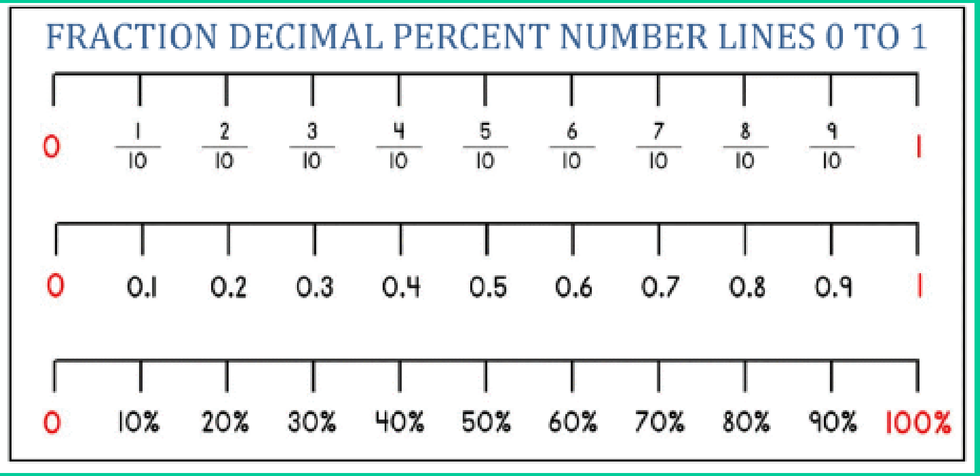
- Use number lines to improve students' understanding of fraction equivalence, fraction density (the concept that there are an infinite number of fractions between any two fractions), and negative fractions.

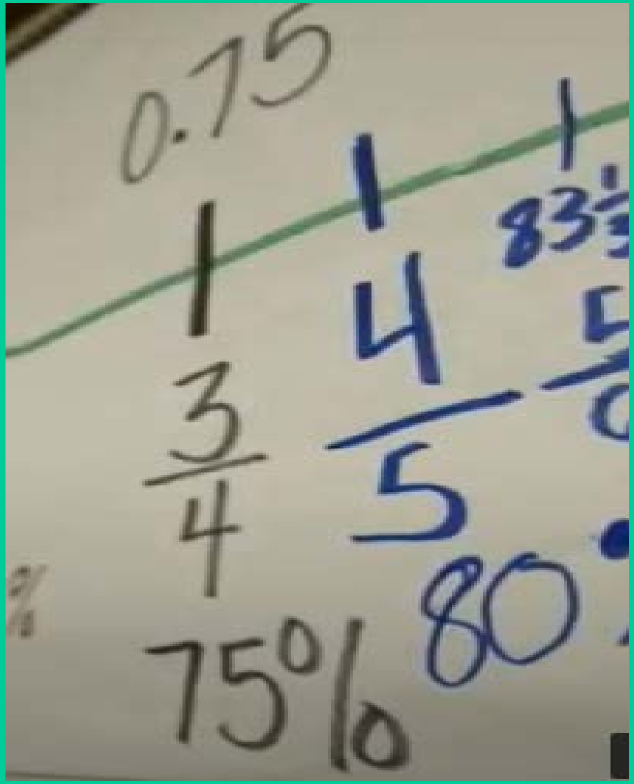
- Help students understand that fractions can be represented as common fractions, decimals, and percentages, and develop students' ability to translate among these forms.

A solid understanding of fractions as numbers allows students to both relate fractions to whole numbers and compare fractions to other fractions.



On a number line, students can represent fractions as decimals and percentages. Different sets of labels help students compare the representations and see that three-fourths, 0.75, and 75% are equivalent.





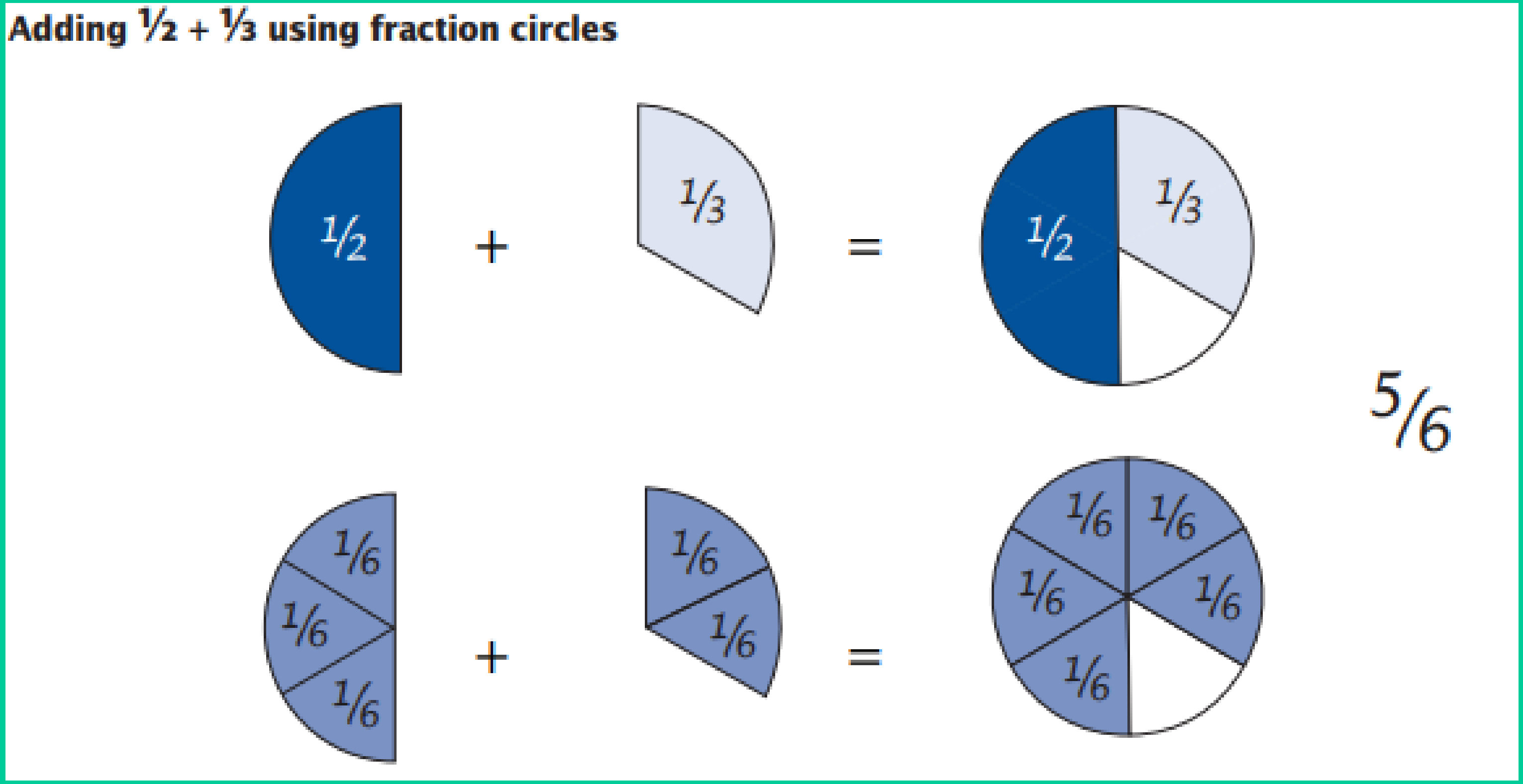
Recommendation 3

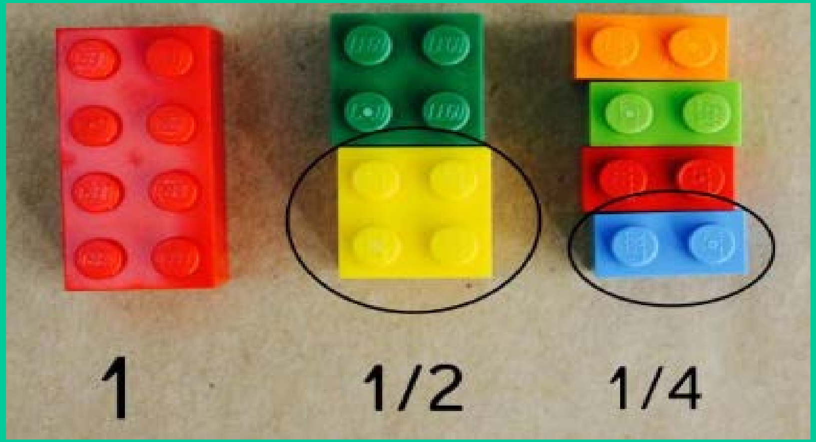
Help students understand why procedures for computations with fractions make sense.

- Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures.
- Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.
- Address common misconceptions regarding computational procedures with fractions.
- Present real-world contexts with plausible numbers for problems that involve computing with fractions.

Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures.

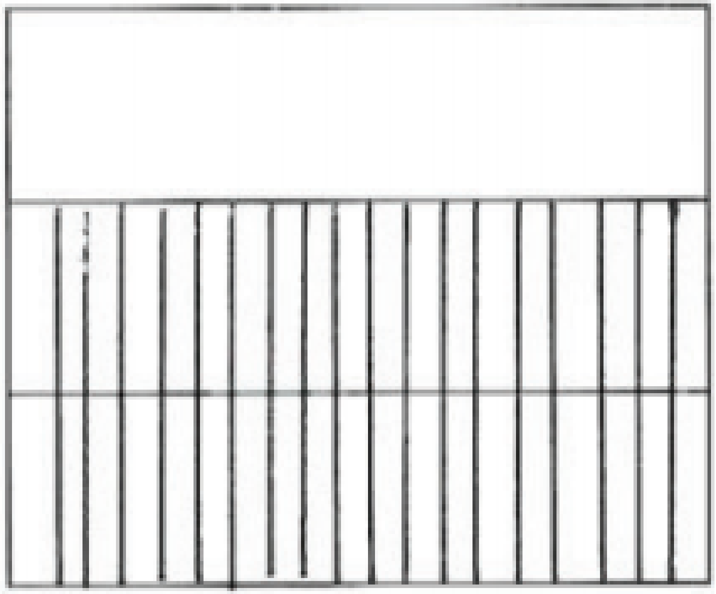
Use representations to find key concepts. Find common denominators when adding and subtracting fractions.



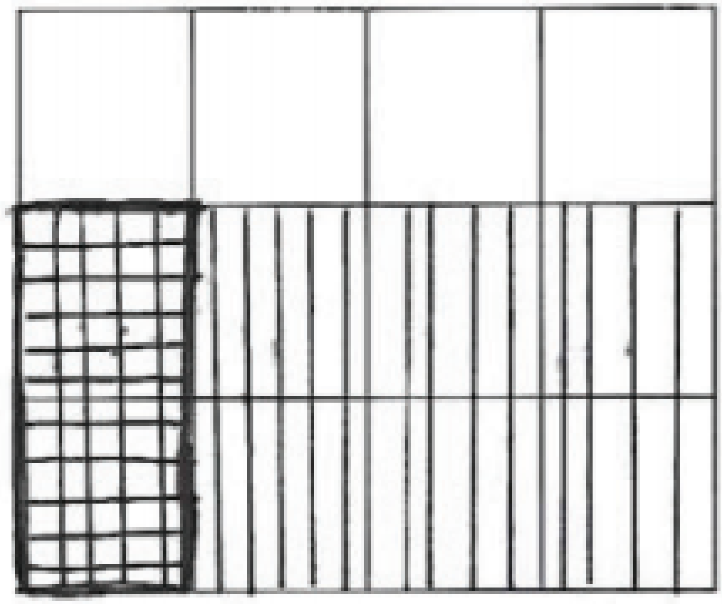


Redefine the unit when multiplying fractions.

Lori is icing a cake. She knows that 1 cup of icing will cover ⅔ of a cake. How much cake can she cover with ¼ cup of icing?



$\frac{2}{3}$ of a cake



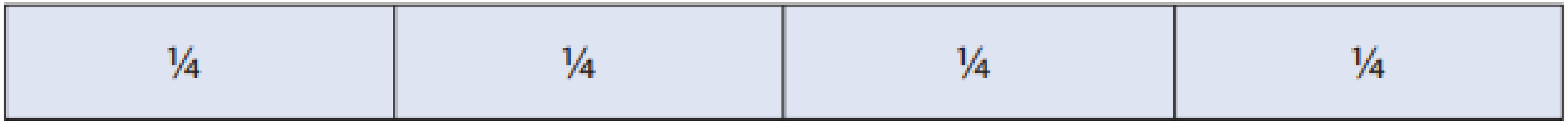
$\frac{1}{4}$ of $\frac{2}{3}$ of a cake

$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$ of a cake

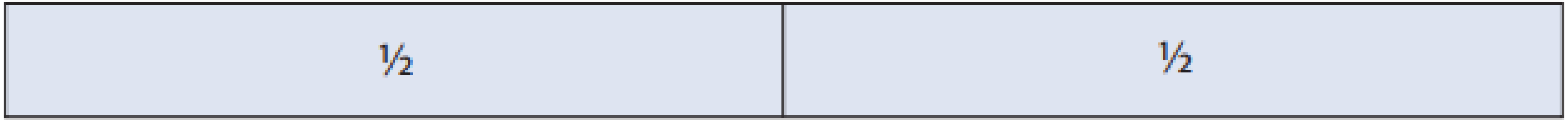
Divide a number into fractional parts.

Students use ribbons to solve $\frac{1}{2} \div \frac{1}{4}$

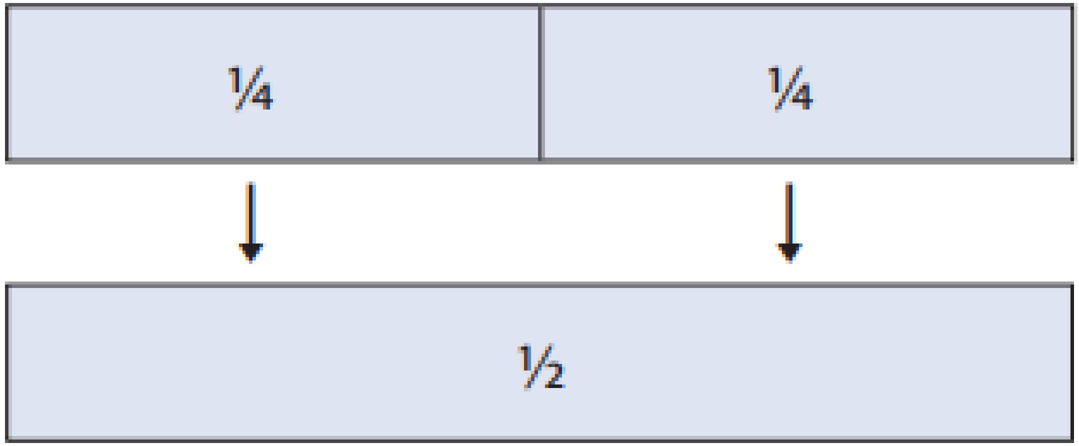
Step 1. Divide a ribbon into fourths.



Step 2. Divide a ribbon of the same length into halves.



Step 3. Find out how many fourths of a ribbon can fit into one-half of the ribbon.



Two fourths fit into one-half of the ribbon.
So, $\frac{1}{2} \div \frac{1}{4} = 2$.

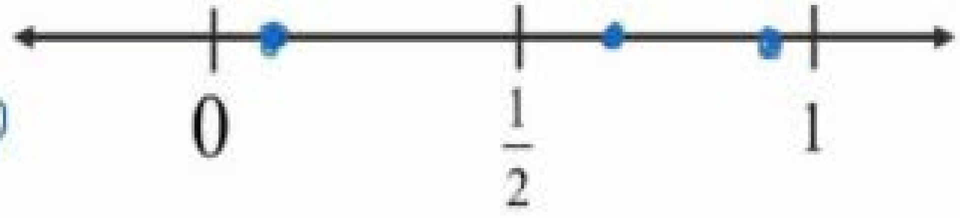
Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.

Estimating by Rounding

- Proper fractions can be rounded to 3 different numbers.

$\frac{6}{10} \approx \frac{1}{2}$

$\frac{2}{10} \approx 0$



- When rounding compare the numerator to the denominator.
  - If the numerator and denominator are far apart in value then the fraction rounds to zero.
  - If the numerator is about half of the denominator then the fraction rounds to half
  - If the numerator and denominator are close in value then the fraction rounds to 1.

$9/10 \approx$

Misconceptions regarding computational procedures with fractions.

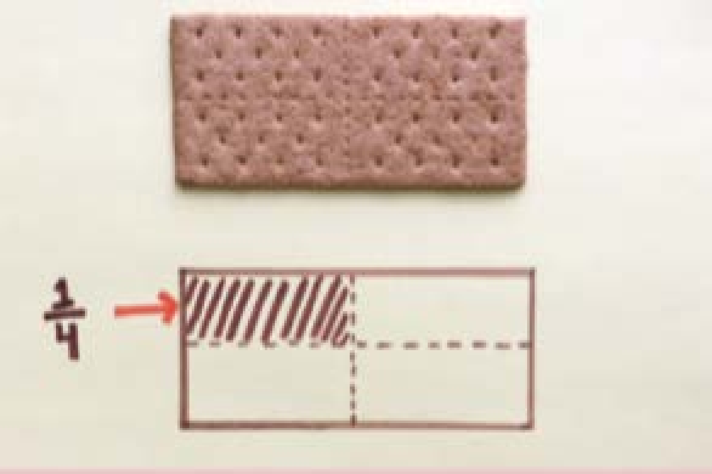
Believing that fractions' numerators and denominators can be treated as separate whole numbers.

Failing to find a common denominator when adding or subtracting fractions with unlike denominators.

Believing that only whole numbers need to be manipulated in computations with fraction greater than on

Treating the denominator the same in fraction addition and multiplications problems

Present real-world contexts with plausible numbers for problems that involve computing with fractions



Recommendation 4

Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.

- Develop students' understanding of proportional relations before teaching computational procedures that are conceptually difficult to understand (e.g., cross-multiplication).
- Build on students' developing strategies for solving ratio, rate, and proportion problems.
- Encourage students to use visual representations to solve ratio, rate, and proportion problems.
- Provide opportunities for students to use and discuss alternative strategies for solving ratio, rate, and proportion problems. (6^{th} grade and higher)

Problem encouraging specific strategies

Ratio, rate, and proportion problems can be solved using many strategies, with some problems encouraging use of particular strategies. Illustrated below are three commonly used strategies and types of problems on which each strategy is particularly advantageous.

Buildup Strategy

Sample problem. If Steve can purchase 3 baseball cards for \$2, how many baseball cards can he purchase with \$10?

Solution approach. Students can build up to the unknown quantity by starting with 3 cards for \$2, and repeatedly adding 3 more cards and \$2, thus obtaining 6 cards for \$4, 9 cards for \$6, 12 cards for \$8, and finally 15 cards for \$10.

Unit Ratio Strategy

Sample problem. Yukari bought 6 balloons for \$24. How much will it cost to buy 5 balloons?

Solution approach. Students might figure out that if 6 balloons costs \$24, then 1 balloon costs \$4. This strategy can later be generalized to one in which eliminating all common factors from the numerator and denominator of the known fraction does not result in a unit fraction (e.g., a problem such as $\frac{6}{15} = \frac{X}{10}$, in which reducing $\frac{6}{15}$ results in $\frac{2}{5}$).

Cross-Multiplication

Sample problem. Luis usually walks the 1.5 miles to his school in 25 minutes. However, today one of the streets on his usual path is being repaired, so he needs to take a 1.7-mile route. If he walks at his usual speed, how much time will it take him to get to his school?

Solution approach. This problem can be solved in two stages. First, because Luis is walking at his "usual speed," students know that $\frac{1.5}{25} = \frac{1.7}{x}$. Then, the equation may be most easily solved using cross-multiplication. Multiplying 25 and 1.7 and dividing the product by 1.5 yields the answer of $28\frac{1}{3}$ minutes, or 28 minutes and 20 seconds. It would take Luis 28 minutes and 20 seconds to reach school using the route he took today.

Encourage students to use visual representations to solve ratio, rate, and proportion problems.

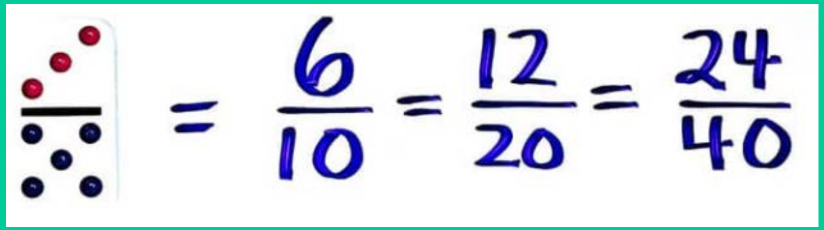
Ratio table for a proportion problem

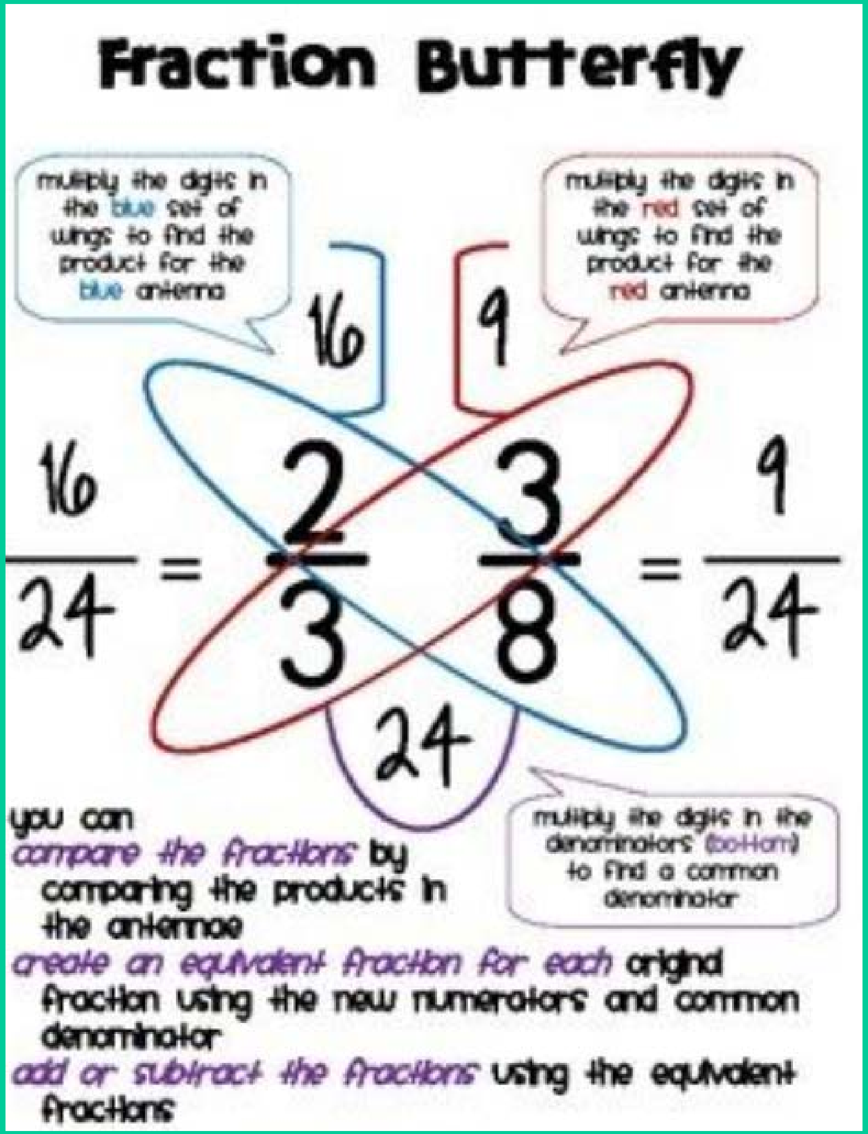
Cups of Flour	1	2	3	**4**
Number of People Served	8	16	24	**32**

Ratio table for exploring proportional relations

Cups of Flour	5	7.5	10	12.5
Number of People Served	2	3	4	5

The Progression of Fractions





https://gfletchy.com/2016/12/08/the-progression-of-fractions/

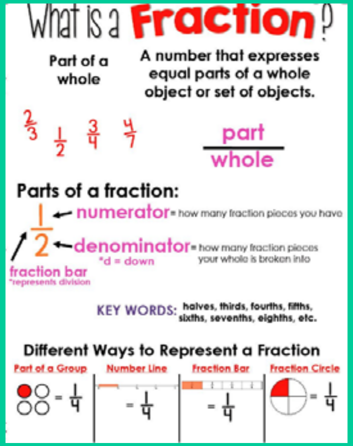
Recommendation 5

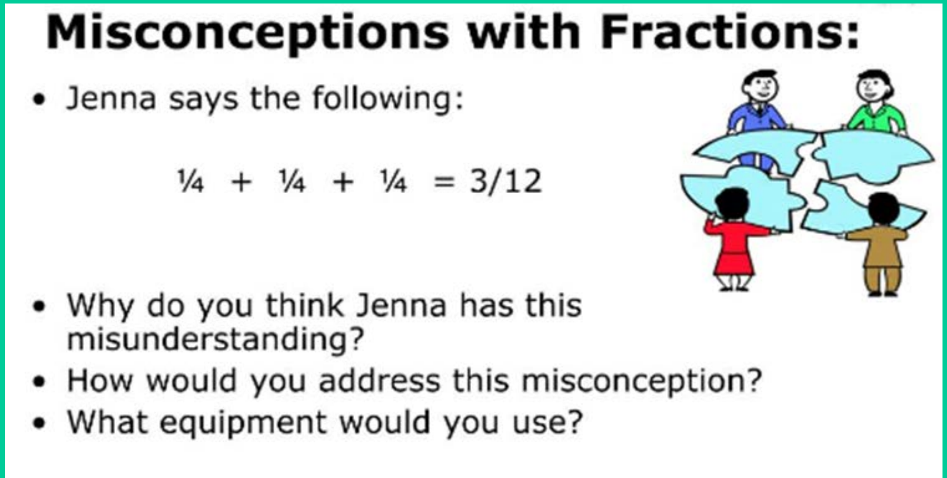
Professional development programs should place a high priority on improving teachers’ understanding of fractions and of how to teach them.

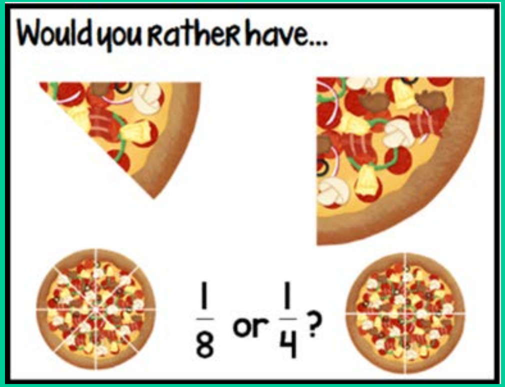
- Build teachers’ depth of understanding of fractions and computational procedures involving fractions.
- Prepare teachers to use varied pictorial and concrete representations of fractions and fraction operations.
- Develop teachers’ ability to assess students’ understandings and misunderstandings of fractions.

Misconceptions

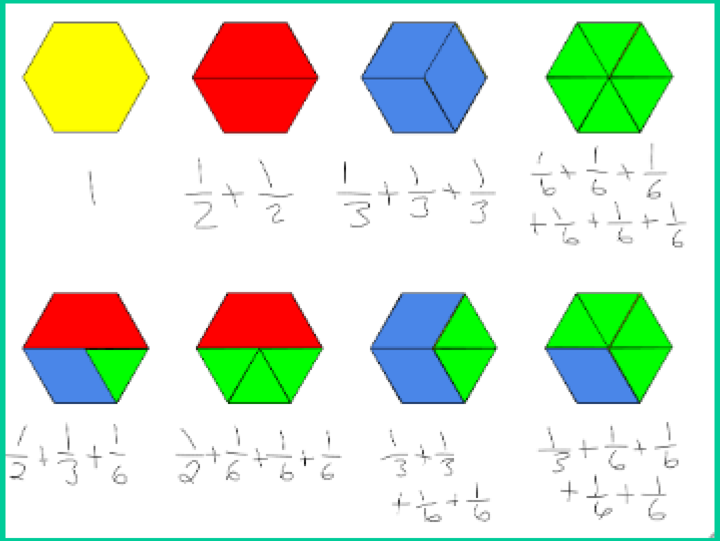
- Top Number and Bottom Number/ “Numerator and Denominator”
- The larger the denominator the larger the fraction
- Big number always goes in the bottom
- **Improper fractions** may change to “Fractions greater than one”



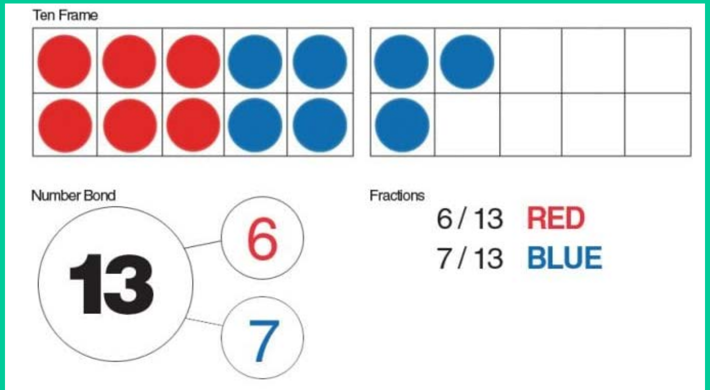




DON’T SKIP THE MANIPULATIVES Fractions are such an abstract concept that without concrete support it is nearly impossible for students to develop conceptual understanding. Manipulatives and concrete learning are for every grade level, not just the primary grades.



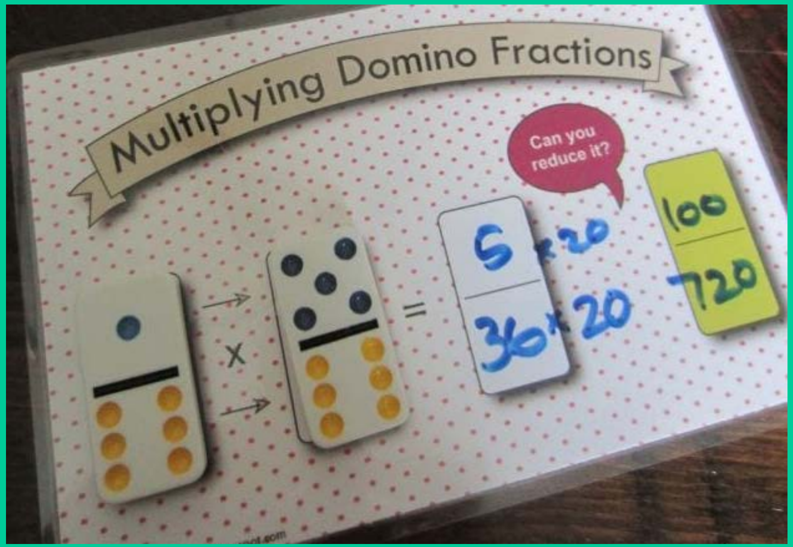
Strategies…



Shake it up with number bonds.

- All you need is a plastic cup and two-sided counters. Kids shake the cup and pour the counters on the table, then count how many of each color landed face up. For example, if 13 counters were rolled, six red and seven blue landed face up. Six and seven are both parts of thirteen (the whole). Have students write the number bond and fractions for each color.

Strategies…



Do the math with domino fractions.

- Dominoes are like ready-made fractions! Multiply (or add, subtract, or divide) them and reduce the results. Turn it into a race to see who can finish—correctly—first.

Awesome math books for fractions!

One Minute Problems
http://www.theteachertoolkit.com/index.php/tool/one-minute-problem

Quiz, Quiz, Trade- Review Math vocabulary
http://www.theteachertoolkit.com/index.php/tool/quiz-quiz-trade

Oxford Owl Resources
https://home.oxfordowl.co.uk/blog/five-easy-ways-to-explore-fractions-with-your-child/

Thank You! ☺